Holography and the Gravitational S-Matrix

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FWF Der Wissenschaftsfonds.

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Outline

1. Motivation
2. The Gravitational S-Matrix
3. AdS/CFT and the Flat-Space S-Matrix
4. Conclusions

Motivation

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- Black hole evaporation and unitarity

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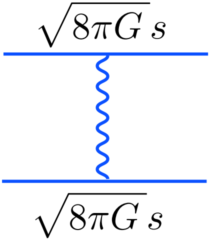
- Strong Quantum Gravity – lessons for cosmology?
- Implications of AdS/CFT for gravity in flat-space

The Gravitational S-Matrix



Born Amplitude

$$S = 1 + i\delta^D \left(\sum p_i \right) (2\pi)^D T(s,t)$$

$$T_{\mathrm{Born}} = -8\pi G \frac{s^2 + ts}{t}$$

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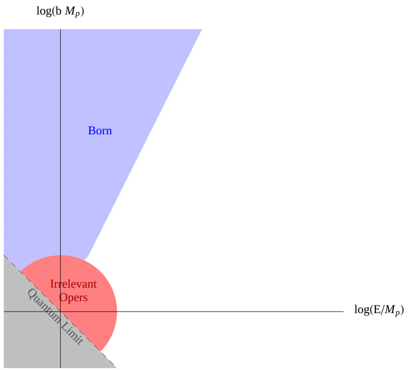

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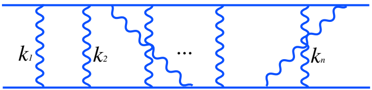

Individual graviton momenta k_i small

Eikonal Amplitude

- Eikonal phase $\chi(b, s) = \frac{1}{2s} \int \frac{d^{D-2}q_\perp}{(2\pi)^{D-2}} e^{i\vec{q}_\perp \cdot \vec{b}} T_{\text{Born}}\left(s, -\vec{q}_\perp^{\,2}\right)$

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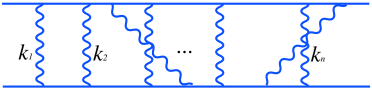

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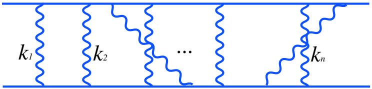

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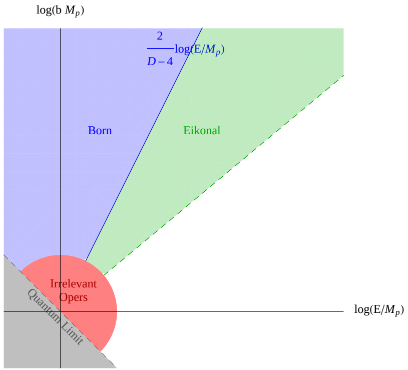

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- Crossover occurs for $\chi \sim 1 \Rightarrow b \sim E^{\frac{2}{D-4}}$

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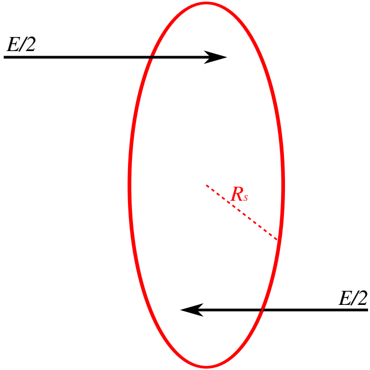

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- Form black hole when $b < R_S(E) = E^{\frac{1}{D-3}}$

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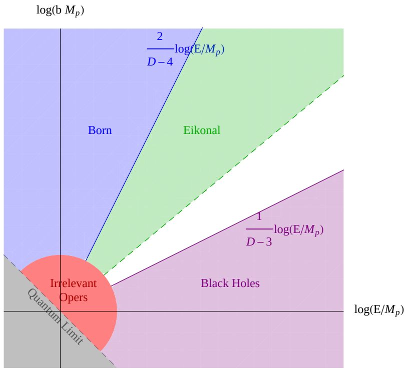

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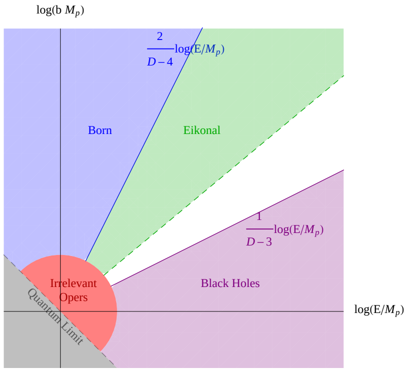

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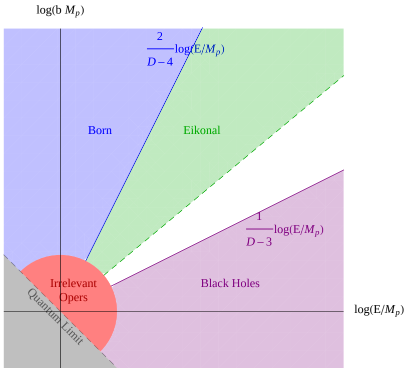

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- Typical energy of Hawking quanta $\langle E \rangle \sim \frac{1}{R_S} \sim E^{\frac{-1}{D-3}}$

The AdS/CFT Correspondence

Gravity in AdS

- Gravitational theory in Asymptotically AdS
  $\frac{R^2}{\cos^2\rho}\left(-d\tau^2 + d\rho^2 + \sin^2\rho d\Omega_{D-2}^2\right)$

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- Masses of fields $m^2 \propto R^{-2}$

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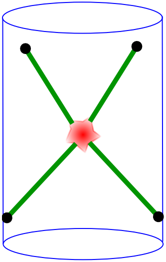
S-Matrix from AdS/CFT



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- Scatter wavepackets in a single, flat region [Polchinski, 99; Susskind, 99]

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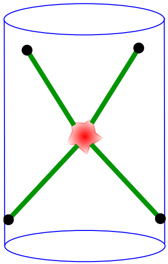

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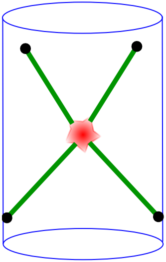

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Solution

- Use boundary-compact sources: compact in both S^{D-2} and time

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- For $R \gg 1$, $N_{\max} \gg \langle N \rangle$

Overcounting

Problem

Reversing this logic, should be able to localize $N_{\mathrm{max}} \sim (\omega R)^{D-2}$ particles within a single R_{AdS}-sized region. R-dependence agrees with the holographic bound, but for $\omega \gg 1$, this grossly violates usual graviatational intuition.

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Solution?

Power law tails–∂-cpct wavepackets have power law tails $\psi \sim \frac{1}{(\omega\delta\theta x_\perp)^\Delta}$. More of the norm is outside of the flat region than would like.

Conclusions

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Difficulties

- BH S-Matrix requires strong coupling CFT calculation
- Too many states corresponding to localized excitations – solution from power law tails?

References

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