The Geometry of Higher Spins Reconsidered

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Outline

1. Motivation
2. Pure Gravity in AdS_3
3. Higher Spin Gravity in AdS_3
4. More General Backgrounds
5. Conclusions

Motivation

- Higher Spin Holography with Lifshitz Scaling
- Role of Geometry in Higher Spin Theories
- Understanding the Mechanisms of Holography

3D Gravity

$$S = \frac{1}{16\pi G_N} \int_{\mathcal{M}} \sqrt{-g}\,(R+2)$$
$$= \frac{k}{4\pi}\left(S_{\mathrm{CS}}[A] - S_{\mathrm{CS}}\left[\overline{A}\right]\right)$$

$$S_{\mathrm{CS}}[A] = \int_{\mathcal{M}} \mathrm{tr}\left(\mathrm{A} \wedge \mathrm{dA} - \frac{2}{3}\mathrm{A}^3\right)$$

$$e = \frac{A - \overline{A}}{2} \qquad \omega = \frac{A + \overline{A}}{2}$$

- Gravity in Asymptotically AdS$_3$ can be formulated as $\mathfrak{sl}_2(\mathbb{R}) \times \mathfrak{sl}_2(\mathbb{R})$ Chern-Simons theory with level $k = \frac{1}{4G_N}$
- Denote $\mathfrak{sl}_2$ generators $L_0, L_{\pm 1}$

Canonical Analysis

- Impose Asymptotic AdS boundary conditions (Brown-Henneaux)

$$A = g^{-1}dg + g^{-1}ag \quad \overline{A} = gdg^{-1} + g\bar{a}g^{-1} \quad g = e^{\rho L_0}$$

$$a = \left(L_1 + \mathcal{L}(x^+)L_{-1}\right) dx^+ + o(1)$$
$$\bar{a} = \left(L_{-1} + \overline{\mathcal{L}}(x^-)L_1\right) dx^- + o(1)$$

$$ds^2 = d\rho^2 - \left(e^{2\rho} + e^{-2\rho}\mathcal{L}\overline{\mathcal{L}}\right) dx^+dx^- + \mathcal{L}^2 dx^{+2} + \overline{\mathcal{L}}^2 dx^{-2} + \cdots$$

- Asymptotic Symmetry Algebra is two copies of Viraosoro with $c_L = c_R = 6k$

Higher Spin Generalization

- Enlarge $\mathfrak{sl}_2$ to $\mathfrak{sl}_N$
- Choice of embedding $\mathfrak{sl}_2 \hookrightarrow \mathfrak{sl}_N$ determines other field content
- Spins of other fields given by weight under gravitational $\mathfrak{sl}_2$ action
- Typical choice: Principal embedding, integer spins $2, \ldots, N$.

$$g_{\mu\nu} = \frac{1}{2}\mathrm{tr}\left[\mathrm{e}_\mu \mathrm{e}_\nu\right]$$

$$\phi_{\mu\nu\rho} = \mathrm{tr}\left[\mathrm{e}_{(\mu}\mathrm{e}_\nu \mathrm{e}_{\rho)}\right]$$

$$\vdots$$

$\mathfrak{sl}_3$ Conventions

- $\mathfrak{sl}_2$ generators $L_0, L_{\pm 1}$
- Spin 3 generators $W_0, W_{\pm 1}, W_{\pm 2}$
- Commutators

$$[L_n, L_m] = (n - m)L_{n+m} \qquad [L_n, W_m] = (2n - m)W_{n+m}$$

$$[W_n, W_m] \propto L_{n+m}$$

- Traces

$$\mathrm{tr}\left(\mathrm{L_n L_m}\right) \propto \delta_{n,-m} \qquad \mathrm{tr}\left(\mathrm{W_n W_m}\right) \propto \delta_{n,-m}$$

$$\mathrm{tr}\left(\mathrm{L_n W_m}\right) = 0$$

AdS Boundary Conditions

$$A = g^{-1} dg + g^{-1} a g \qquad \overline{A} = g dg^{-1} + g \bar{a} g^{-1} \qquad g = e^{\rho L_0}$$

$$a = \left(L_1 + \mathcal{L}(x^+) L_{-1} + \mathcal{W}(x^+) W_{-2}\right) dx^+ + o(1)$$
$$\bar{a} = \left(L_{-1} + \overline{\mathcal{L}}(x^-) L_1 + \overline{\mathcal{W}}(x^-) W_2\right) dx^- + o(1)$$

- Asymptotic Symmetry Algebra: two copies of $\mathcal{W}_3$ with central charges $c_L = c_R = 6k$
- Vacuum: metric is AdS_3, spin-3 field is 0, invariant under $\mathfrak{sl}_3 \times \mathfrak{sl}_3$ symmetry

General Procedure

- Add boundary term to cancel variation of the action

$$S_{\mathrm{CT}} = -\frac{k}{4\pi}\int_{\partial\mathcal{M}} \mathrm{tr}\left(\mathrm{A}^2 - \overline{\mathrm{A}}^2\right)$$

- Split connection into background and fluctuations
- Impose consistent boundary conditions on fluctuations. In particular
  - Find closed set of boundary condition preserving gauge transformations
  - Require finite, conserved, integrable asymptotic charges
- Determine Asymptotic Symmetry Algebra by computing Poisson Brackets and quantizing

Examples

- Lobachevsky boundary conditions

$$\widehat{ds}^2 = d\rho^2 \pm dt^2 + \sinh^2 \rho d\phi^2$$

$\mathcal{W}_N^{(2)} \times \hat{\mathfrak{u}}(1)$ Asymptotic Symmetry Algebra

[1209.2860], [1211.4454]

- Null-Warped AdS_3 boundary conditions

$$\widehat{ds}^2 = d\rho^2 + e^{2\rho} dt d\phi + \frac{9}{4} e^{4\rho} d\phi^2$$

$W_3^{(2)}$ Asymptotic Symmetry Algebra

(Unpublished, with E Perlmutter and D Grumiller)

$z = 2$ Lifshitz Background

- Background connection

$$\hat{a} = L_1 dx + \frac{4}{9} W_2 dt$$
$$\hat{\bar{a}} = L_{-1} dx + W_{-2} dt$$

- Background metric

$$ds^2 = d\rho^2 - e^{4\rho} dt^2 + e^{2\rho} dx^2$$

- Non-trivial background spin-3 field

$$\phi_{xxt} = -\frac{5}{12} e^{4\rho} = \phi_{xtx} = \phi_{txx}$$

Higher Spin Fluctuations on Lifshitz Background

$$a^{(0)} = \left(4t\mathcal{W}L_0 - \mathcal{L}L_{-1} - \frac{16t^2}{9}\mathcal{W}W_2 + \frac{16t}{9}\mathcal{L}W_1 + \mathcal{W}W_{-2}\right) dx + o(1)$$

$$\bar{a}^{(0)} = \left(-\overline{\mathcal{L}}L_1 - 9t\overline{\mathcal{W}}L_0 + \overline{\mathcal{W}}W_2 + 4t\overline{\mathcal{L}}W_{-1} - 9t^2\overline{\mathcal{W}}W_{-2}\right) dx + o(1)$$

- Theory includes states with metrics that would not typically be called asymptotically Lifshitz
- Some of these non-Lifshitz states have metrics which are "asymptotically AdS"
- On-shell solutions have explicit t-dependence

ASA and Symmetries of the Vacuum

- Asymptotic charges are $\mathcal{L}(x), \mathcal{W}(x), \overline{\mathcal{L}}(x), \overline{\mathcal{W}}(x)$, t-independent
- Asymptotic Symmetry Algebra: two copies of $\mathcal{W}_3$ with central charges $c_L = c_R = 6k$
- Background is invariant under $\mathfrak{sl}_3 \times \mathfrak{sl}_3$ symmetry and is thus dual to the CFT vacuum
- No other states are invariant under the full wedge algebra

Interpretations and Duality

- Perturbative spectrum is equivalent to that of spin-3 gravity in AdS_3: duality?
- Unclear if duality holds also at non-perturbative level
- Polynomial x and t dependence of boundary condition preserving gauge transformations seems to imply boundary should be non-compact

Conclusions

- Also looser boundary conditions for Lifshitz background, possibly related to $\mathcal{W}_3^{(2)}$
- Are there other backgrounds which are also related by duality?
- Do such dualities exist for more general higher spin theories?
- Metric and higher spin fields need to be placed on equal footing—all massless degrees of freedom
- To really talk about geometry, we should use a local probe (e.g. scalar field in $HS(\lambda)$ theory)