$W_N^{(2)}$ Gravity

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3d Gravity, First Order Formalism

- Dreibein e^a_μ and spin connection ω^{ab}_μ independent variables
- 3D trick: use invariant antisymmetric rank 3 symbol ϵ^{abc} to construct ω^a_μ
- Action becomes a difference of two $\mathfrak{sl}_2$ Chern-Simons theories

$$I = \frac{k}{4\pi}\left(\int \mathrm{tr}\left[\mathrm{A}\wedge \mathrm{dA} + \frac{2}{3}\mathrm{A}^3\right] - \int \mathrm{tr}\left[\bar{\mathrm{A}}\wedge \mathrm{d}\bar{\mathrm{A}} + \frac{2}{3}\bar{\mathrm{A}}^3\right]\right)$$

  where $A = \frac{1}{2}(\omega + e), \bar{A} = \frac{1}{2}(\omega - e),\ k = \frac{1}{4G_N}$
- $g_{\mu\nu} = \frac{1}{2}\mathrm{tr}\left[(\mathrm{A}-\bar{\mathrm{A}})_\mu(\mathrm{A}-\bar{\mathrm{A}})_\nu\right]$

3d Higher Spin Gravity

- Enlarge $\mathfrak{sl}_2$ to $\mathfrak{sl}_N$ (or other gauge group containing $\mathfrak{sl}_2$)
- Choice of embedding $\mathfrak{sl}_2 \hookrightarrow \mathfrak{sl}_N$ determines other field content
- Spins of other field content given by weight under gravitational $\mathfrak{sl}_2$ action
- Principal embedding: integer spins $2, \ldots, N$
- Next-to-principal embedding: integer spins $1, \ldots, N-1$ and two spin $\frac{N}{2}$ bosons

Next-to-Principal Higher Spin Gravity

- Asymptotic Symmetry Algebra is Feigin-Semikhatov Algebra $W_N^{(2)}$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}$$
$$[J_n, J_m] = \kappa n \delta_{n+m,0}$$
$$[J_n, L_m] = nJ_{n+m} \qquad \left[J_n, G_m^{\pm}\right] = \pm G_{n+m}^{\pm}$$
$$\left[G_n^{+}, G_m^{-}\right] = \lambda f(n)\delta_{n+m,0} + \dots$$
$$\left[L_n, G_m^{\pm}\right] = \left(n(\frac{N}{2} - 1) - m\right) G_{n+m}^{\pm}$$

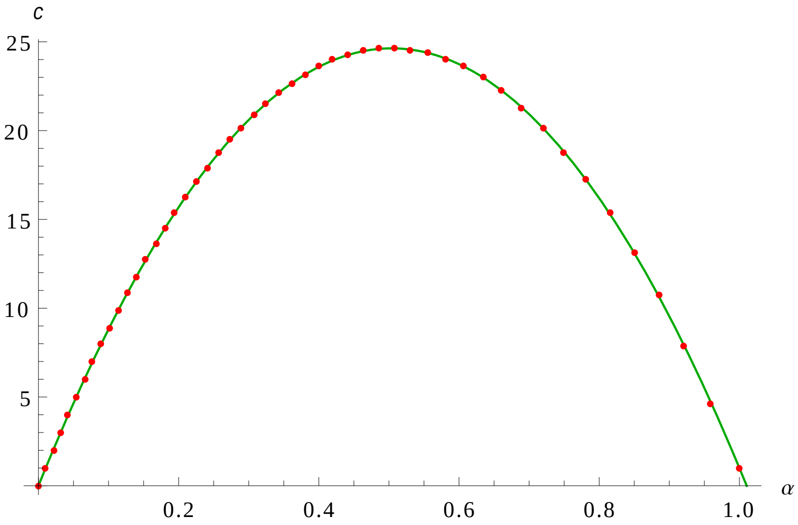
- $G^{\pm}$ central term

$$\lambda = \prod_{m=1}^{N-1} (m(N+k-1) - 1)$$

- Norm of G^{+} and G^{-} descendents have opposite sign. Unitarity $\Rightarrow \lambda = 0$

Unitary Represenations of $W_{100}^{(2)}$

Allowed values of central charge c for $N = 100$



Unitary Representations of $W_N^{(2)}$

- Demaning unitarity $\to$ Newton's constant automatically quantized
- Critical values $\alpha = \frac{\hat{N}}{N-\hat{N}-1}$ where $\hat{N} \in \mathbb{N}, \hat{N} \leq \frac{N-1}{2}$
- Allowed values of central charge (let $m = N - 2\hat{N} - 1$)

$$c(\hat{N}, m) - 1 = (\hat{N} - 1)\left(1 - \frac{\hat{N}(\hat{N}+1)}{(m+\hat{N})(m+\hat{N}+1)}\right)$$

- Exactly values of central charge $W_{\hat{N}}$ minimal models, up to shift by 1 due to $\hat{\mathfrak{u}}(1)$ current algebra
- Small α quantum regime with $c \sim \mathcal{O}(1)$
- Intermediate α semiclassical regime with $c \sim \mathcal{O}(\frac{N}{4})$
- $\alpha \sim \mathcal{O}(1)$ dual quantum regime with $c \sim \mathcal{O}(1)$