12 July, 2011
CERN

Constraints on a Fine-Grained AdS/CFT Correspondence

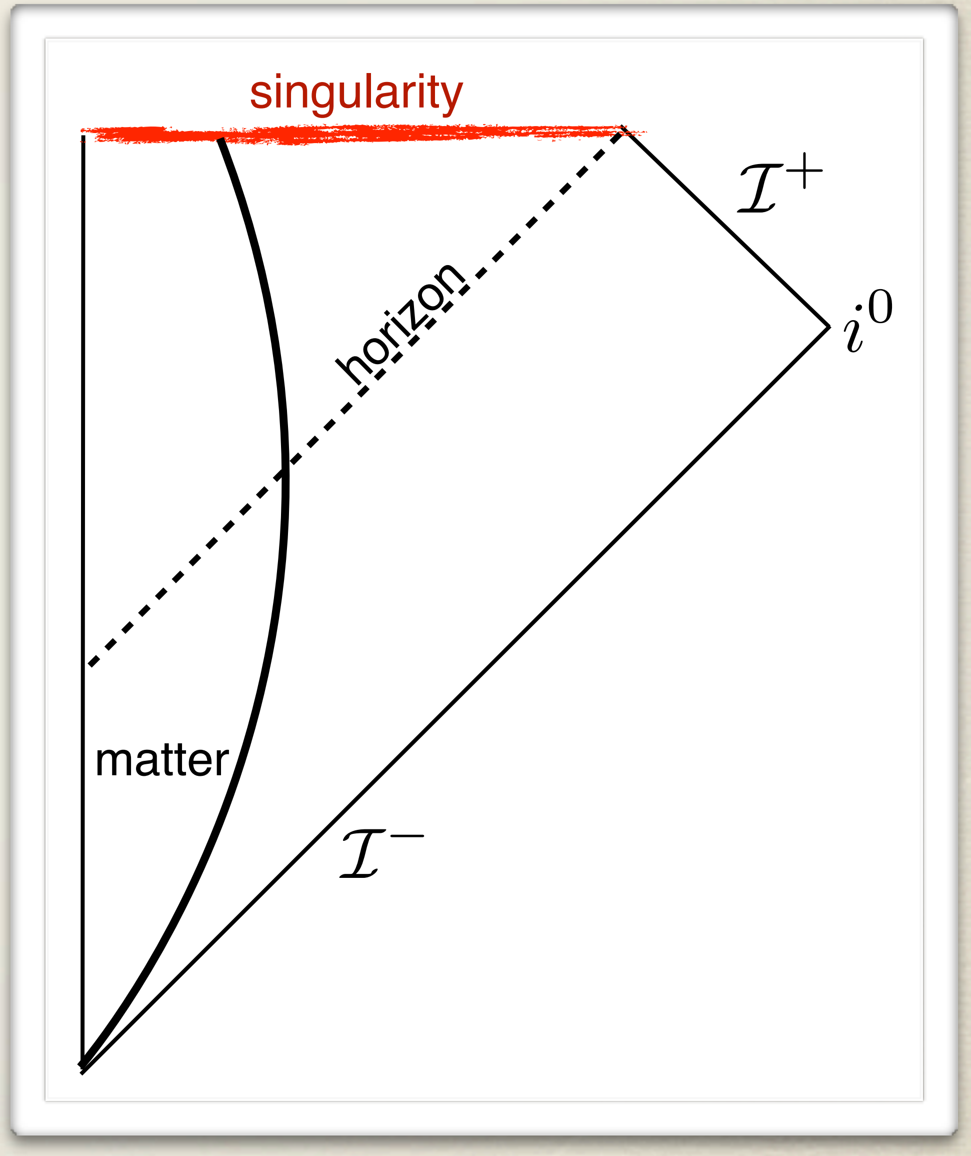
arXiv:0903.4437 (MG, Giddings, Penedones)
arXiv:0904.3544 (MG, Giddings)
arXiv:1106.3553 (MG, Giddings)

Overview

* Black hole evaporation
* The AdS/CFT correspondence
* Scattering in the large R limit
  * Necessity of wavepackets
  * Construction
  * Singularity structure
* Limits on locality
  * Can we do better?
* Understanding Bulk Unitarity
* Conclusions & Open Questions

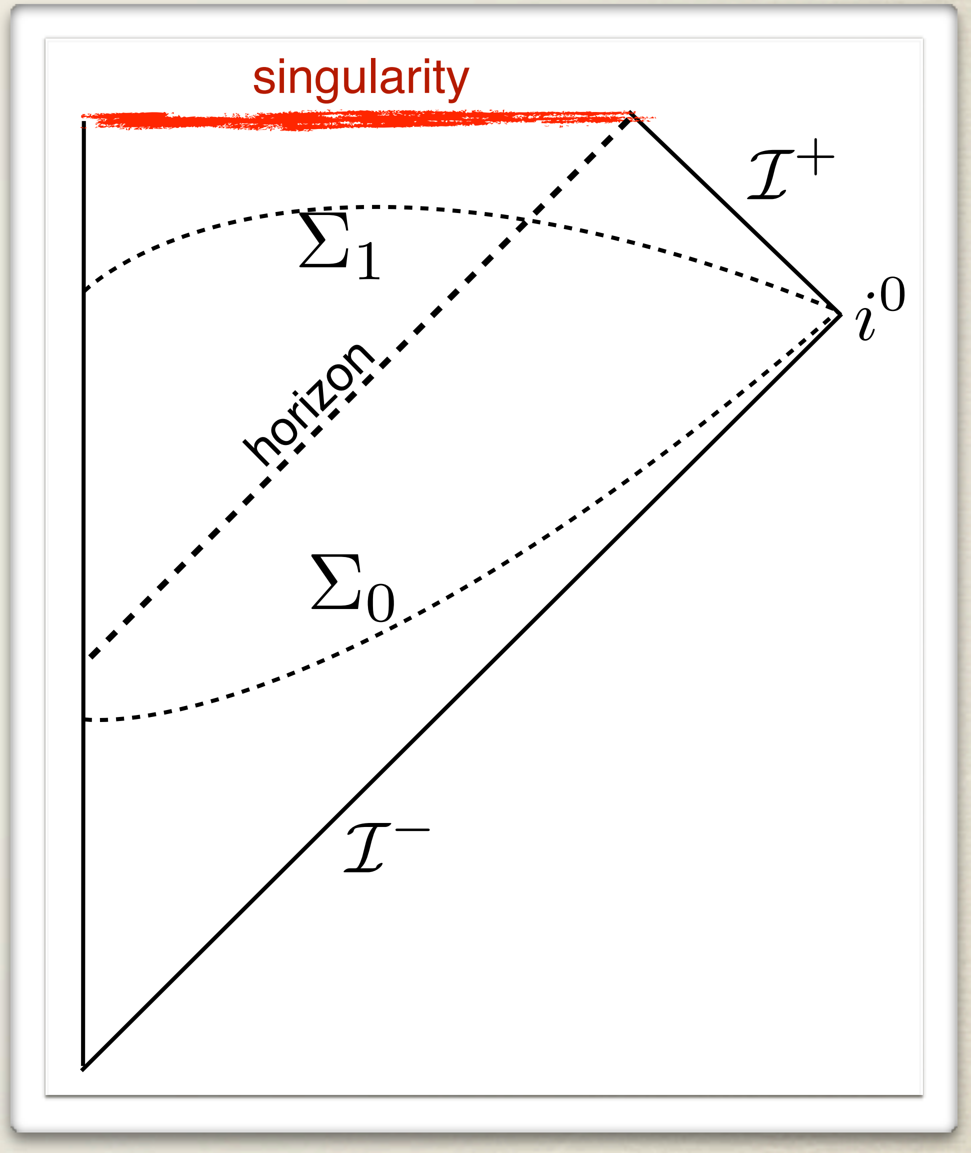
Evaporating Black Holes

- Classically, everything that falls in is lost forever.
- Hawking: Black holes aren't completely black.
- Hawking Radiation ⇒ black holes evaporate.



Evaporating Black Holes

- State on Σ_0 can be pure.
- Locality $\Rightarrow$ trace over sub-sector of Σ_1 inside horizon ➟ mixed state outside.
- Pure state evolving into mixed state violates unitarity.



Evaporating Black Holes

* 3 Possible Resolutions:
  * Non-Unitary evolution
    $\rho \mapsto \$\rho$
  * Remnants
    Black hole only evaporates to Planck scale
  * Non-locality

Non-Unitary Evolution?

$$\rho \mapsto \$\rho$$

* Information transfer requires energy.
* Information loss ⇔ energy loss.
* Virtual effects ⇒ Planck-scale energy non-conservation.
* Banks, Peskin, Susskind (1984):
  Nonunitarity ⇒ thermal bath at $T \sim M_P$

Remnants?

* Long lived Planck-scale remnant: if remnant decays and information gets out, takes

$$t_{\rm decay} = \frac{S^2}{M_P}$$

* Form black holes from arbitrarily many initial states, so arbitrarily many remnant species ⇒ arbitrarily large production cross-section.

Non-Locality?

* Physical observables must be gauge invariant.
* In gravity, this means observables must be diffeomorphism invariant.
* There are no diffeomorphism invariant local observables in gravity. (Torre 1993)

$$\delta\mathcal{O}(x) = \epsilon^{\mu}\nabla_{\mu}\mathcal{O}(x)$$

Holographic Principle

- Hints that locality should be given up.
  - BH Entropy grows with bounding area, not volume.
  - Bousso (1999): trying to access too many states in a fixed volume → black hole formation.
- Look for a non-local formulation of Quantum Gravity.
- Area law ⇒ should look for a theory in one fewer dimensions.

AdS/CFT

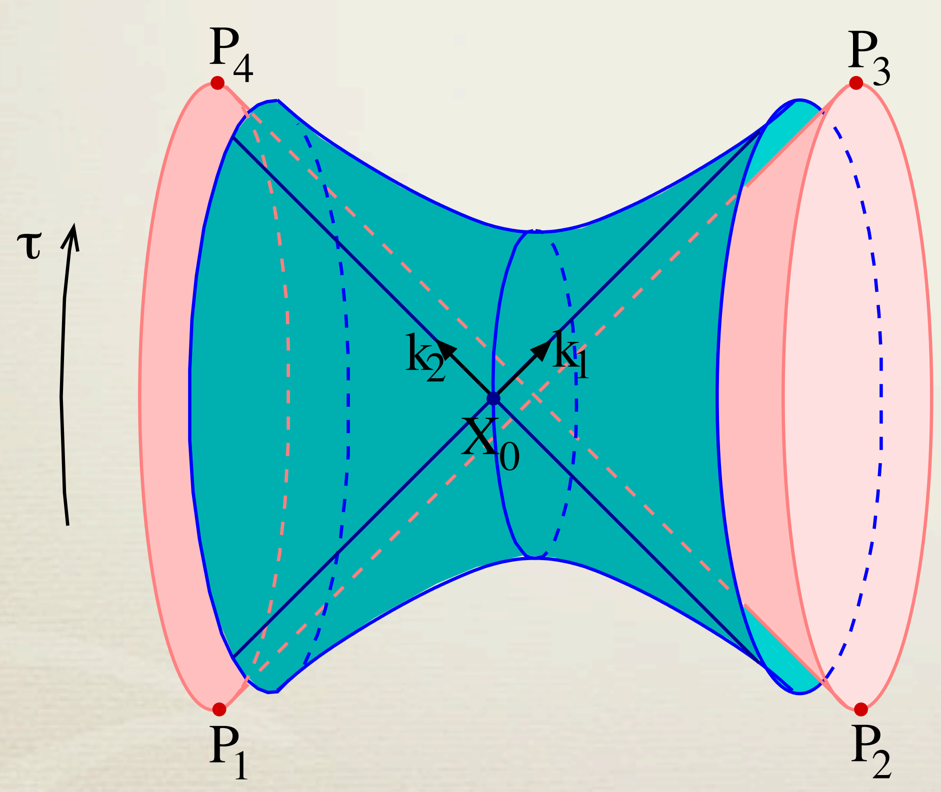
* Quantum Gravity in asymptotically Anti-de Sitter space (AdS) is conjectured to be dual to a Conformal Field Theory (CFT) living on the boundary. (Maldacena 1997)

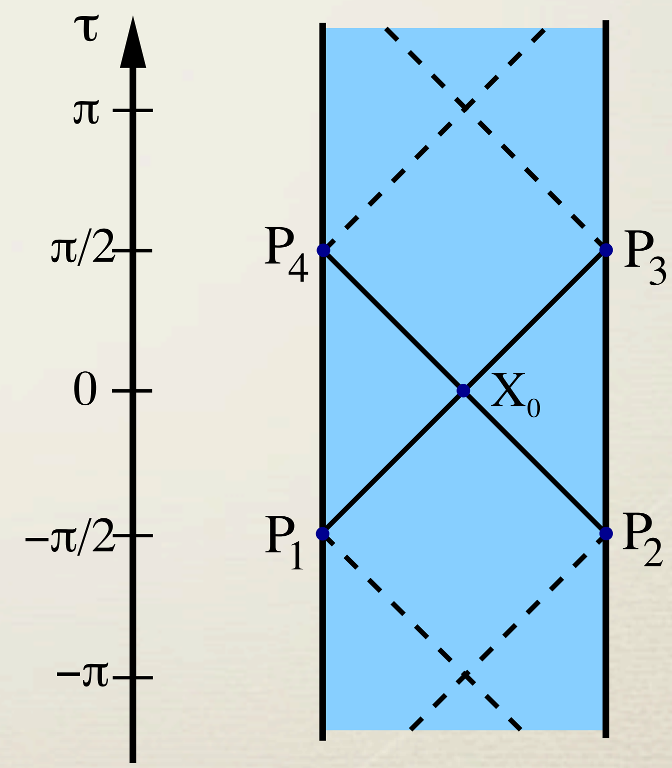
$$\left\langle \exp\left(i\int_{\partial \mathrm{AdS}} \alpha_\phi \mathcal{O}_\phi\right)\right\rangle_{\mathrm{CFT}} = Z_S\left[\alpha_\phi\right]$$

* Operator insertions in the CFT ↔ boundary conditions for fields in AdS.

AdS Geometry

- $ds^2 = \dfrac{R^2}{\cos^2\rho}\left(-d\tau^2 + d\rho^2 + \sin^2\rho\, d\Omega^2_{d-1}\right)$
- Universal cover of hyperboloid of radius R in $M_{2,d}$



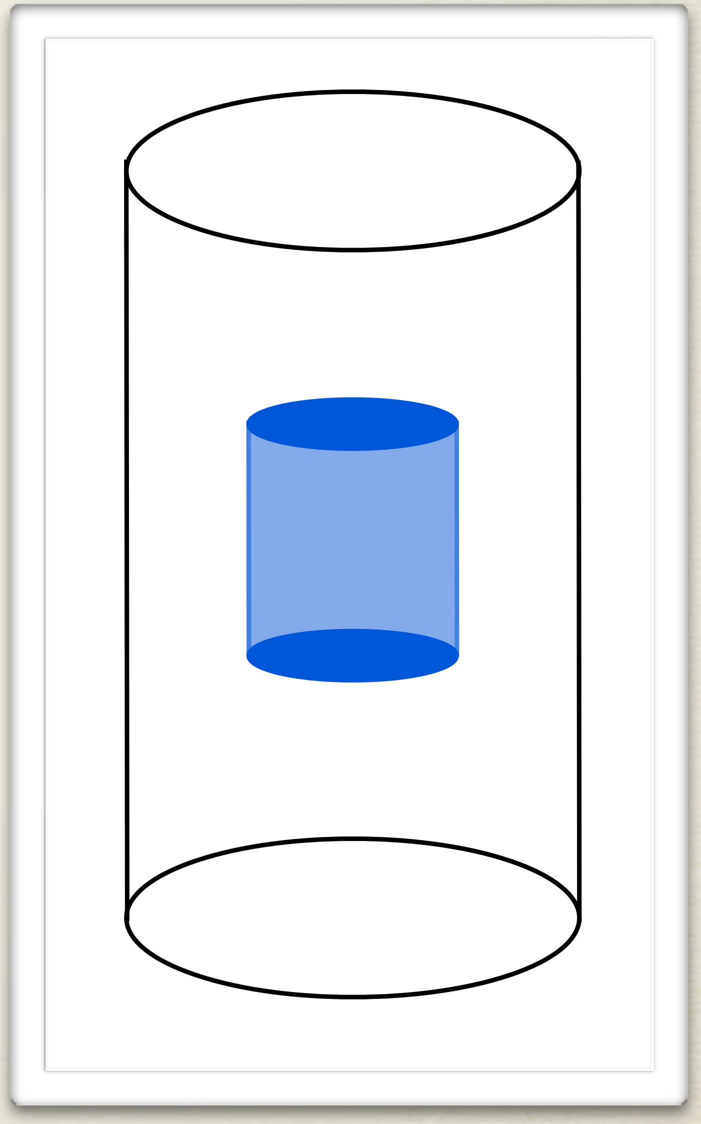


AdS/CFT Dictionary

- Gubser, Klebanov, Polyakov; Witten (1998): Boundary conditions on fields in AdS ↔ operator insertions & VEVs in dual CFT.
  - Fields in AdS have normalizable & non-normalizable modes:
  $$\phi \sim \cos^{2h_-} \rho \ \alpha(\tau, \Omega) + \cdots + \cos^{2h_+} \rho \ \beta(\tau, \Omega)$$
  - Non-normalizable mode ↔ operator insertion:
  $$\mathcal{L}_{\text{CFT}} \mapsto \mathcal{L}_{\text{CFT}} + \alpha_\phi \mathcal{O}_\phi$$
  - Normalizable mode ↔ operator expectation value:
  $$\langle \mathcal{O}_\phi \rangle = \beta_\phi$$

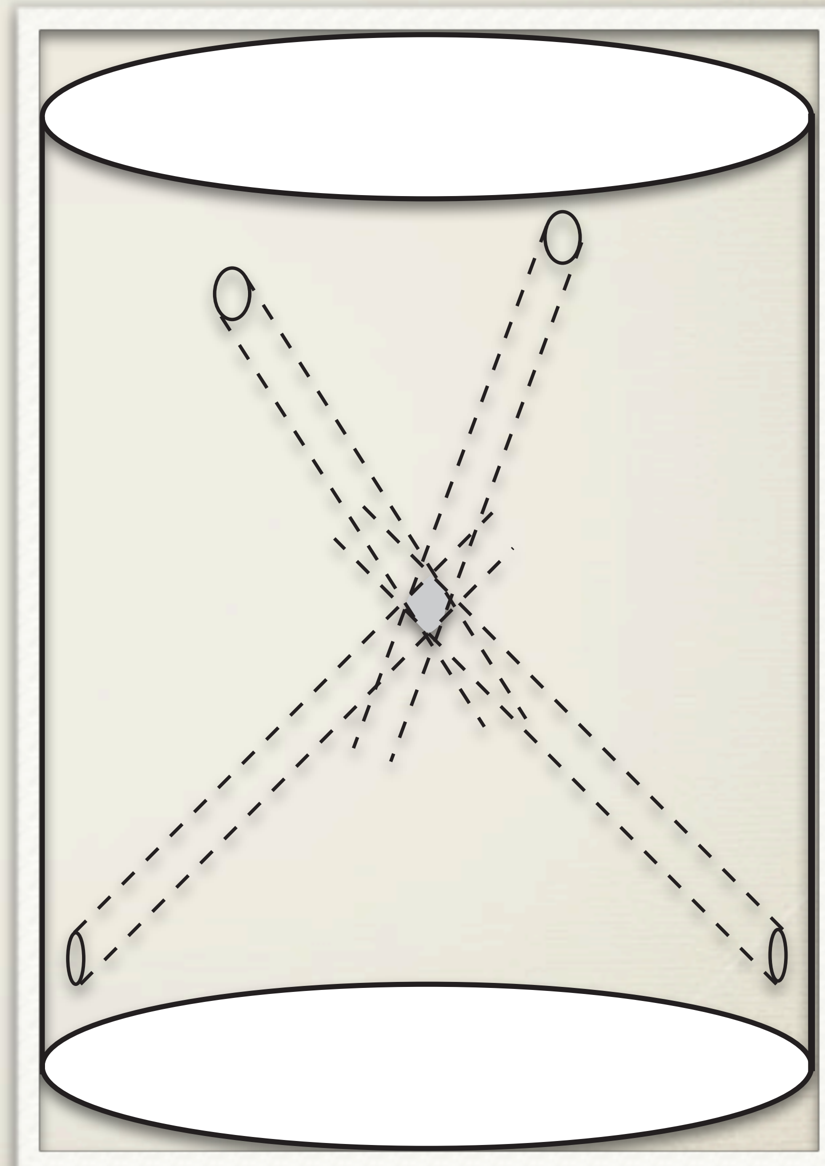
Large R Limit

- $t = R\tau \quad r = R\rho \quad R \to \infty$
- Approximately flat
  $ds^2 \to -dt^2 + dr^2 + r^2 d\Omega^2$
- Normalizable frequencies
  $\omega_{nl} \to \omega R$
- Normalizable wavefunctions
  $$\phi_{nl\vec{m}} \to \frac{\sqrt{2\omega^{d-1}}}{(\omega r)^{\frac{d}{2}-1}} J_{l+\frac{d}{2}-1}(\omega r) Y_{l\vec{m}}(\Omega)$$



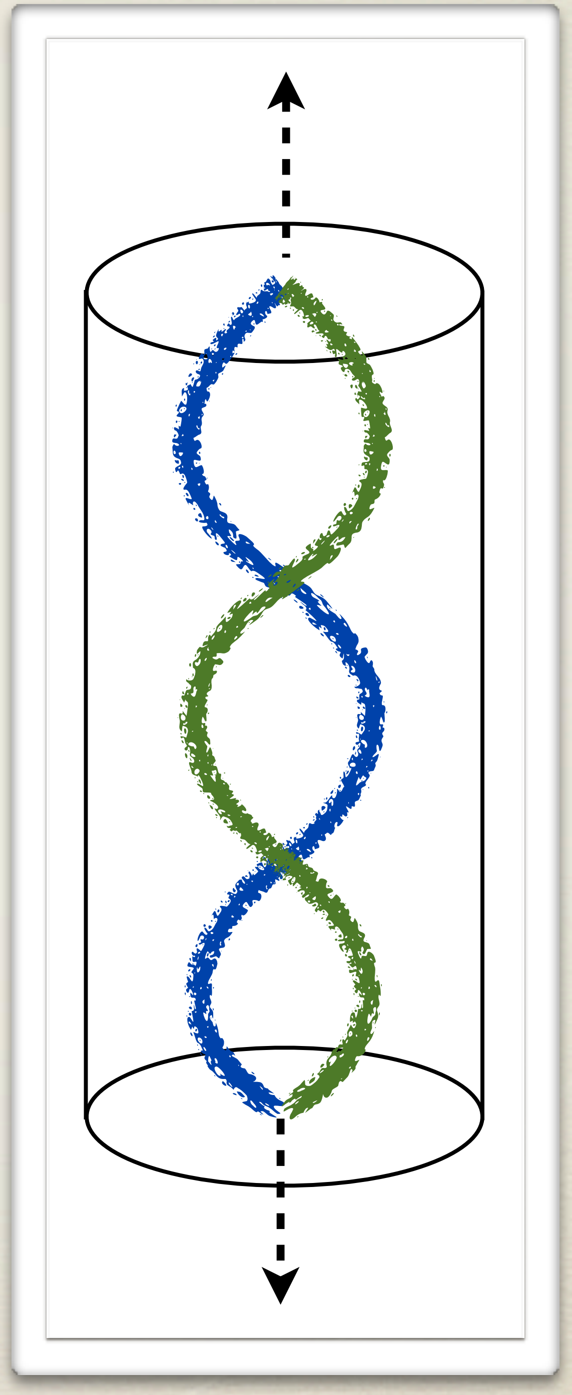
Scattering in the Flat Region

- Scattering in the flat region should approximate local physics in our universe.
- Need to construct wavepackets to localize scattering to a single flat region of AdS.



Multiple Scattering

* Free fields in AdS are periodic.
* Purely normalizable states will interact infinitely many times.
* Can't isolate contribution from one scattering experiment.



Interactions Near the Boundary

- Boundary sources → infinite particle production near the boundary.
- Single particle states not well defined when boundary sources turned on.

$$N = \int_{t_0} d^d\vec{x}\sqrt{-g}(\phi^* \overleftrightarrow{\partial_t} \phi) = \infty$$

- Difficult to isolate scattering in flat region from scattering near the boundary.
- Sources should be compact and non-overlapping to avoid infinite interactions near the boundary and normalize states.

Boundary-Compact Wavepackets

* Construct using Bulk-Boundary Propagator.

$$\phi_f(x) = \int db f(b) G_{B\partial}(b, x)$$

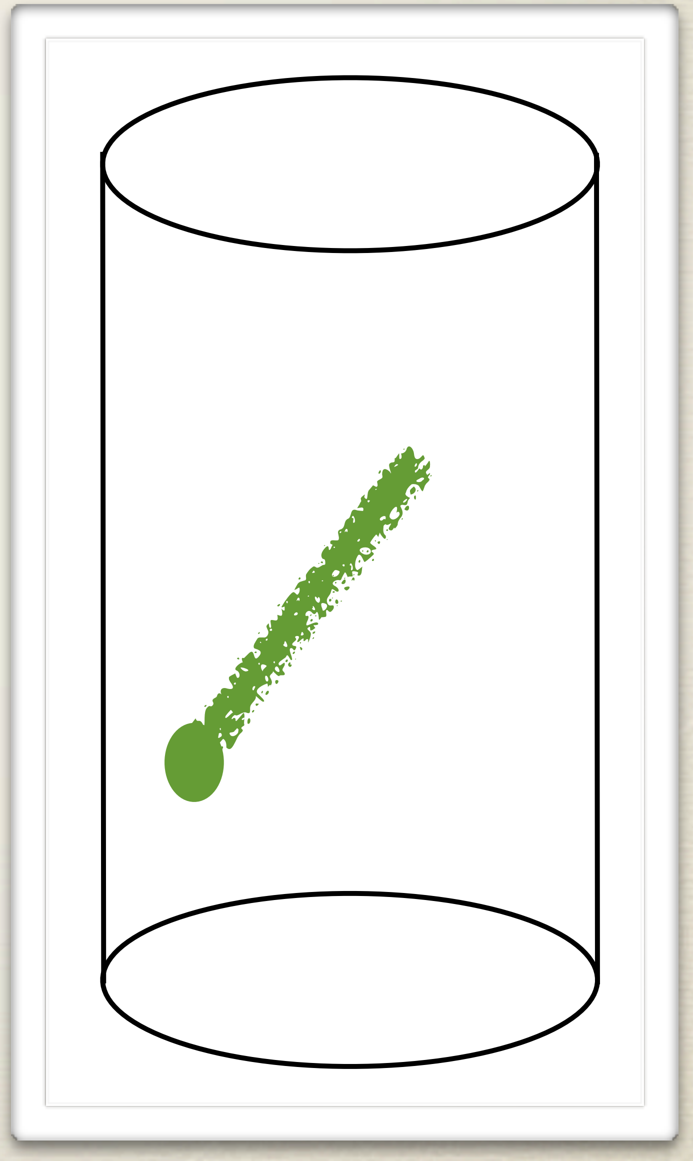
* Compactly supported sources

$$f(b) = L\left(\frac{\tau - \tau_0}{\Delta\tau}\right) L\left(\frac{\theta}{\Delta\theta}\right) e^{-i\omega R(\tau - \tau_0)}$$

of size $\Delta\tau$, $\Delta\theta$.

* $\frac{1}{\omega R} \ll \Delta\tau, \Delta\theta \ll 1$

* Scatter when sources turned off.



Wavepackets in the Scattering Region

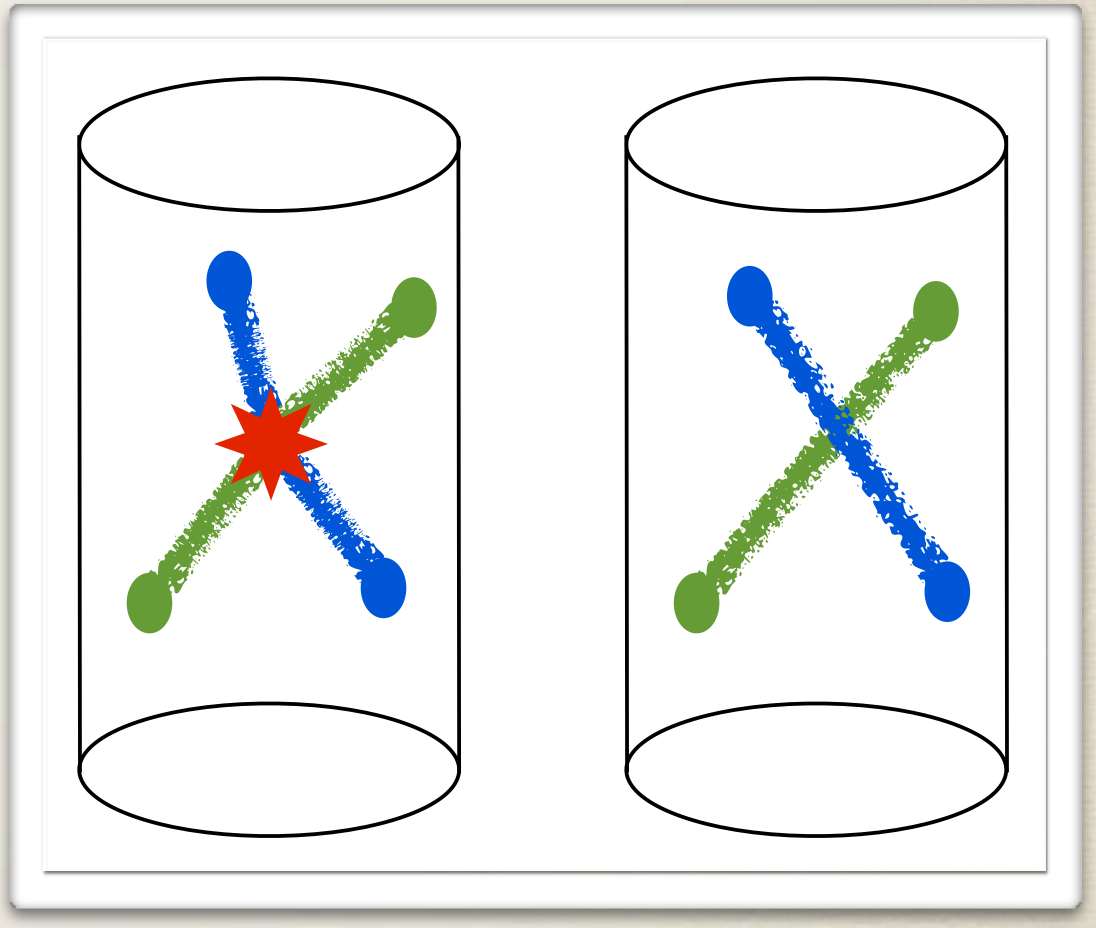
- Near the center of AdS,

$$\phi_f(x) \approx \phi_f(0) \frac{\tilde{L}_{d-1}(x_\perp \omega \Delta\theta)}{\tilde{L}_{d-1}(0)} L\left(\frac{u}{\Delta t}\right) e^{-i\omega u} .$$

- Longitudinal width $\Delta t \sim R\Delta\tau$.
- Transverse width $\Delta x_\perp \sim 1/(\omega\Delta\theta)$.
- Well localized for $1/\omega \ll \Delta t, \Delta x_\perp \ll R$, equivalent to earlier requirement.

Singularity Structure

* Signal of interaction from intersecting wavepackets: *local bulk physics!*

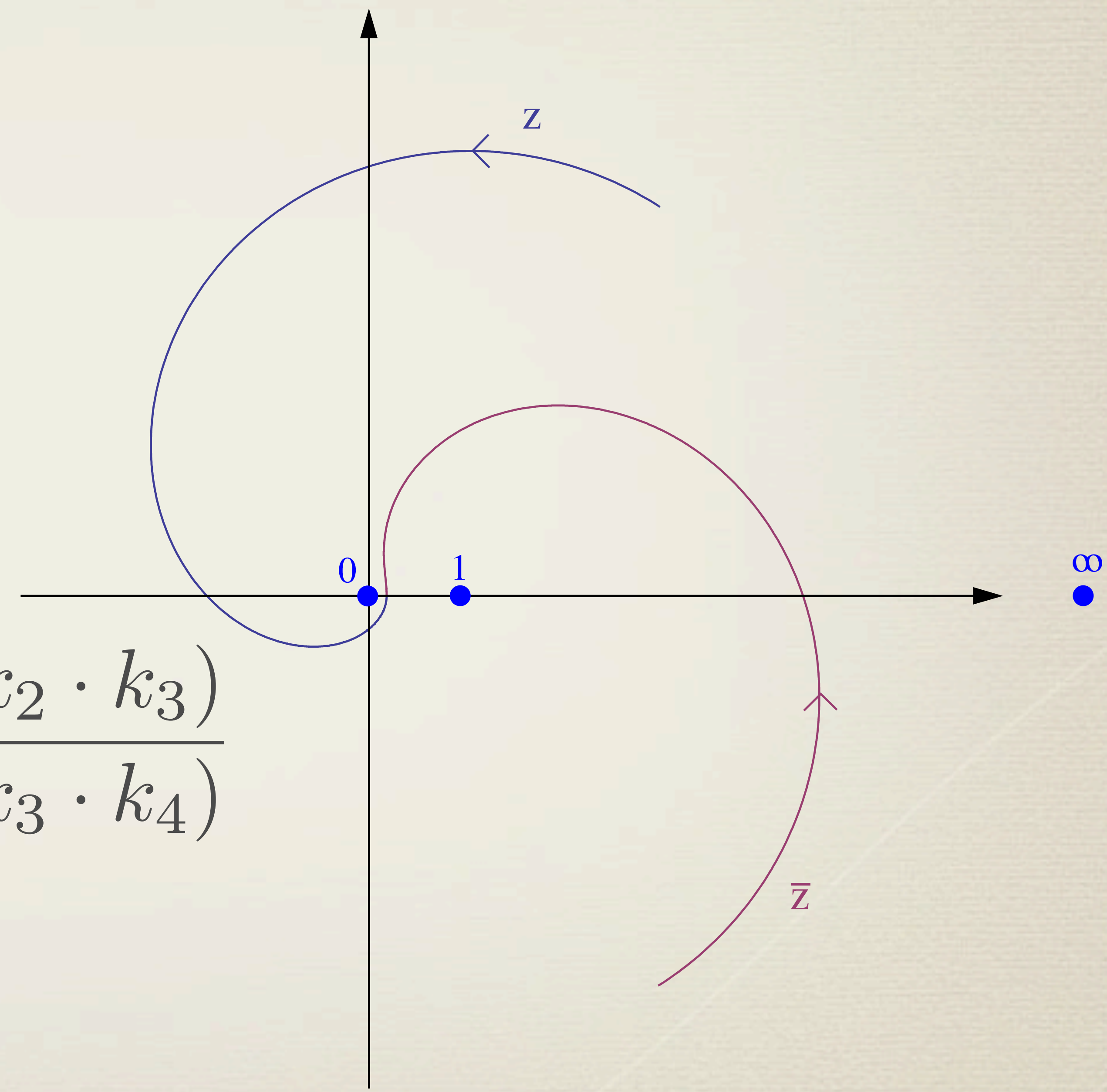


Analytic Continuation

- Singularity only present in physical Lorentzian continuation.
- $z\bar{z} = \dfrac{(k_1 \cdot k_3)(k_2 \cdot k_4)}{(k_1 \cdot k_2)(k_3 \cdot k_4)}$

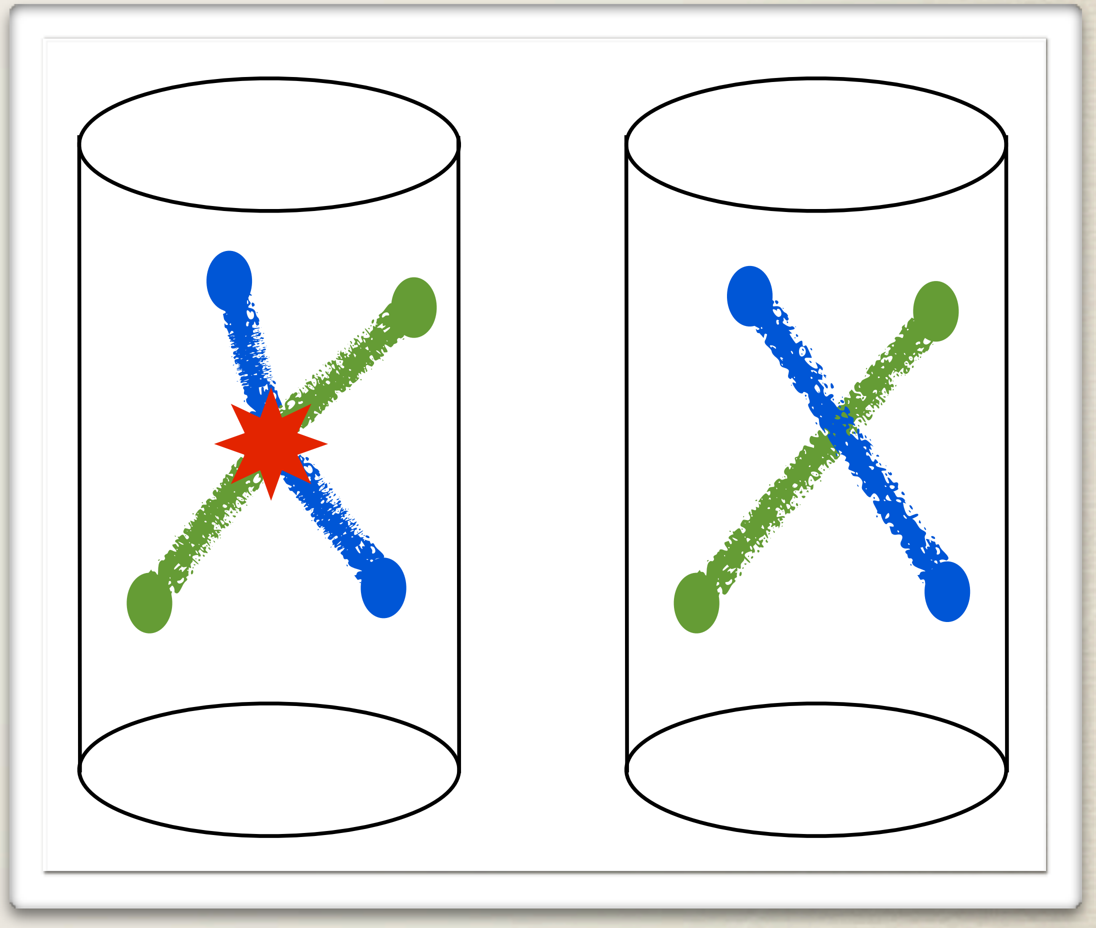
$$(1 - z)(1 - \bar{z}) = \frac{(k_1 \cdot k_4)(k_2 \cdot k_3)}{(k_1 \cdot k_2)(k_3 \cdot k_4)}$$

- $z \longrightarrow \bar{z}$



Singularity Structure

- Signal of interaction from intersecting wavepackets: *local bulk physics!*
- $z = \sigma e^{-\rho}\ ,\ \bar{z} = \sigma e^{\rho}$
- $\mathcal{A}(z, \bar{z}) \approx g^2 R^{5-d-2j} \dfrac{F(\sigma)}{(-\rho^2)^{\beta}}$
- $\beta = \Delta_1 + \Delta_2 + j - 5/2$



Momentum Conservation

- Flat space S-Matrix conserves momentum:

$$S = 1 + i(2\pi)^D \delta^D \left(\sum k_i \right) \mathcal{T}(s,t)$$

- Momentum conserving δ-function must emerge from CFT amplitude in appropriate limit.

- Delta function does emerge from form of boundary compact sources and singularity structure:

$$\lim_{R\to\infty} \int d\nu \frac{R^n e^{-i\nu}}{(R^2\kappa^2 - (\nu + i\epsilon)^2)^\beta} \propto \delta^n(\vec{\kappa})$$

The S-Matrix

* Determine flat space scattering amplitude from residue of CFT singularity:

$$\mathcal{A}(z,\bar{z}) \to g^2 R^{5-d-2j} \frac{\mathcal{F}(\sigma)}{(-\rho^2)^\beta}$$

$$i\mathcal{T}(s,t) = \mathcal{K} g^2 s^{j-1} \left(\frac{-t}{s}\right)^{j-2} \left(\frac{-u}{s}\right)^{3-j-\Delta_1-\Delta_2} \mathcal{F}\left(\frac{-s}{t}\right)$$

Examples

- Compute CFT correlators using AdS/CFT dictionary (D'Hoker *et al.*, 1999), read off scattering amplitudes.
- Scalar exchange:

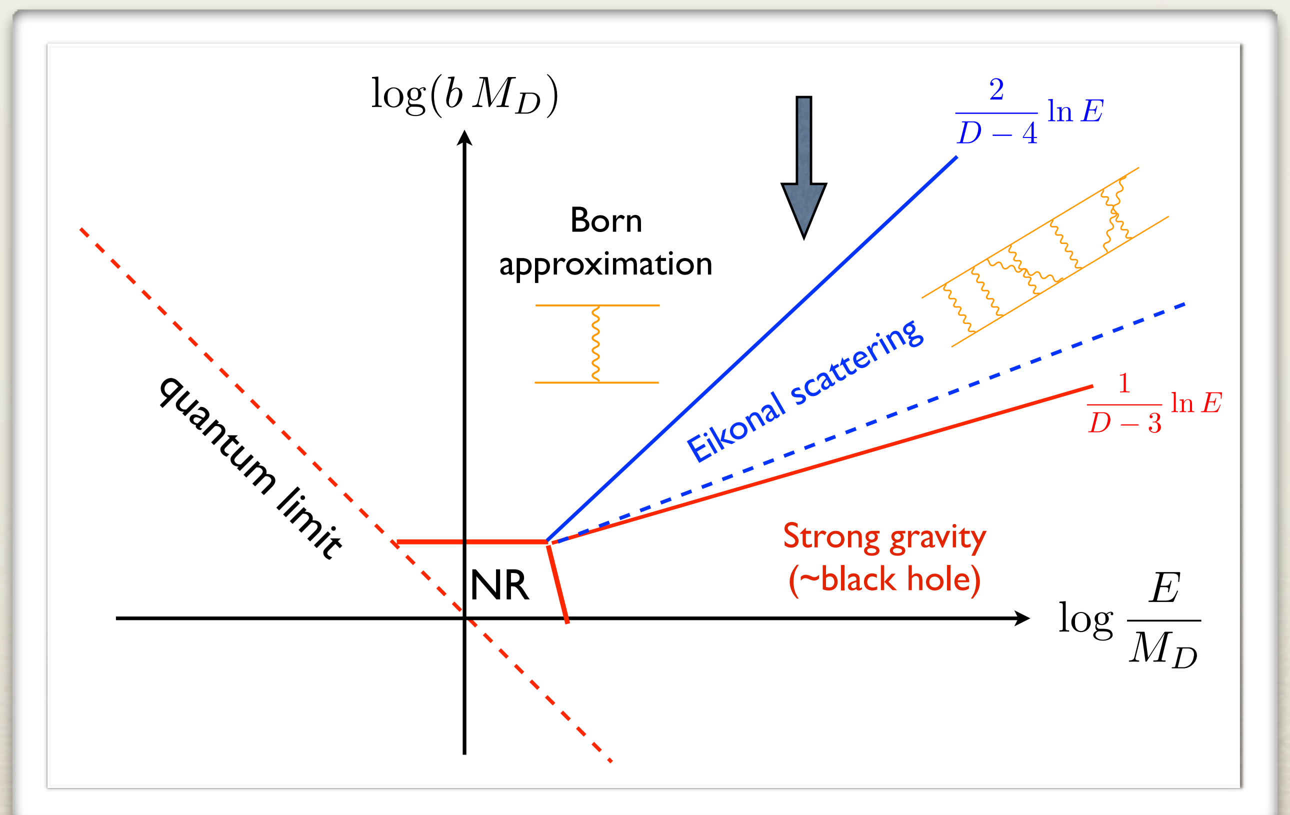
$$\mathcal{F}(\sigma) \propto \sigma(1-\sigma)^{\Delta_1+\Delta_2-3} \quad \rightarrow \quad \mathcal{T}(s,t) = \frac{g^2}{-t}$$

- Graviton exchange:

$$\mathcal{F}(\sigma) \propto \frac{(1-\sigma)^8}{\sigma} \quad \rightarrow \quad \mathcal{T}(s,t) = 8\pi G_5 \frac{s^2+ts}{-t}$$

Source of Singularity?

- Cause of the singularity unclear from CFT perspective.
- From bulk, singularity occurs when all four boundary sources are at light-like separation from a common bulk point.
- How are different scattering regimes encoded in the CFT?



Limits on Resolution

* Well behaved wavepackets are critical for derivation of LSZ decomposition.
* Bulk-Boundary propagator maps boundary-compact sources to non-compact wavepackets in the bulk.
* Cannot construct regular wavepackets, typically used for formal derivations of LSZ decomposition.
* What about Schwartz wavepackets?

Power-Law Tails

- *NO*: boundary-compact wavepackets have power-law tails in flat region.
- $x_\perp \gg \Delta t/\Delta\theta,\ u/\Delta\theta$:

$$\phi_f(x) \approx \phi_f(0)\frac{\omega\Delta t\tilde{L}(\omega\Delta t)\hat{L}}{(x_\perp\omega\Delta\theta)^\Delta}$$

- $u \gg \Delta t$

$$\phi_f(x) \approx \phi_f(0)\frac{\omega\Delta t\tilde{L}(\omega\Delta t)\Gamma(\Delta)}{2\pi(i\omega u)^\Delta}$$

Recovering the S-Matrix?

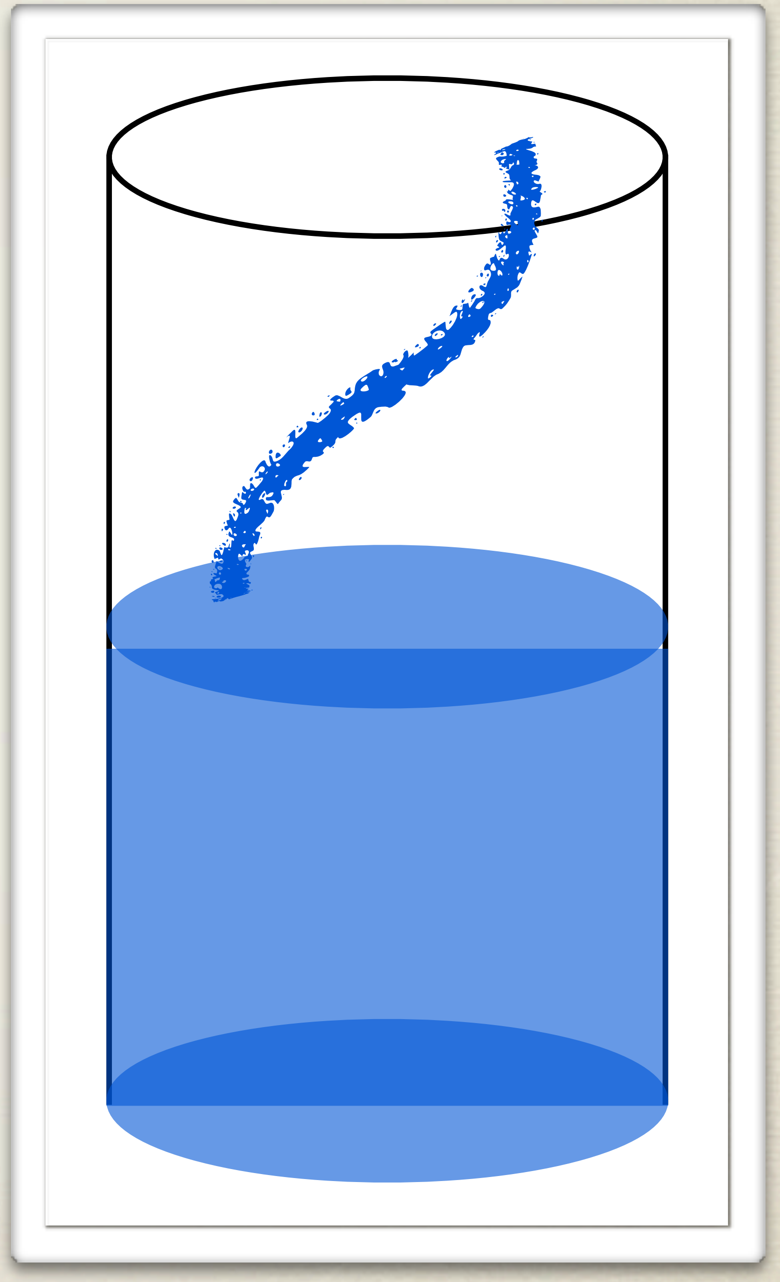
- Tails from direct contribution interfere with scattered contribution to amplitude.
- Scattered contribution doesn't always go in the correct direction: haze of order $\tilde{L}(\omega\Delta t)$.
- We cannot recover the full flat space S-Matrix this way.
- Can we build better wavepackets?

Resonant Wavepackets

* Use resonant structure of AdS to build any normalizable wavefunction.

$$\phi_f(x) = \sum_{nl\vec{m}} c_{nl\vec{m}} \phi_{nl\vec{m}}(x)$$

* Source compactly supported for 1/2 AdS time.
* Well-behaved in interaction region.
* What about while the source is turned on?



Multiple Interactions and Power Law Tails

- While source is on, two terms contribute in the large R limit.
- First term: wavepacket builds/decays linearly in time,

$$\phi_f(x)\left(1+\frac{\tau}{\pi}\right)$$

$\rightarrow$secondary interactions while wavepackets built/decay.

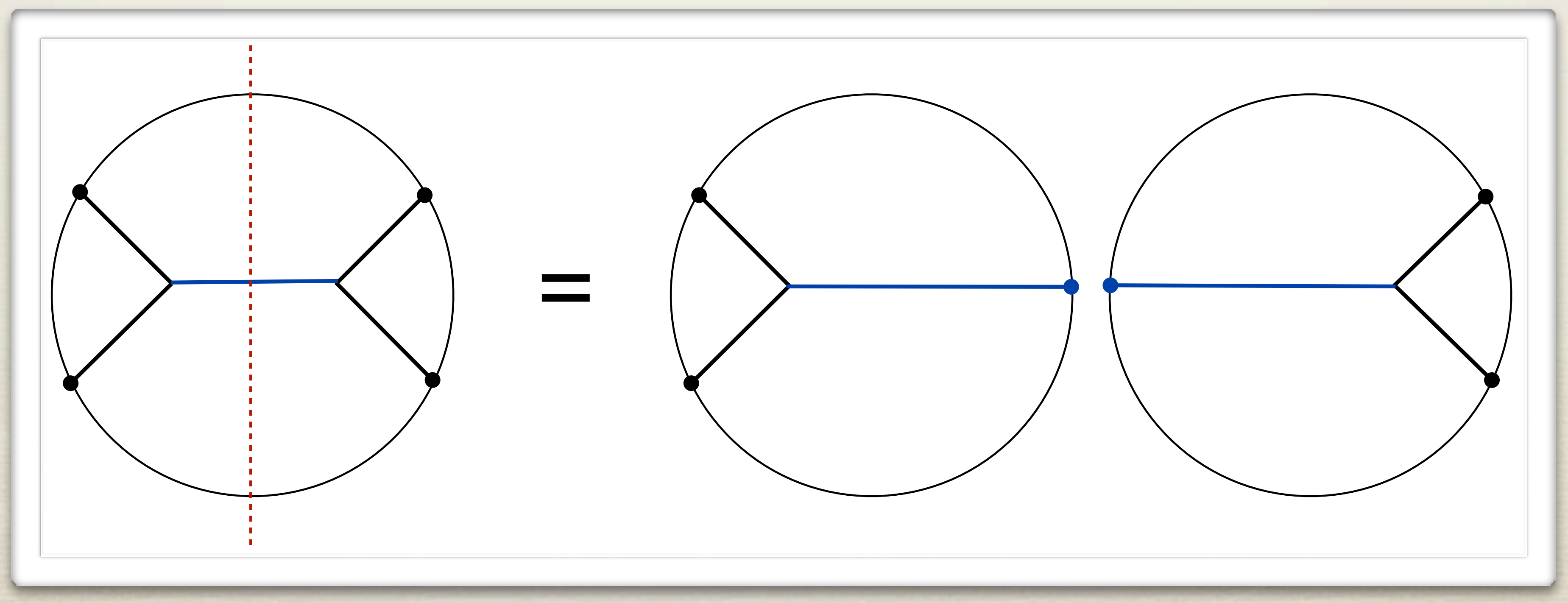
- Second term: power law tail in time.

$$\int d\omega\phi_f(\omega)\frac{\omega^{\frac{d-1}{2}}e^{-i\omega R\tau}}{(-i\omega R(\pi+\tau))^{\Delta}}$$

- Not quite a No-Go theorem, but hinders S-Matrix.

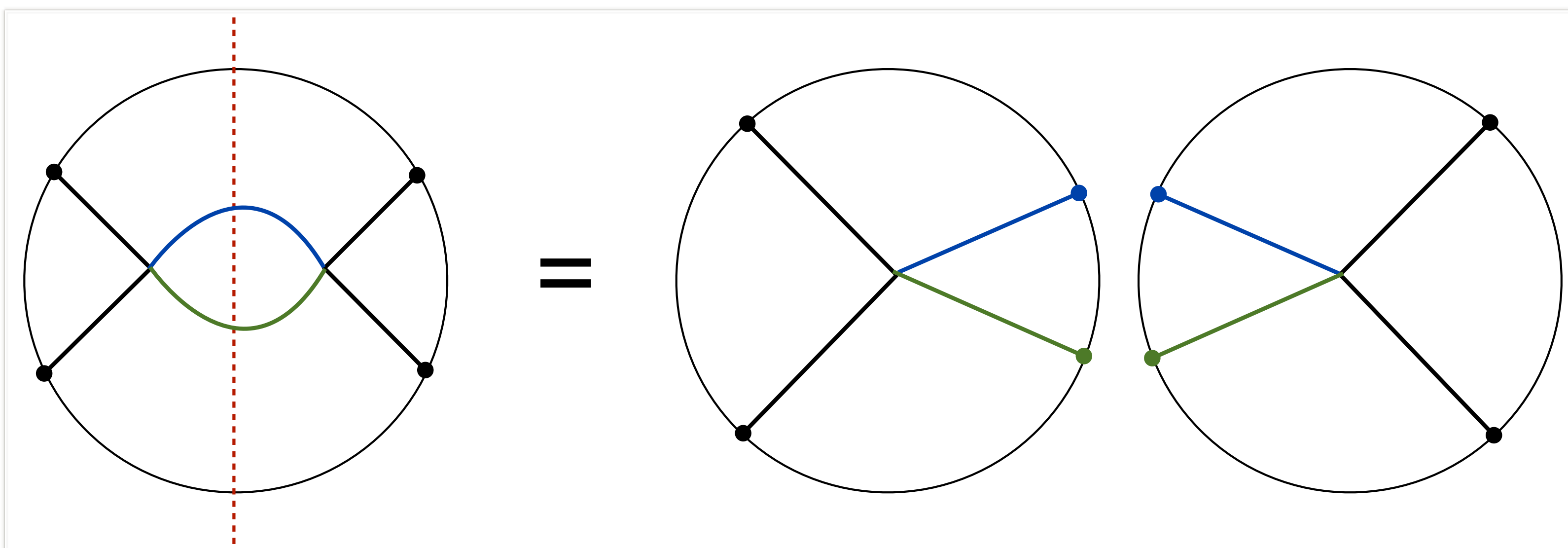
Bulk Unitarity

- Often claimed: Unitary evolution in the CFT ⇒ unitary evolution in the bulk.
- Can we see bulk unitarity from the CFT?
- Look at pertubative bulk unitarity. At tree level, just the OPE:



Bulk Unitarity

* What about loops?
* Seem to need new relations:



$$\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle_{1\text{loop}} = \sum_{A,B\neq 1} \langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_A\mathcal{O}_B\rangle_{\text{tree}}\langle \mathcal{O}_A\mathcal{O}_B\mathcal{O}_3\mathcal{O}_4\rangle_{\text{tree}}$$

* Are these relations present? What does this imply?

Conclusions

- CFT singularity signature of fine-grained locality.
- Boundary-compact wavepackets are not as well-localized as flat space wavepackets.
- Power-law tails can hide important physics.
  - Signature of black hole formation is exponentially suppressed $2 \to 2$ amplitude, swamped by tails.
- Can build better wavepackets in AdS from compact sources on the boundary, but have multiple scattering.

Unresolved Questions

* What in the CFT is responsible for the appearance of the singularity?
* How are different scattering regimes encoded in the CFT?
* Is there a way to build nice wavepackets that don't have multiple scattering or power-law tails?

Unresolved Questions

* How is bulk unitarity encoded in the boundary CFT?
  * Necessary for understanding black hole evaporation & resolving information problem.
* Are power-law tails a signature of inherent non-locality?
  * Does the CFT really capture everything in the gravitational theory?

Thank You