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Beyond AdS Holography in 3d Higher Spin Gravity

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Work in Progress

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Overview

* Motivation
* Intro to Higher Spin Theories in 3D
* Boundary Conditions & Asymptotics beyond Brown-Henneaux AdS/CFT
* Canonical Analysis
* Conclusions & Future Directions

Why Higher Spin?

* Between pure Einstein Gravity and String Theory
* Related to tensionless limit of String Theory
* In some cases dual to soluble/free field theories

Why Non-AdS?

* Want to understand more generic holography
  * Would like holographic duals for flat space, dS
* Some CFT applications require non-AdS asymptotics
  * Cold Atoms
  * Other non-relativistic systems

Why in 3D?

* Dual 2D CFTs are often solvable
* 3D Gravity is topological
  * No local DoF
  * Can be formulated as a Chern-Simons Theory

Non-Principal Embeddings?

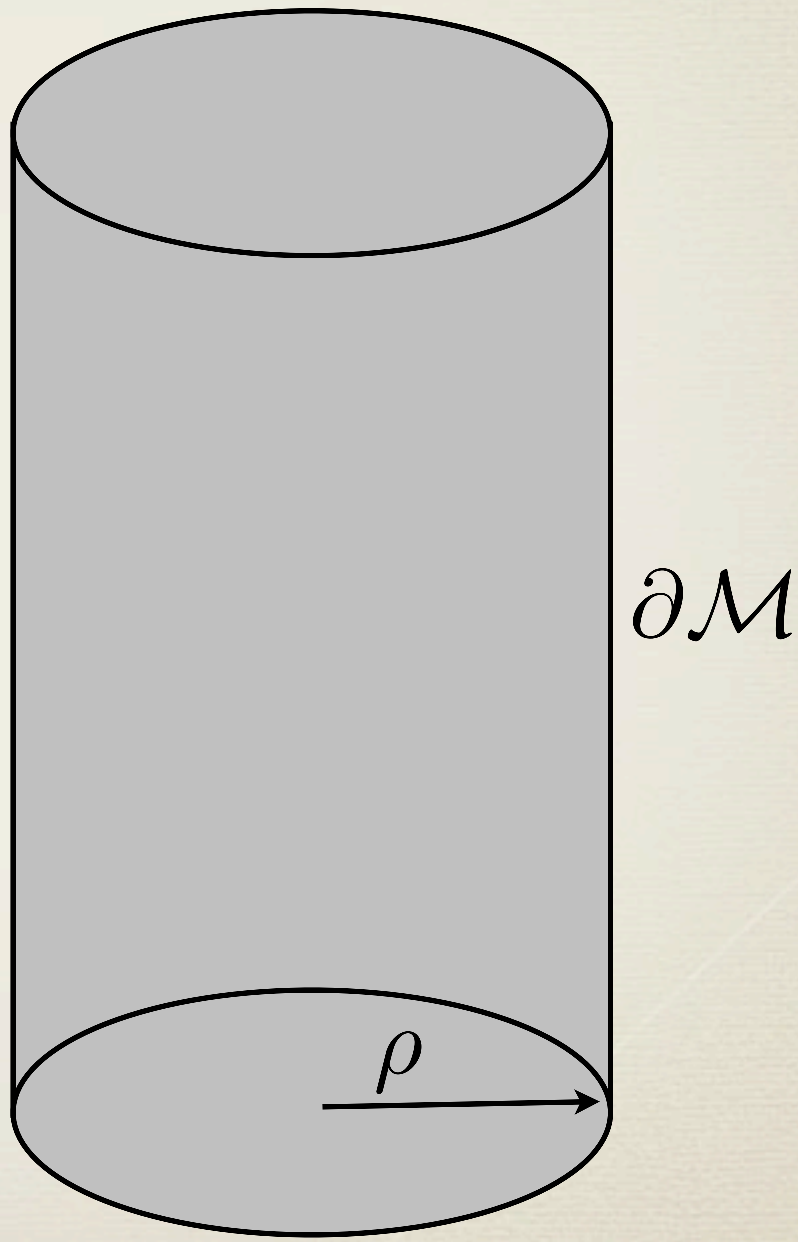
* Puzzle relating to RG flow in Kraus *et al.*
  * Flow from low to high central charge
  * Triggered by deformation - state or new theory?
* What aspects of geometry are gauge invariant?
  * Gauge transformations apparently change number of horizons - are they small, large, singular?

Geometry

* Take space-time to have topology of a solid cylinder

* Often useful to choose coordinates ϱ, $x^\pm$

* $\partial\mathcal{M}$ has topology $S^1 \times \mathbb{R}$



CS-Gravity in 3D

* Gravity in AdS_3 can be formulated as $sl_2 \times sl_2$ Chern-Simons theory

* $$S_{\text{bulk}} = \frac{k}{4\pi} \int_{\mathcal{M}} \text{tr}\left[CS(A) - CS(\bar{A})\right]$$

$$CS(A) = A \wedge dA + \frac{2}{3} A \wedge A \wedge A$$

$$\delta S_{\text{bulk}} \propto \int_{\partial\mathcal{M}} \text{tr}\left[A_+ \wedge \delta A_- - A_- \wedge \delta A_+\right] - (\text{bar})$$

* Vielbein: $e = A - \bar{A}$ Spin-connection: $\omega = A + \bar{A}$

Metric: $g_{\mu\nu} = \frac{1}{2} \text{tr}\left[e_\mu e_\nu\right]$

CS-Gravity in 3D

* Supplement bulk action by boundary term

$$S = S_{\text{bulk}} + \frac{k}{8\pi}\int_{\partial\mathcal{M}} \text{tr}\left[A \wedge A + \bar{A} \wedge \bar{A}\right]$$

* Variation: $\delta S \propto \int_{\partial\mathcal{M}} \text{tr}\left[A_+ \wedge \delta A_-\right] - (\text{bar})$

* Allows for more general boundary conditions

Generalizing to HS

- Enlarge gauge group sl_2 to sl_N
- Vielbein becomes Zuvielbein $e_\mu^a = A_\mu^a - \bar{A}_\mu^a$

$$g_{\mu\nu} = \frac{1}{2}\mathrm{tr}\left[(A-\bar{A})_\mu (A-\bar{A})_\nu\right]$$

- Definition of trace depends on choice of gravitational sector (choice of embedding of sl_2 into sl_N)
- Asymptotic symmetry algebra enhanced from Virasoro to W-algebra

Embedding sl_2 in sl_N

* Choose an embedding $sl_2 \rightarrow sl_N$, label generators $L_0, L_\pm$
* Other generators W_h labeled by sl_2 weight
  $[W_h, L_0] = hW_h$
* More generally, $\left[L_n, W_m^{l[a]}\right] = (nl - m)W_{n+m}^{l[a]}$
* Embeddings given by partitions of N (with a condition)
  * 3 = 3, 2+1
  * 4 = 4, 3+1, 2+2, 2+1+1

Embeddings of sl_2 in sl_4

- Principal embedding: 4 = 4
  - sl_2 multiplets: $\mathbf{3} \oplus \mathbf{5} \oplus \mathbf{7}$
- 4 = 3 + 1
  - sl_2 multiplets: $\mathbf{3} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \mathbf{5} \oplus \mathbf{1}$
- 4 = 2 + 2
  - sl_2 multiplets: $\mathbf{3} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \mathbf{3} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$
- 4=2 + 1 + 1
  - sl_2 multiplets: $\mathbf{3} \oplus \mathbf{2} \oplus \mathbf{2} \oplus \mathbf{2} \oplus \mathbf{2} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$

AdS Beyond Brown-Henneaux

- Brown-Henneaux is an allowed set of boundary conditions that leads to non-trivial asymptotic symmetry algebra
- Unknown if most general allowed boundary conditions for higher spin AdS theories
- Possibility that looser boundary conditions allow more states - should explore
  - See for example AdS / log CFT

AdS Beyond Brown-Henneaux

- Consider an embedding with generators $W_{\pm h}$ such that $h > 0,\ \ h \neq 1$, and $\operatorname{tr}\left[W_h W_{-h}\right] \neq 0$
- Connection

$$A = L_0 d\rho + \left[e^{\rho} L_+ + e^{h\rho} w_+(x^+) W_h\right] dx^+$$
$$\bar{A} = -L_0 d\rho + \left[e^{\rho} L_- + e^{h\rho} w_-(x^-) W_{-h}\right] dx^-$$

- Metric given by

$$ds^2 = a_0 d\rho^2 + \left[a_1 e^{2\rho} + a_2 e^{2h\rho} w_+(x^+) w_-(x^-)\right] dx^+ dx^-$$

Fefferman-Graham expansion goes beyond B-H

$AdS_2 \times R$ Geometry

- Metric: $ds^2 = d\rho^2 - ae^{2\rho}dt^2 + dx^2$
  - $AdS_2 \times R$ or $H_2 \times R$ depending on sign of a
- Applications to condensed matter
  - Strange Metals seem to be dual to $AdS_2 \times R^2$, closely related to high-T_c superconductors

$AdS_2 \times R$ Background

* Embedding with at least one sl_2 singlet S with $\mathrm{tr}\left[S^2\right] \neq 0$.
* Connection
$$A = L_0 d\rho + a_1 e^{\rho} L_+ dt$$
$$\bar{A} = -L_0 d\rho + e^{\rho} L_- dt + S dx$$
* Metric
$$g_{\rho\rho} = 2\mathrm{tr} L_0^2$$
$$g_{tt} = -a_1 \mathrm{tr}\left(L_+ L_-\right) e^{2\rho}$$
$$g_{xx} = \frac{1}{2}\mathrm{tr} S^2$$

Schrödinger Spacetime

- Metric: $ds^2 = \ell^2 \left[\frac{dr^2 \pm 2dtd\xi}{r^2} - \frac{dt^2}{r^{2z}} \right]$
- Applications to holography of systems with anisotropic scaling exponents.
    - Strongly correlated electrons & other non-relativistic condensed matter systems at quantum critical points
    - Similar applications for asymptotic Lifshitz geometries

Schrödinger Background

- Generators $W_{\pm z}$ with sl2 weight ±z such that $[W_{-z}, L_-] = 0$ and $\text{tr}\,(W_{+z}W_{-z}) \neq 0.$
- Connection
  $$A = L_0 + (a_1 e^{\rho} L_+ + a_2 e^{z\rho} W_z)\, dt$$
  $$\bar{A} = -L_0 + e^{z\rho} W_{-z} dt + e^{\rho} L_- d\xi$$
- Metric: set $r = e^{-z\rho}$
  $$ds^2 = \ell^2 \left[\frac{dr^2 \pm 2dtd\xi}{r^2} - \frac{dt^2}{r^{2z}}\right]$$

Lifshitz Background

* Generators $W_{\pm z}$ with sl2 weight ±z such that $[W_{-z}, L_-] = 0$ and $\mathrm{tr}\,(W_{+z}W_{-z}) \neq 0$.
* Connection
$$A = L_0 d\rho + a_1 e^{z\rho} W_z dt + e^{\rho} L_+ dx$$
$$\bar{A} = -L_0 d\rho + e^{z\rho} W_{-z} dt + a_2 e^{\rho} L_- dx$$
* Metric: set $r = e^{-z\rho}$
$$ds^2 = \ell^2 \left[\frac{dr^2 + dx^2}{r^2} - \frac{dt^2}{r^{2z}} \right]$$

Warped AdS

* Space-like WAdS Metric:

$$ds^2 = \frac{\ell^2}{\nu^2+3}\left(d\rho^2 - \cosh^2\rho\, dt^2 + \frac{4\nu^2}{\nu^2+3}\left(dx + \sinh\rho\, dt\right)^2\right)$$

  * Stretched: $\nu^2 > 1$
  * Squashed: $\nu^2 < 1$
  * Null-warped AdS: $\nu^2 \to 1$

Applications of WAdS

- Generalization of AdS with less symmetry
  - $sl_2 \times sl_2$ reduced to $sl_2 \times U(1)$
- Similarities to Kerr/CFT correspondence
  - Application of holography to more realistic gravitational systems

WAdS Background

- Generators $W_{\pm\frac{1}{2}}, S$ with $\mathrm{tr}\left(W_{\frac{1}{2}}W_{-\frac{1}{2}}\right) \neq 0$
  $\mathrm{tr}S^2 \neq 0$.

- Connection

$$A = L_0 d\rho + \left(a_1 e^{\rho} L_+ + a_2 e^{\rho/2} W_{\frac{1}{2}}\right) dx^+$$

$$\bar{A} = -L_0 d\rho + e^{\rho} L_- dx^+ + \left(e^{\rho/2} W_{-\frac{1}{2}} + \mu S\right) dx^-$$

- Metric

$$g_{\rho\rho} = 2\mathrm{tr}L_0^2 \qquad g_{+-} = -\frac{a_2 a_3}{2} e^{\rho} \mathrm{tr}\left(L_+ L_-\right)$$

$$g_{++} = -a_1 e^{2\rho} \mathrm{tr}\left(L_+ L_-\right) \qquad g_{--} = \mu^2 \frac{a_4}{2} \mathrm{tr}\left(L_+ L_-\right)$$

Canonical Analysis

* Canonical Analysis underway
  * Necessary to understand asymptotic symmetry algebra and charges
* To go beyond AdS, necessary to perform analysis without partial gauge-fixing
  * New surprises already for sl_2 case

Canonical Anlysis

* Define canonical variables
* Determine constraints and algebra
* Construct Dirac bracket $\{\ ,\ \}_{DB}$
* Use Castellani procedure to find gauge generator
* Find charges by demanding variation of gauge generator is functionally differentiable

Canonical Analysis

* Make an Ansatz for boundary conditions
  * Fall-off for each component of the connection A_μ^a
* Find boundary condition preserving gauge transformations $\delta A = d\epsilon + [A, \epsilon]$
* Compute canonical charges $Q \propto \lim_{\rho \to \infty} \oint d\phi \; \mathrm{tr}\,[\epsilon A_\phi]$
  * Need to be finite, conserved, and integrable
  * Charges should be state-dependent for non-trivial theory

sl_2 Canonical Analysis

* Boundary conditions

$$A^1_\rho = \mathcal{O}\left(e^{-\rho}\right) \qquad A^0_\rho = 1 + \mathcal{O}\left(e^{-2\rho}\right) \qquad A^{-1}_\rho = \mathcal{O}\left(e^{-3\rho}\right)$$

$$A^1_+ = e^\rho + \mathcal{O}\left(e^{-\rho}\right) \qquad A^0_+ = \mathcal{O}\left(e^{-2\rho}\right) \qquad A^{-1}_+ = a(x^+)e^{-\rho} + \mathcal{O}\left(e^{-3\rho}\right)$$

$$A^1_- = \mathcal{O}\left(e^{-\rho}\right) \qquad A^0_- = \mathcal{O}\left(e^{-2\rho}\right) \qquad A^{-1}_- = \mathcal{O}\left(e^{-3\rho}\right)$$

* Boundary condition preserving gauge transformations

$$\epsilon^1 = \epsilon(x^+)e^\rho + \mathcal{O}\left(e^{-\rho}\right)$$

$$\epsilon^0 = -\epsilon'(x^+) + \mathcal{O}\left(e^{-2\rho}\right)$$

$$\epsilon^{-1} = \left[\frac{1}{2}\partial_+^2 + a(x^+)\right]\epsilon(x^+)e^{-\rho} + \mathcal{O}\left(e^{-3\rho}\right)$$

sl_2 Canonical Analysis

- Un-integrated canonical charges

$$\delta Q = -\frac{k}{\pi}\left[\epsilon(x^+)a(x^+) - \epsilon''(x^+)\right]$$

- Integrated canonical charges

$$Q = -\frac{k}{\pi}\oint d\phi\ \epsilon(x^+)a(x^+)$$

  - Charge is chiral, even though g_{++} can depend on x^-

Questions

* Asymptotic symmetry algebras of non-AdS geometries?
  * Expect generalizations of W-algebra to include current algebras, Schrödinger algebra, Galilean conformal algebra, etc.
* Relations between embeddings?
  * RG flow or something else
* Understand geometry & higher spin symmetries
  * Causal structure vs. larger gauge symmetry

Conclusions

* Straightforward to go beyond standard AdS holography with Brown-Henneaux boundary conditions
  * non Brown-Henneaux AdS
  * $AdS_2 \times R$
  * Schrödinger/Lifschitz
  * WAdS
* Canonical analysis underway, necessary to understand
  * Asymptotic symmetry algebra
  * Central charges
  * Valid boundary conditions

Thank You