# Relational Observables in 2-D Quantum Gravity

## *Based on hep-th/0612191 M.G. and S.B. Giddings*

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# Motivation

- Want to make local observations in Quantum Gravity
- Singularities are a local phenomena
- We are local observers
- Is locality fundamental?

# The Problem

- In a QFT, observables $\mathcal{O}(x)$ must be gauge invariant
- In Gravity, gauge symmetry $\supset$ diffeomorphisms

$$\delta_\xi \mathcal{O}(x) = \xi^\mu \partial_\mu \mathcal{O}(x) \neq 0$$

  Therefore $\mathcal{O}(x)$ is not an observable
- $\int_M \sqrt{g}\mathcal{O}(x)$ is gauge invariant, but non-local.

# A Solution

- Localize relative to some background state (e.g. us, a detector, earth, etc.).
- Requires special observables and a special background.
- Observables are diffeomorphism invariant.
- Such observables approximately reduce to "semi-local" observables only in special backgrounds.
- Leads to limiations on locality.

# The Z-Model

- Proposed in hep-th/0512200 S.B. Giddings, D. Marolf, J. Hartle
- Consider a QFT with operator $\mathcal{O}(x)$
- Introduce $D$ fields $Z^i$ and state $|\Psi\rangle$ such that

$$\langle\Psi|\, Z^i \,|\Psi\rangle = \lambda\delta^i_\mu x^\mu$$

  in some region of spacetime
- Define operators $\mathcal{O}_\xi =$ "$\int_M \sqrt{g}\delta(Z^i - \xi^i)\mathcal{O}(x)$"
- $\langle\Psi|\, \mathcal{O}_{\xi_1} \cdots \mathcal{O}_{\xi_N} \,|\Psi\rangle \approx \mathcal{O}(x^\mu_1) \cdots \mathcal{O}(x^\mu_N)$ where $x^\mu_A = \lambda^{-1}\delta^i_\mu \xi^i_A$

# 2-D Quantum Gravity

- Consider 2-D Liouville Gravity with metric $g$ and conformally coupled matter $m$ with anomaly $c$

$$Z = \int \frac{\mathcal{D}g\mathcal{D}m}{\mathrm{Vol(diff)}} e^{iS[m,g]}$$

- Locally, metric is of form $ds^2 = e^{\phi}\eta_{ab}dx^a dx^b$
- Gauge fix to a metric $\hat{g}$ of this form, giving

$$S_L[\phi,\hat{g}] = \int_\Sigma \sqrt{\hat{g}}\left(\frac{\hat{g}^{ab}}{2}\partial_a\phi\partial_b\phi + \hat{R}\phi\right)$$

$$Z = \int \mathcal{D}_{\hat{g}}\phi\mathcal{D}_{\hat{g}}m\,|det_{\hat{g}}P|\exp\left(i\frac{c-25}{48\pi}S_L[\phi,\hat{g}] + iS[m,\hat{g}]\right)$$

- For $c = 25$, $S_L$ reduces to a free scalar action.

# 2-D Relational Observables

- For simplicity, assume matter is 2 free scalar fields $X^0, X^1$ and $m'$ with conformal anomaly $c' = 23$.
- $c = 2 + c' = 25$, so let $\hat{X} = \sqrt{\alpha' \frac{25-c}{24}}\phi$.
- $\Sigma = S^1 \times \mathbb{R}$ with coordinates $-\infty < t < \infty$ and $0 \leq \theta < 2\pi$
- In analogy to the Z-model, let

$$
\begin{array}{rcl}
Z^i & \leftrightarrow & X^0, X^1 \\
\mathcal{O}(x) & \leftrightarrow & \mathcal{O}\left[m'\right] \\
|\Psi\rangle & \leftrightarrow & ??? \\
\mathcal{O}_\xi & \leftrightarrow & ???
\end{array}
$$

# Background State

- Background $|\Psi\rangle$

$$\langle\Psi|\,X^0\,|\Psi\rangle = p^0 t, \langle\Psi|\,X^1\,|\Psi\rangle = R\theta$$

  with $X^1 \simeq X^1 + 2\pi R$

- Winding state of the string!
- Wheeler-deWitt equation (=Virasoro constraints)

$$(p^0)^2 = \frac{R^2}{4} - 2$$

# Semi-local Observable

- Let $\mathcal{O}\,[m']$ have conformal dim $\Delta \ll 1$, then

$$\hat{\mathcal{O}}(k_0, k_1) = \int d^2x \sqrt{\hat{g}}\,\mathcal{O}\, e^{ik_0 X^0 + ik_1 X^1 + i\hat{k}\hat{X}}$$

is diff invariant iff $\hat{k} = \pm 2\sqrt{\frac{1-\Delta}{2} - \frac{k_a k^a}{4}}$

- Fourier transforming w.r.t. $k_0, k_1$

$$\rightarrow \hat{\mathcal{O}}(\hat{t}, \hat{\theta}) = \int d^2k\, e^{-\sigma^2 k_a^2} e^{2ik^0 p^0 \hat{t} - ik^1 R\hat{\theta}}\, \hat{\mathcal{O}}(k_0, k_1)$$

where the “resolution” is $\sigma \gtrsim 1$

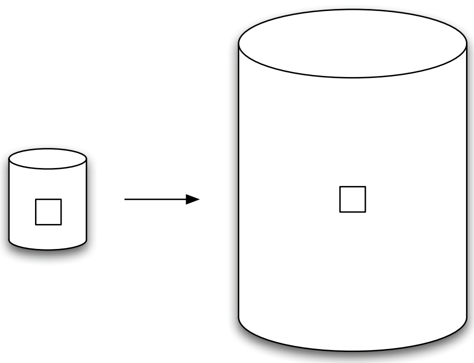
# Localization

$$\langle\Psi|\,\hat{\mathcal{O}}_1(\hat{t}_1,\hat{\theta}_1)\cdots\hat{\mathcal{O}}_N(\hat{t}_N,\hat{\theta}_N)\,|\Psi\rangle$$

$$\approx\int\prod_{i=1}^{N}d^2x_i e^{-(\frac{p^0}{\sigma})^2(t_i-\hat{t}_i)^2-(\frac{R}{2\sigma})^2(\theta_i-\hat{\theta}_i)^2}f(x_i)\left\langle\prod_{i=1}^{N}\mathcal{O}_i(x_i)\right\rangle$$

when $\sigma\gg 1$

- Localized with resolution $\Delta t\sim\sigma/p^0$, $\Delta\theta\sim\sigma/R$.
- Increasing $p^0$, $R$ increases resolution.



As $p^0$, $R$ are increased with $\ell_s$ held fixed, a string-scale area on the worldsheet becomes smaller relative to the worldsheet area.

# Limitations

- Resolution limited by value of $p^0, R$, which can be taken arbitrarily large, giving arbitrary resolution.
- In a theory with a high energy cutoff $\Lambda$, resolution would be limited by the cutoff, since $p$ could not be taken to be larger than $\Lambda$.
- In higher dimensions, gravitational backreaction limits the size of field momenta: if the field momenta become too large, curvature becomes strong and a horizon forms.