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Bulk Locality in AdS/CFT

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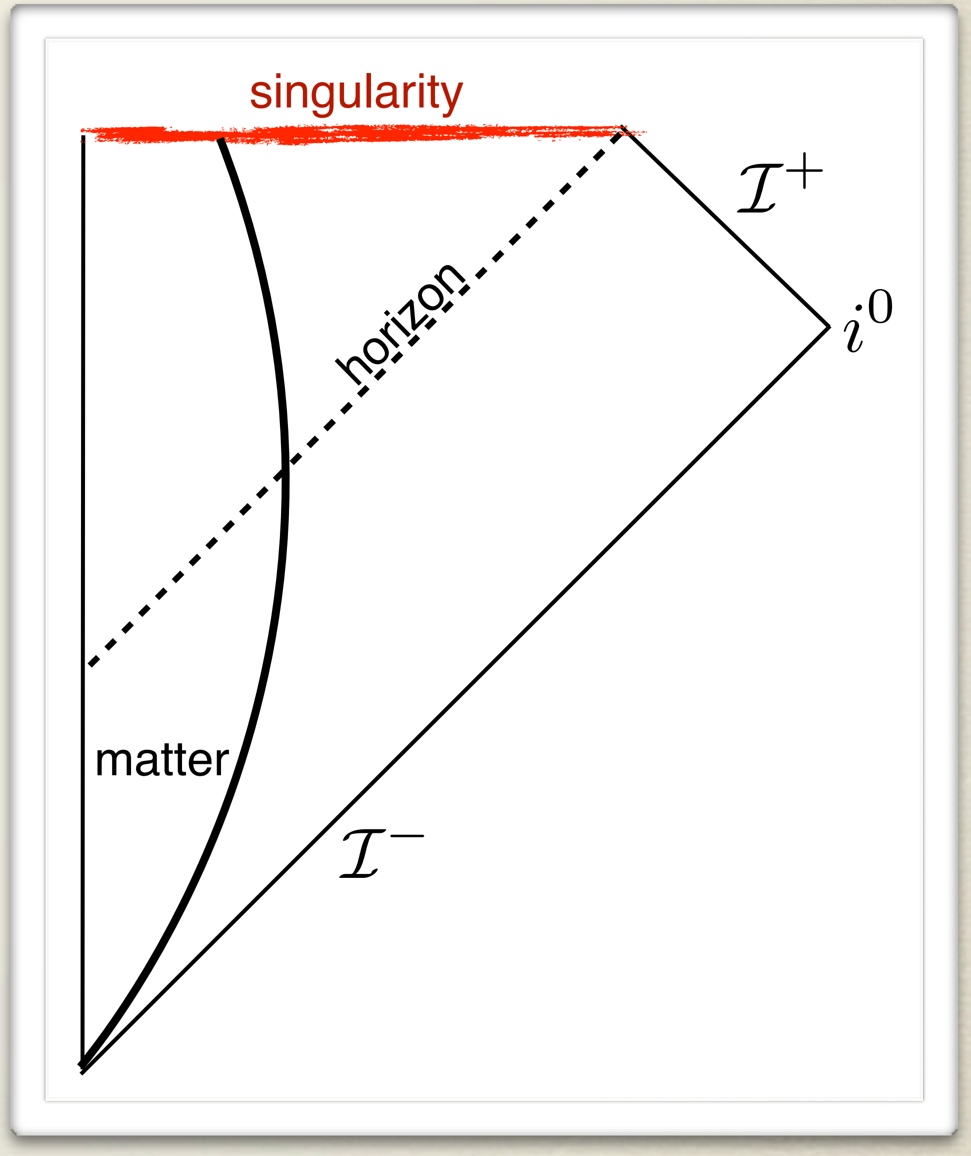
arXiv:0903.4437 (MG, Giddings, Penedones)
arXiv:0904.3544, 1106.3553 (MG, Giddings)

Overview

* Black hole evaporation & the information problem
* Intro to the AdS/CFT correspondence
* A test of bulk locality

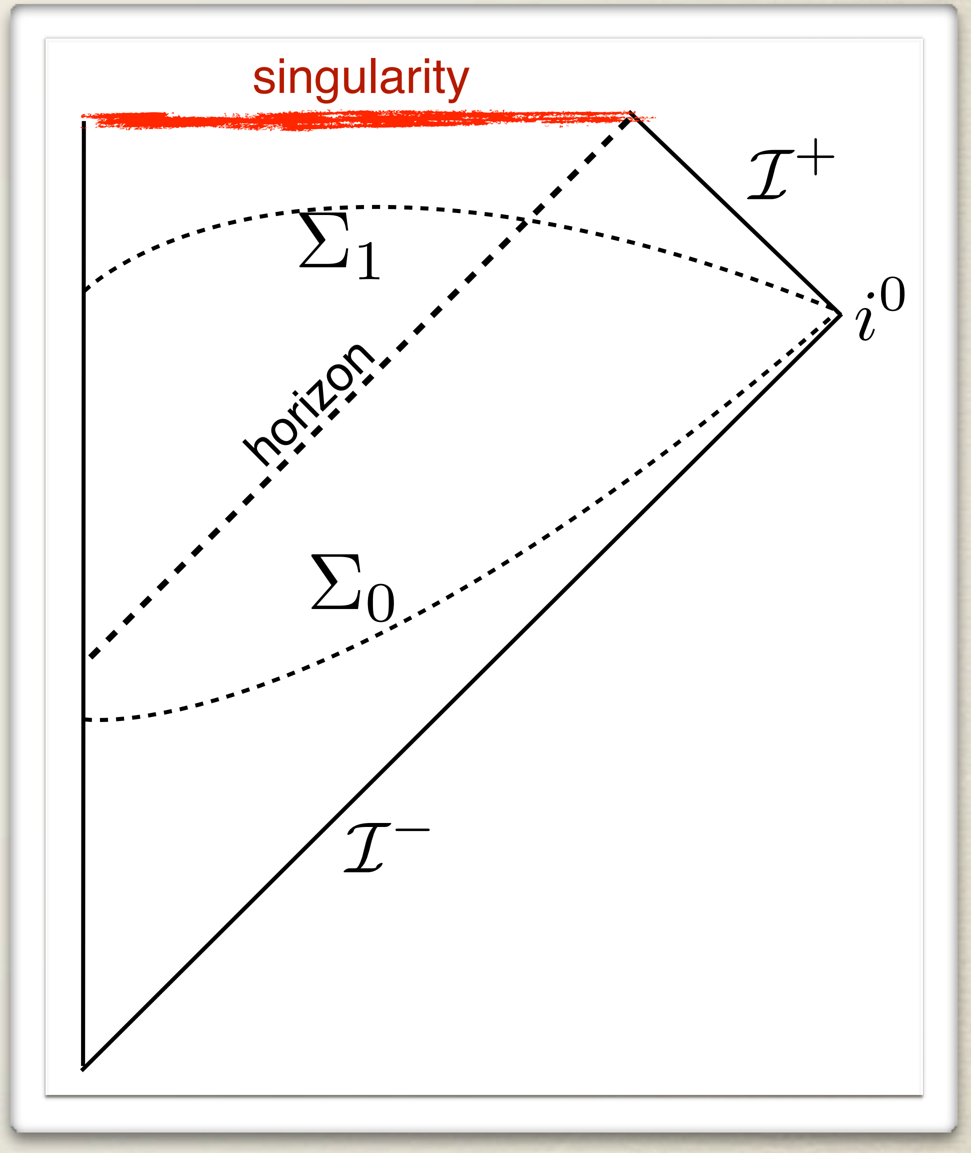
Evaporating Black Holes

- Classically, everything that falls in is lost forever.
- Hawking: Black holes aren't completely black.
- Hawking Radiation ⇒ black holes evaporate.



Evaporating Black Holes

* State on Σ_0 can be pure.
* Locality $\Rightarrow$ trace over sub-sector of Σ_1 inside horizon ➟ mixed state outside.
* Pure state evolving into mixed state violates unitarity.



Evaporating Black Holes

* 3 Possible Resolutions:
  * Non-Unitary evolution
    $$\rho \mapsto \$\rho$$
  * Remnants
    Black hole only evaporates to Planck scale
  * Non-locality

Non-Unitary Evolution?

$$\rho \mapsto \$\rho$$

* Information transfer requires energy.
* Information loss ⇔ energy loss.
* Virtual effects ⇒ Planck-scale energy non-conservation.
* Banks, Peskin, Susskind (1984):
  Nonunitarity ⇒ thermal bath at $T \sim M_P$

Remnants?

* Long lived Planck-scale remnant: if remnant decays and information gets out, takes

$$t_{\rm decay} = \frac{S^2}{M_P}$$

* Form black holes from arbitrarily many initial states, so arbitrarily many remnant species $\Rightarrow$ arbitrarily large production cross-section.

Non-Locality?

- Physical observables must be gauge invariant.
- In gravity, this means observables must be diffeomorphism invariant.
- There are no diffeomorphism invariant local observables in gravity. (Torre 1993)

$$\delta\mathcal{O}(x) = \epsilon^{\mu}\nabla_{\mu}\mathcal{O}(x)$$

Holographic Principle

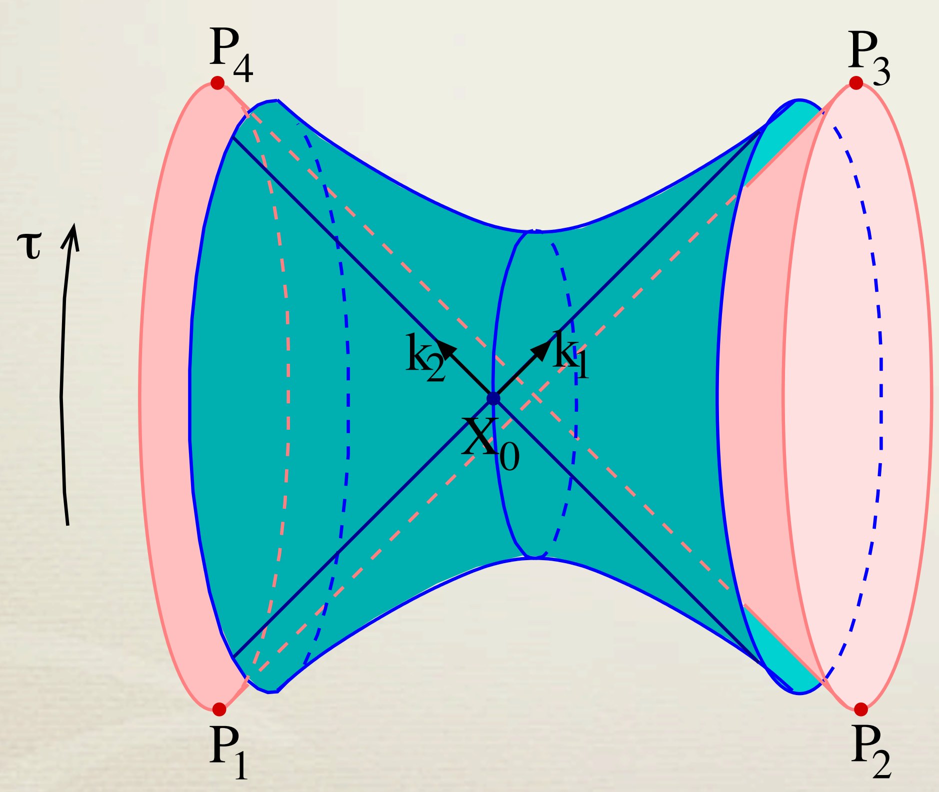
* Hints that locality should be given up.
  * BH Entropy grows with bounding area, not volume.
  * Bousso (1999): trying to access too many states in a fixed volume $\rightarrow$ black hole formation.
* Look for a non-local formulation of Quantum Gravity.
* Area law $\Rightarrow$ should look for a theory in one fewer dimensions.

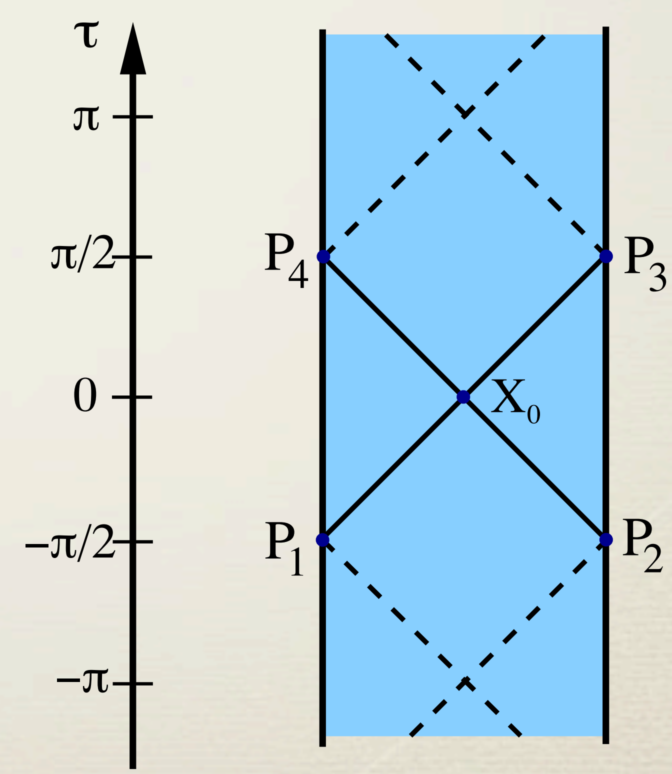
AdS/CFT

* Quantum Gravity in asymptotically Anti-de Sitter space (AdS) is conjectured to be dual to a Conformal Field Theory (CFT) living on the boundary. (Maldacena 1997)
* Simplest example: stack of N D3-branes. As $N \to \infty$, geometry back-reacts, near-horizon geometry becomes $AdS_5 \times S^5$, with SUGRA as LEFT. World-volume LEFT of branes is $\mathcal{N} = 4$ SYM with gauge group SU(N).
* Example of open/closed string duality.

AdS Geometry

- $ds^2 = \frac{R^2}{\cos^2 \rho} \left(-d\tau^2 + d\rho^2 + \sin^2 \rho \, d\Omega^2_{d-1}\right)$
- Universal cover of hyperboloid of radius R in $M_{2,d}$.
- Asymptotic Boundary $S^{d-1} \times \mathbb{R}$.





AdS/CFT Dictionary

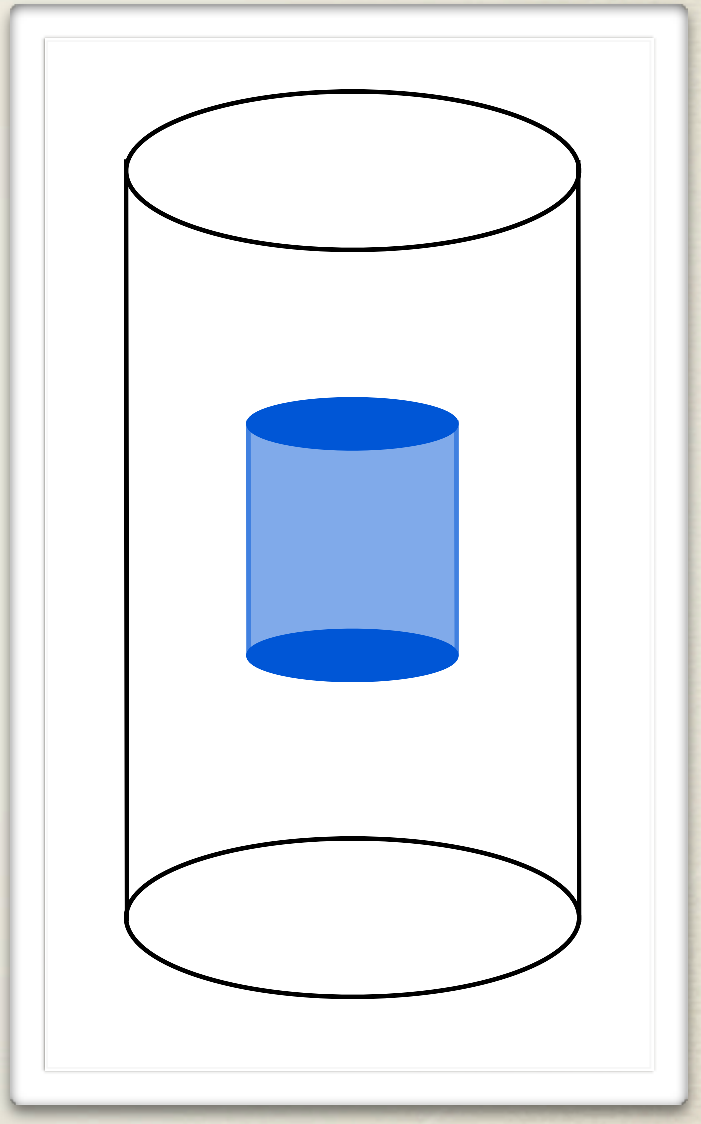
* Gubser, Klebanov, Polyakov; Witten (1998): Boundary conditions on fields in AdS ⇔ operator insertions & VEVs in dual CFT.
  * Fields in AdS have normalizable & non-normalizable modes:
  $$\phi \sim \cos^{2h_-} \rho \ \alpha(\tau, \Omega) + \cdots + \cos^{2h_+} \rho \ \beta(\tau, \Omega)$$
  * Non-normalizable mode ⇔ operator insertion:
  $$\mathcal{L}_{\rm CFT} \mapsto \mathcal{L}_{\rm CFT} + \alpha_\phi \mathcal{O}_\phi$$
  * Normalizable mode ⇔ operator expectation value:
  $$\langle \mathcal{O}_\phi \rangle = \beta_\phi$$

Physical Interpretation

* AdS/CFT is a geometrization of the Renormalization Group.
    * Radial direction ↔ Energy Scale
* Locality in Energy leads to coarse locality in radius (up to corrections of order R_{AdS}).
* How does fine-grained locality emerge?
    * Look for a test of fine-grained locality: scattering.

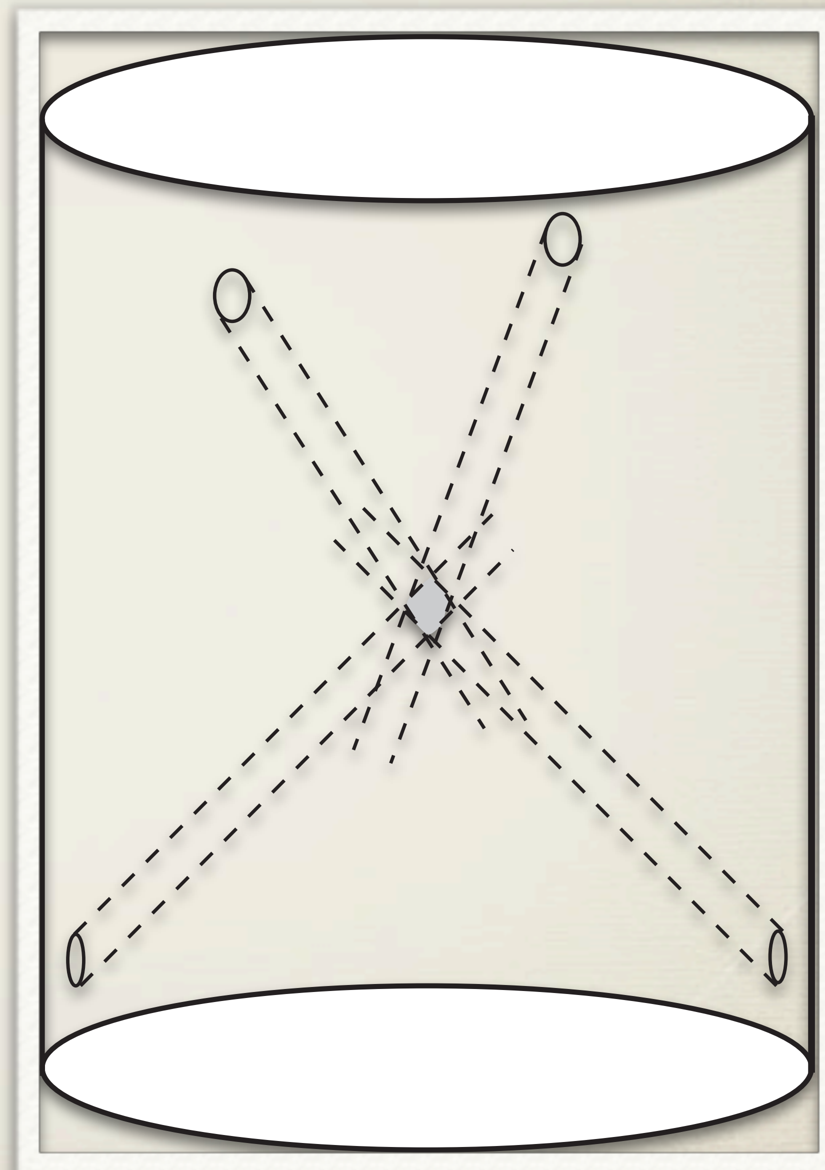
Large R Limit

- $t = R\tau \quad r = R\rho \quad R \to \infty$
- Approximately flat
  $ds^2 \to -dt^2 + dr^2 + r^2 d\Omega^2$
- Normalizable frequencies
  $\omega_{nl} \to \omega R$
- Normalizable wavefunctions
  $$\phi_{nl\vec{m}} \to \frac{\sqrt{2\omega^{d-1}}}{(\omega r)^{\frac{d}{2}-1}} J_{l+\frac{d}{2}-1}(\omega r) Y_{l\vec{m}}(\Omega)$$



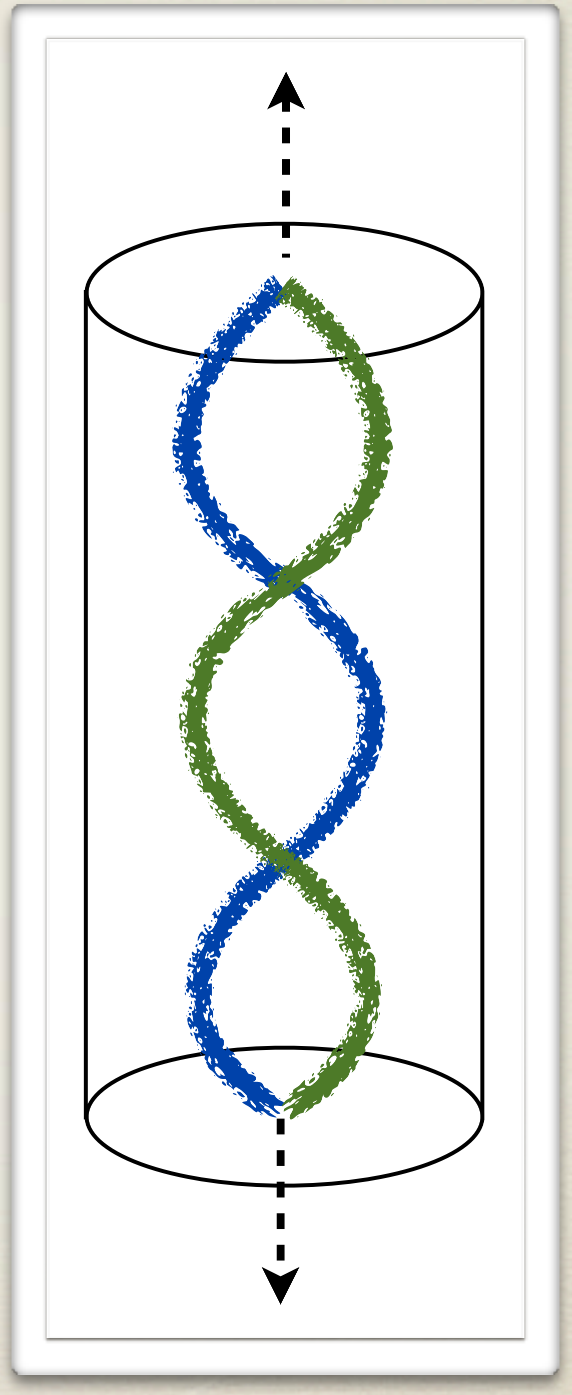
Scattering in the Flat Region

* Scattering in the flat region should approximate local physics in our universe.
* Need to construct wavepackets to localize scattering to a single flat region of AdS.



Multiple Scattering

- Free fields in AdS are periodic.
- Purely normalizable states will interact infinitely many times.
- Can't isolate contribution from one scattering experiment.



Interactions Near the Boundary

- Boundary sources → infinite particle production near the boundary.
- Single particle states not well defined when boundary sources turned on.

$$N = \int_{t_0} d^d\vec{x}\sqrt{-g}(\phi^* \overleftrightarrow{\partial_t} \phi) = \infty$$

- Difficult to isolate scattering in flat region from scattering near the boundary.
- Sources should be compact and non-overlapping to avoid infinite interactions near the boundary and normalize states.

Boundary-Compact Wavepackets

* Construct using Bulk-Boundary Propagator.

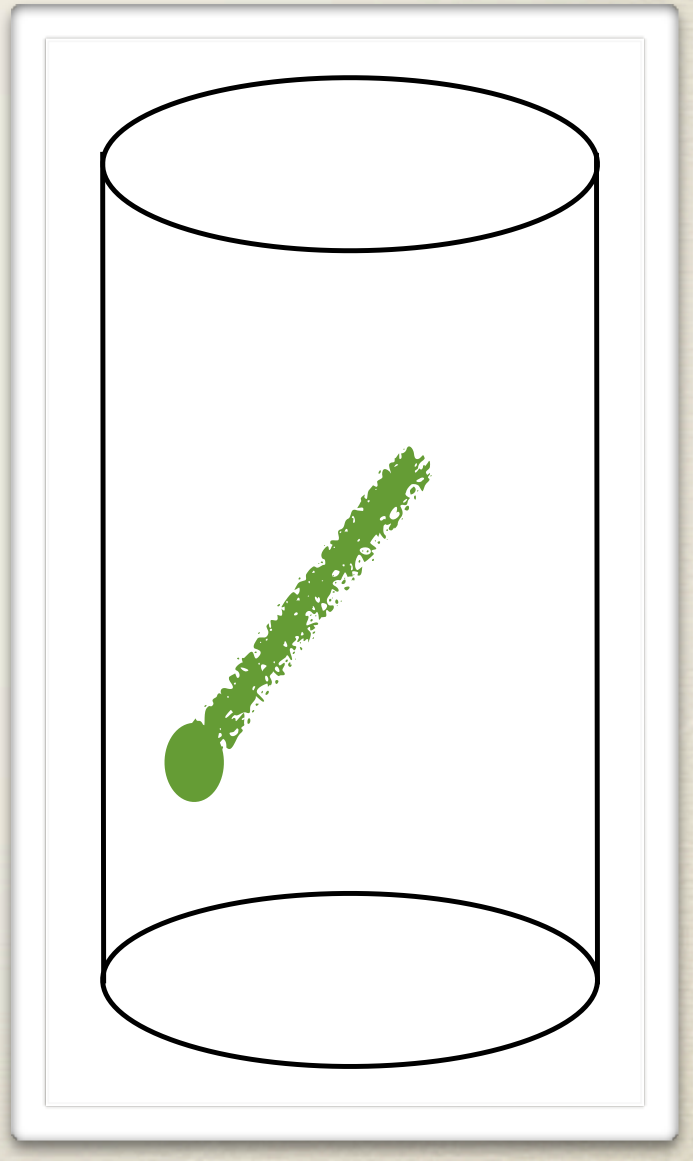
$$\phi_f(x) = \int db f(b) G_{B\partial}(b,x)$$

* Compactly supported sources

$$f(b) = L\left(\frac{\tau-\tau_0}{\Delta\tau}\right) L\left(\frac{\theta}{\Delta\theta}\right) e^{-i\omega R(\tau-\tau_0)}$$

of size $\Delta\tau$, $\Delta\theta$.

* $\frac{1}{\omega R} \ll \Delta\tau, \Delta\theta \ll 1$
* Scatter when sources turned off.



Wavepackets in the Scattering Region

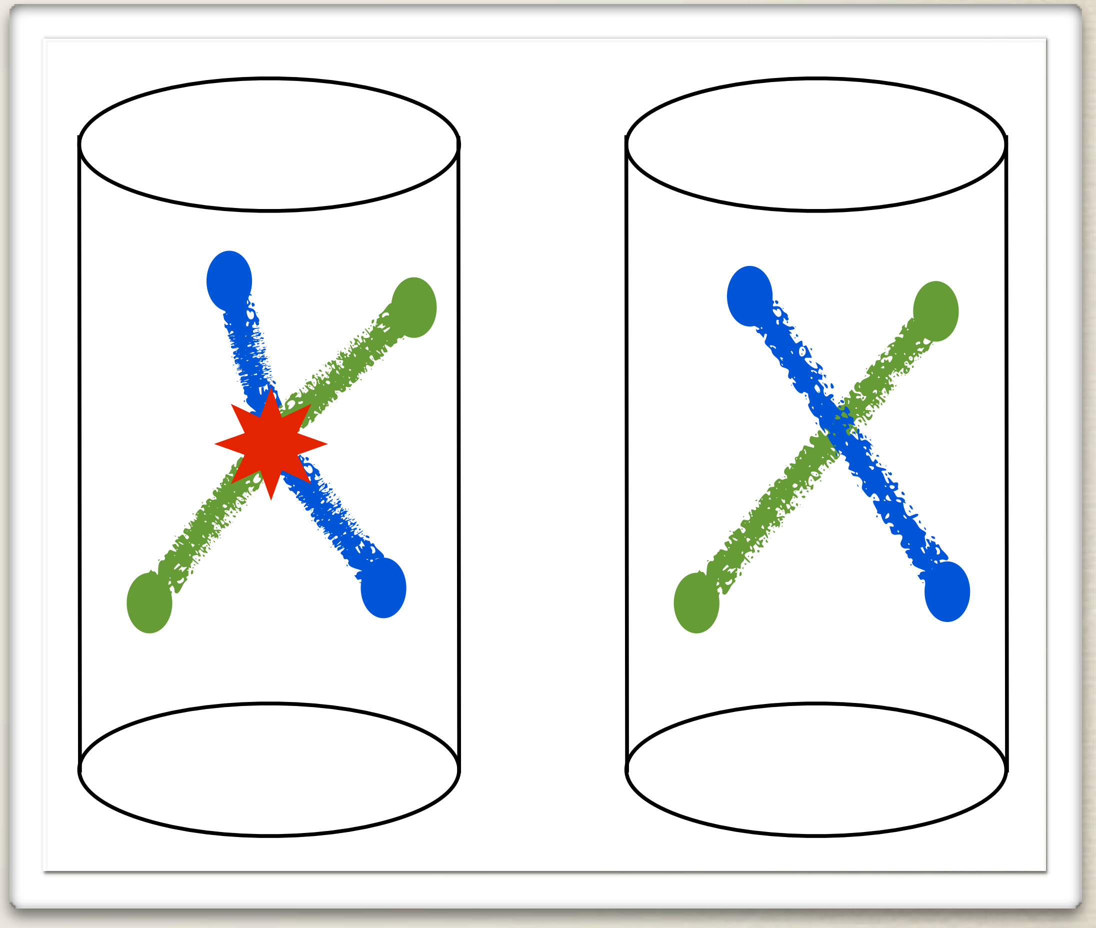
- Near the center of AdS,

$$\phi_f(x) \approx \phi_f(0)\frac{\tilde{L}_{d-1}(x_\perp \omega \Delta\theta)}{\tilde{L}_{d-1}(0)} L\left(\frac{u}{\Delta t}\right) e^{-i\omega u} \, .$$

- Longitudinal width $\Delta t \sim R\Delta\tau$.
- Transverse width $\Delta x_\perp \sim 1/(\omega\Delta\theta)$.
- Well localized for $1/\omega \ll \Delta t, \Delta x_\perp \ll R$, equivalent to earlier requirement.

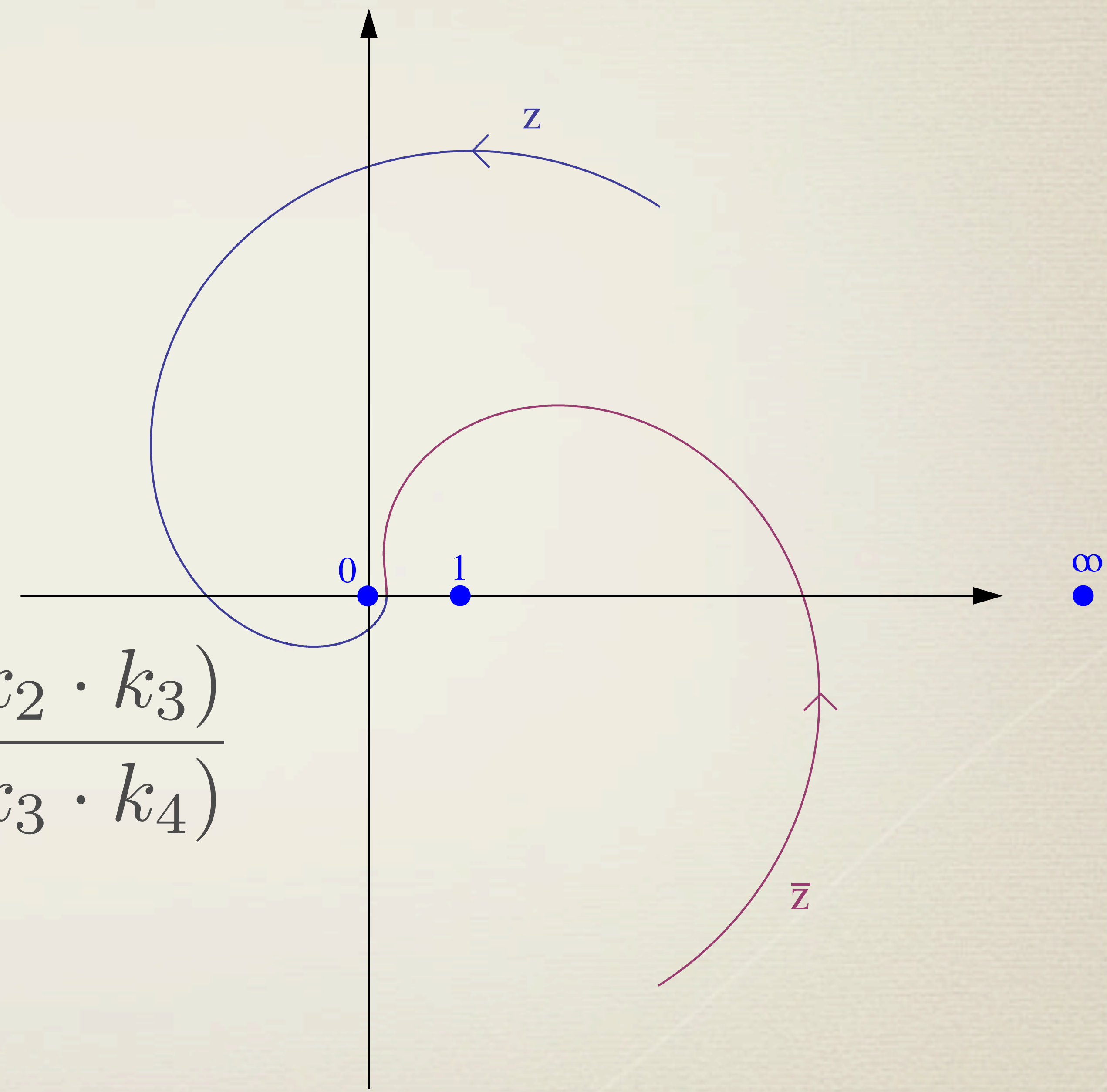
Singularity Structure

* Signal of interaction from intersecting wavepackets: *local bulk physics!*



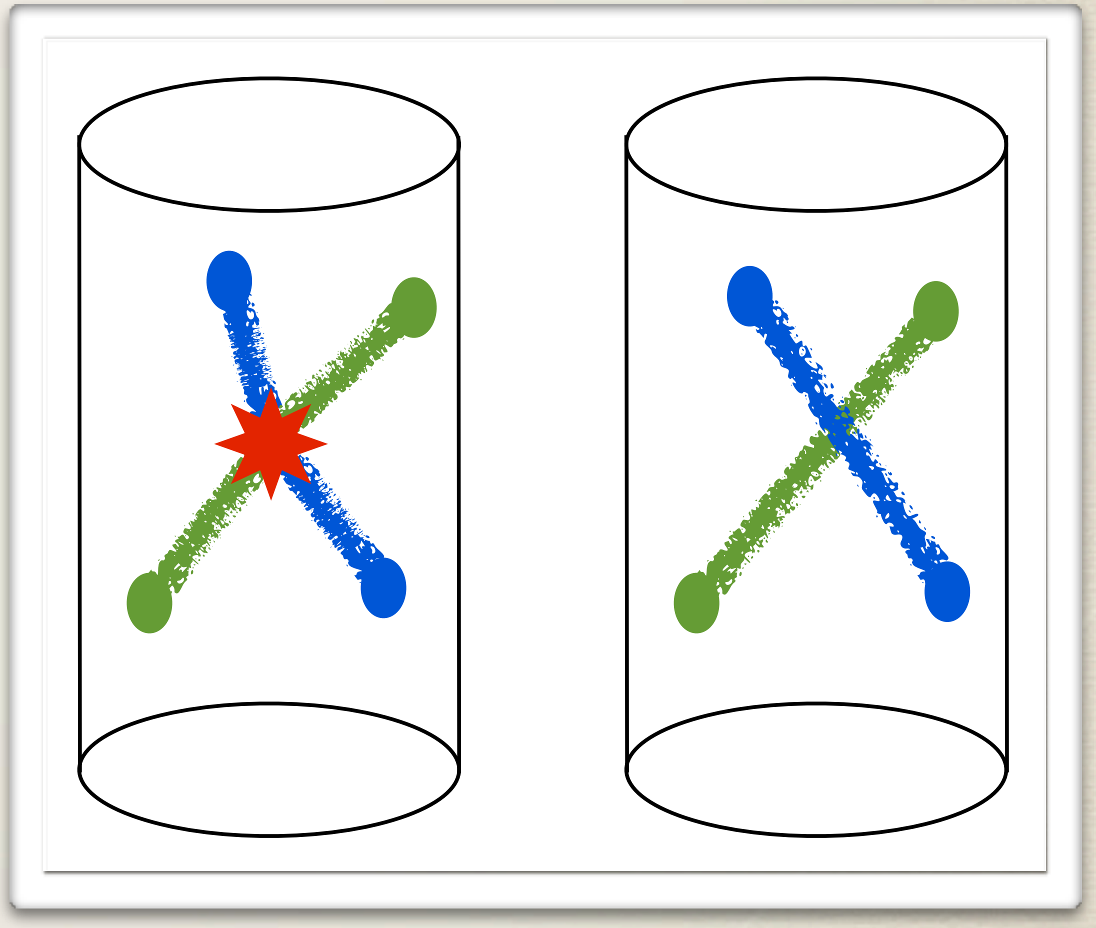
Analytic Continuation

- Singularity only present in physical Lorentzian continuation.
- $$z\bar{z} = \frac{(k_1 \cdot k_3)(k_2 \cdot k_4)}{(k_1 \cdot k_2)(k_3 \cdot k_4)}$$
  $$(1-z)(1-\bar{z}) = \frac{(k_1 \cdot k_4)(k_2 \cdot k_3)}{(k_1 \cdot k_2)(k_3 \cdot k_4)}$$
- $z \longrightarrow \bar{z}$



Singularity Structure

* Signal of interaction from intersecting wavepackets: *local bulk physics!*
* $z = \sigma e^{-\rho}\ ,\ \bar{z} = \sigma e^{\rho}$
* $\mathcal{A}(z, \bar{z}) \approx g^2 R^{5-d-2j} \dfrac{F(\sigma)}{(-\rho^2)^{\beta}}$
* $\beta = \Delta_1 + \Delta_2 + j - 5/2$



Momentum Conservation

* Flat space S-Matrix conserves momentum:

$$S = 1 + i(2\pi)^D \delta^D \left(\sum k_i\right) \mathcal{T}(s,t)$$

* Momentum conserving δ-function must emerge from CFT amplitude in appropriate limit.

* Delta function does emerge from form of boundary compact sources and singularity structure:

$$\lim_{R\to\infty} \int d\nu \frac{R^n e^{-i\nu}}{(R^2\kappa^2 - (\nu + i\epsilon)^2)^\beta} \propto \delta^n(\vec{\kappa})$$

The S-Matrix

* Determine flat space scattering amplitude from residue of CFT singularity:

$$\mathcal{A}(z,\bar{z}) \to g^2 R^{5-d-2j} \frac{\mathcal{F}(\sigma)}{(-\rho^2)^{\beta}}$$

$$i\mathcal{T}(s,t) = \mathcal{K} g^2 s^{j-1} \left(\frac{-t}{s}\right)^{j-2} \left(\frac{-u}{s}\right)^{3-j-\Delta_1-\Delta_2} \mathcal{F}\left(\frac{-s}{t}\right)$$

Examples

* Compute CFT correlators using AdS/CFT dictionary (D'Hoker *et al.*, 1999), read off scattering amplitudes.

* Scalar exchange:

$$\mathcal{F}(\sigma) \propto \sigma(1-\sigma)^{\Delta_1+\Delta_2-3} \quad \rightarrow \quad \mathcal{T}(s,t) = \frac{g^2}{-t}$$

* Graviton exchange:

$$\mathcal{F}(\sigma) \propto \frac{(1-\sigma)^8}{\sigma} \quad \rightarrow \quad \mathcal{T}(s,t) = 8\pi G_5 \frac{s^2+ts}{-t}$$

Conclusions

* AdS/CFT ⇒ BH Evaporation is Unitary & non-local.
* CFT singularity signature of fine-grained locality.
  * AdS/CFT really is holographic in the large N limit.
  * Locality should only be approximate, should go away when stringy corrections are included.

Thank You