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Non-AdS Holography in Higher Spin 3d Gravity

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Work in Progress

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Overview

- Motivation
- Intro to Higher Spin Theories in 3D
- Boundary Conditions & Asymptotics
  - AdS Beyond Brown-Henneaux
  - $AdS_2 \times R$
  - Schrödinger/Lifshitz
  - Warped AdS
- Conclusions & Future Directions

Why Higher Spin?

- Between pure Einstein Gravity and String Theory
- Related to tensionless limit of String Theory
- In some cases dual to soluble/free field theories

Why Non-AdS?

* Want to understand more generic holography
  * Would like holographic duals for flat space, dS
* Some applications require non-AdS asymptotics
  * Cold Atoms
  * Other non-relativistic systems

Why in 3D?

* Dual 2D CFTs are often solvable
* 3D Gravity is topological
  * No local DoF
  * Can be formulated as a Chern-Simons Theory

CS-Gravity in 3D

* Gravity in AdS_3 can be formulated as $sl_2 \times sl_2$ Chern-Simons theory
* $$S_{\rm bulk} = \frac{k}{4\pi}\int_{\mathcal{M}} {\rm tr}\left[CS(A) - CS(\bar{A})\right]$$
  $$CS(A) = A \wedge dA + \frac{2}{3} A \wedge A \wedge A$$
* $$S = S_{\rm bulk} + \frac{k}{8\pi}\int_{\partial\mathcal{M}} {\rm tr}\left[A \wedge A + \bar{A} \wedge \bar{A}\right]$$
* $$e = A - \bar{A} \qquad \omega = A + \bar{A}$$
  $$g_{\mu\nu} = \frac{1}{2}{\rm tr}\left[e_\mu e_\nu\right]$$

Generalizing to HS

- Enlarge gauge group sl_2 to sl_N
- Vielbein becomes Zuvielbein $e^a_\mu = A^a_\mu - \bar{A}^a_\mu$

$$g_{\mu\nu} = \frac{1}{2}\mathrm{tr}\left[(A-\bar{A})_\mu (A-\bar{A})_\nu\right]$$

- Definition of trace depends on choice of gravitational sector (choice of embedding of sl_2 into sl_N)
- Asymptotic symmetry algebra enhanced from Virasoro to W-algebra

Embedding sl_2 in sl_N

- Choose an embedding $sl_2 \rightarrow sl_N$, label generators $L_0, L_\pm$
- Other generators W_h labeled by sl_2 weight
  $[W_h, L_0] = hW_h$
- Embeddings given by partitions of (N-1)

AdS Beyond Brown-Henneaux

- Consider an embedding with generators $W_{\pm h}$ such that $h > 0,\ \ h \neq 1$, and $\mathrm{tr}\left[W_h W_{-h}\right] \neq 0$

- Connection

$$A = L_0 d\rho + \left[e^{\rho} L_+ + e^{h\rho} w_+(x^+) W_h\right] dx^+$$
$$\bar{A} = -L_0 d\rho + \left[e^{\rho} L_- + e^{h\rho} w_-(x^-) W_{-h}\right] dx^-$$

- Metric given by

$$ds^2 = a_0 d\rho^2 + \left[a_1 e^{2\rho} + a_2 e^{2h\rho} w_+(x^+) w_-(x^-)\right] dx^+ dx^-$$

Fefferman-Graham expansion goes beyond B-H

$AdS_2 \times R$ Background

- Embedding with at least one sl_2 singlet S with $\mathrm{tr}\left[S^2\right] \neq 0$.
- Connection
  $A = L_0 d\rho + a_1 e^\rho L_+ dt$
  $\bar{A} = -L_0 d\rho + e^\rho L_- dt + S dx$
- Metric $g_{\rho\rho} = 2\mathrm{tr} L_0^2$

$$g_{tt} = -a_1 \mathrm{tr}\left(L_+ L_-\right) e^{2\rho}$$

$$g_{xx} = \frac{1}{2}\mathrm{tr} S^2$$

Schrödinger Background

- Generators $W_{\pm z}$ with sl2 weight ±z such that $[W_{-z}, L_-] = 0$ and $\mathrm{tr}\,(W_{+z}W_{-z}) \neq 0$.
- Connection
$$A = L_0 + (a_1 e^\rho L_+ + a_2 e^{z\rho} W_z)\, dt$$
$$\bar{A} = -L_0 + e^{z\rho} W_{-z} dt + e^\rho L_- d\xi$$
- Metric: set $r = e^{-z\rho}$
$$ds^2 = \ell^2 \left[\frac{dr^2 \pm 2dtd\xi}{r^2} - \frac{dt^2}{r^{2z}} \right]$$

Lifshitz Background

- Generators $W_{\pm z}$ with sl2 weight ±z such that $[W_{-z}, L_-] = 0$ and $\mathrm{tr}\left(W_{+z}W_{-z}\right) \neq 0.$

- Connection
$$A = L_0 d\rho + a_1 e^{z\rho} W_z dt + e^{\rho} L_+ dx$$
$$\bar{A} = -L_0 d\rho + e^{z\rho} W_{-z} dt + a_2 e^{\rho} L_- dx$$

- Metric: set $r = e^{-z\rho}$
$$ds^2 = \ell^2 \left[\frac{dr^2 + dx^2}{r^2} - \frac{dt^2}{r^{2z}} \right]$$

WAdS Background

* Generators $W_{\pm\frac{1}{2}}, S$ with $\mathrm{tr}\left(W_{\frac{1}{2}} W_{-\frac{1}{2}}\right) \neq 0$
  $\mathrm{tr} S^2 \neq 0$.

* Connection

$$A = L_0 d\rho + \left(a_1 e^{\rho} L_+ + a_2 e^{\rho/2} W_{\frac{1}{2}}\right) dx^+$$

$$\bar{A} = -L_0 d\rho + e^{\rho} L_- dx^+ + \left(e^{\rho/2} W_{-\frac{1}{2}} + \mu S\right) dx^-$$

* Metric

$$g_{\rho\rho} = 2\mathrm{tr} L_0^2$$

$$g_{+-} = -\frac{a_2 a_3}{2} e^{\rho} \mathrm{tr}\left(L_+ L_-\right)$$

$$g_{++} = -a_1 e^{2\rho} \mathrm{tr}\left(L_+ L_-\right)$$

$$g_{--} = \mu^2 \frac{a_4}{2} \mathrm{tr}\left(L_+ L_-\right)$$

Questions & Conclusions

* Straightforward to go beyond standard AdS holography with Brown-Henneaux boundary conditions
* Canonical analysis underway, necessary to understand
  * Asymptotic symmetry algebra
  * Central charges
  * Valid boundary conditions
* Relations between embeddings?
* Understand geometry & higher spin symmetries
  * Causal structure vs. larger gauge symmetry

Thank You