Relational Observables in 2-D Quantum Gravity

Based on hep-th/0612191 M.G. and S.B. Giddings

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Summary

- Want to see locality in a complete theory of Quantum Gravity.
- Work in 2d where we understand the theory well.
- Use Z-model from hep-th/0512200 by S.B. Giddings, D. Marolf, and J. Hartle.

The Z-Model

- Consider a QFT with operator $\mathcal{O}(x^\mu)$.
- Introduce fields Z^i and state $|\Psi\rangle$ such that

$$\langle\Psi|\, Z^i \,|\Psi\rangle = \lambda \delta^i_\mu x^\mu$$

  in some region of spacetime.
- Localize operator with respect to $\langle\Psi|\, Z^i \,|\Psi\rangle$.

$$\mathcal{O}_\xi = \int_M \sqrt{g}\,''\delta(Z^i - \xi^i)''\mathcal{O}(x)$$

- Since $\langle Z^i \rangle \sim x^\mu$, this localizes $\mathcal{O}$ near position ξ^μ, up to quantum fluctuations.

2-D Liouville Gravity

- Consider 2-D Liouville Gravity with metric g and conformally coupled matter m with anomaly c

$$Z = \int \frac{\mathcal{D}g\mathcal{D}m}{\mathrm{Vol(diff)}} e^{iS[m,g]}.$$

- Gauge fix to metric $\hat{g} = e^{\phi}\eta_{ab}dx^a dx^b$, giving

$$S_L[\phi, \hat{g}] = \int_\Sigma \sqrt{\hat{g}} \left(\frac{\hat{g}^{ab}}{2} \partial_a\phi\partial_b\phi + \hat{R}\phi \right)$$

$$Z = \int \mathcal{D}_{\hat{g}}\phi\mathcal{D}_{\hat{g}}m \left| det_{\hat{g}}P \right| e^{i\frac{c-25}{48\pi}S_L[\phi,\hat{g}]+iS[m,\hat{g}]}.$$

- For $c = 25$, S_L reduces to a free scalar action.

2-D Relational Observables

- For simplicity, assume matter is 2 free scalar fields X^0, X^1 and m' with conformal anomaly $c' = 23$.
- $c = 2 + c' = 25$, so let $\hat{X} = \sqrt{\alpha' \frac{25-c}{24}}\phi$.
- Spacetime $\Sigma = S^1 \times \mathbb{R}$ with coordinates $-\infty < t < \infty$ and $0 \leq \theta < 2\pi$ with Lorentzian signature. (Euclidean version conformally equivalent to a sphere with 2 punctures).
- Localize with respect to fields X^0, X^1.

Background State

Want background $|\Psi\rangle$ so that

- $\langle\Psi| X^0 |\Psi\rangle = p^0 t$
- $\langle\Psi| X^1 |\Psi\rangle = R\theta$ with $X^1 \simeq X^1 + 2\pi R$.

Such states exist. This is a winding state of the bosonic string if we take m' to be 23 free scalars.

- Wheeler-deWitt equation (=Virasoro constraints) $(p^0)^2 = \frac{R^2}{4} - 2$.

Semi-Local Observables

- Let $\mathcal{O}\,[m']$ have weight $\Delta \ll 1$.
- $\hat{\mathcal{O}}(k_a) = \int d^2x \sqrt{\hat{g}}\mathcal{O}e^{ik_0X^0+ik_1X^1+i\hat{k}\hat{X}}$ has weight 1 ($\Leftrightarrow$ Diff invt.) if $\hat{k} = \pm 2\sqrt{\frac{1-\Delta}{2} - \frac{k_a k^a}{4}}$.
- Fourier transform w.r.t. k_0, k_1 against a gaussian of width $\sigma \geq 1$.

$$\hat{\mathcal{O}}(\hat{t},\hat{\theta}) = \int d^2k e^{-\sigma^2 k_a^2} e^{2ik^0p^0\hat{t}-ik^1R\hat{\theta}}\hat{\mathcal{O}}(k_0,k_1)$$

- $\hat{\mathcal{O}}(\hat{t},\hat{\theta})$ is approximately $\mathcal{O}$ times a spatial gaussian centered at $(\hat{t},\hat{\theta})$.

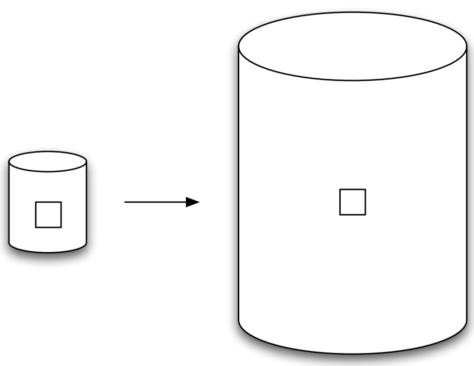
The Amplitude

- Want to calculate $\mathcal{A} = \langle \Psi | \prod_i \hat{\mathcal{O}}_i | \Psi \rangle$.
- Conformal symmetry $\rightarrow$ fix position of one operator (no extra work to localize).
- Diff. invariance $\rightarrow$ integrate over positions of remaining operators.
- $\Rightarrow$ gaussians fix positions of operators on Σ.

Localization

$$\mathcal{A} \approx \frac{1}{(4\pi\sigma^2)^N} \int \left(\prod_{i=1}^N d^2 x_i e^{2it_i} e^{-(\frac{p_0}{\sigma})^2 (t_i - \hat{t}_i)^2 - (\frac{R}{2\sigma})^2 (\theta_i - \hat{\theta}_i)^2} \right) \times f(x_i) \left\langle \prod_{i=0}^N \mathcal{O}_i(x_i) \right\rangle$$

- Localized with resolution $\Delta t \sim \sigma/p^0, \Delta\theta \sim \sigma/R$.
- Increasing p^0, R increases resolution.



As p^0, R are increased with ℓ_s held fixed, a string-scale area on the worldsheet becomes smaller relative to the worldsheet area.

Approximations

- For $|k_i^a| \ll 1, k_i \cdot k_j = 2 + \mathcal{O}((k_i^a)^2)$, $f(k_i, x_i) \approx f(x_i)$.
- For $\sigma \gg 1$, k-space gaussians sharply peaked, $\Rightarrow |k_i^a| \ll 1$, so integrand is rapidly oscillating away from region of interest and integrals converge.
- $p^0, R \gg 1$ so that sum over momentum modes (n/R) can be approximated as an integral.
- Subleading terms in $1/R$, $1/p_0$ expansions also dropped.

Limitations

- Resolution limited by value of p^0, R, which can be taken arbitrarily large, giving arbitrary resolution.
- Note difference from higher dimensions because no bakcreaction, no UV cutoffs.
- Operators only localize for $\sigma \gg 1$.
- Operators well defined independent of background, only localize in background $|\Psi\rangle$.