The Noncommutative Standard Model: Phenomenology & Beyond $\mathcal{O}(\theta)$

Thorsten Ohl

Ana Alboteanu, Johannes Rauh, Reinhold Rückl, Christian Speckner, Jörg Zeiner
Institute for Theoretical Physics, University of Würzburg

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- most motivation can be skipped for this audience: preaching to the choir ...
  - still
    - possible low energy manifestation of string theory [Seiberg/Witten] (how low is low?)
    - surprisingly hard to detect (see below) deformation of standard model particle physics
  - Framework: canonical noncommutativity

$$[\hat{x}_\mu, \hat{x}_\nu] = \mathrm{i}\theta_{\mu\nu} = \mathrm{i}\frac{C_{\mu\nu}}{\Lambda_{\text{NC}}^2} = \mathrm{i}\frac{1}{\Lambda_{\text{NC}}^2}\begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{pmatrix}$$

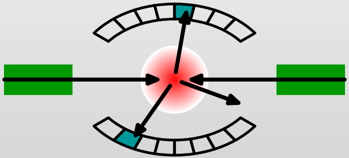
corresponding to a minimal area in the $e_\mu \wedge e_\nu$-plane

$$a_{\text{NC}} = l_{\text{NC}}^2 = 1/\Lambda_{\text{NC}}^2$$

- complicated enough for me ...

typical collider experiment:

- accelerators prepare initial state,
- that is transformed by the interaction under study,
- a detector registers the resulting final state:



∴ experiments do not study the coordinates $\hat{x}_\mu$ directly, but functions on them: asymptotic states and fields

∴ results of observations encoded in effective lagrangians as products of functions:

$$\mathcal{L}_{\text{eff.}}(x) = \cdots + g_2 \bar\psi(x)\gamma_\mu(1-\gamma_5)\psi'(x)W^\mu(x) + g_3 \sum_{a,b,c} f_{abc} \frac{\partial A^a_\nu}{\partial x^\mu}(x) A^{b,\mu}(x) A^{c,\nu}(x) + \cdots$$

- ☺ simpler, but equivalent realization: replace all point products of functions of noncommuting coordinates

$$(f\cdot g)(\hat{x}) = f(\hat{x})g(\hat{x})$$

by Moyal-Weyl-$*$-products of functions of commuting coordinates:

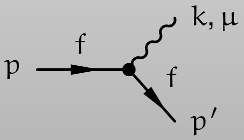
$$(f*g)(x) = f(x)e^{\frac{i}{2}\overleftarrow{\partial^\mu}\theta_{\mu\nu}\overrightarrow{\partial^\nu}}g(x) = f(x)g(x)+\frac{i}{2}\theta_{\mu\nu}\frac{\partial f(x)}{\partial x_\mu}\frac{\partial g(x)}{\partial x_\nu}+\mathcal{O}(\theta^2)$$

- then $(x_\mu * x_\nu)(x) = x_\mu x_\nu + \frac{i}{2}\theta_{\mu\nu}$ and in particular

$$[x_\mu \overset{*}{,} x_\nu](x) = (x_\mu * x_\nu)(x) - (x_\nu * x_\mu)(x) = \mathrm{i}\theta_{\mu\nu}$$

NB: higher orders in $\theta_{\mu\nu}$ required for associativity: $(f*g)*h = f*(g*h)$

- Nonlocal due to Moyal-phases, e. g. for “naive” NCQED



$$= \mathrm{i}Q_f e\gamma_\mu \to \mathrm{i}Q_f e\gamma_\mu \mathrm{e}^{\frac{i}{2}p_\mu\theta^{\mu\nu}p'_\nu}$$

most obvious noncommutative extension of gauge theories:

$$\psi \to \psi' = e^{ig\eta *}\psi = \psi + ig\eta * \psi + \frac{(ig)^2}{2!}\eta * \eta * \psi + \mathcal{O}(\eta^3)$$

$$\begin{aligned} A_\mu \to A'_\mu &= e^{ig\eta *} A_\mu e^{-ig\eta *} + \frac{i}{g} e^{ig\eta *}\left(\partial_\mu e^{-ig\eta *}\right) \\ &= A_\mu + ig[\eta \overset{*}{,} A_\mu] + \partial_\mu \eta + ig[\eta \overset{*}{,} \partial_\mu \eta] + \mathcal{O}(\eta^2) \end{aligned}$$

no difference of abelian and non abelian couplings:

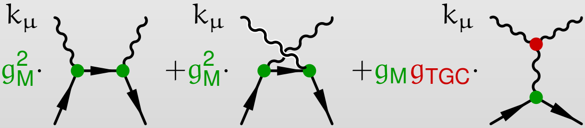
- $\therefore$ $A'_\mu \neq A_\mu + \partial_\mu \eta$ even if $[\eta, A_\mu] = 0$, because $[\eta \overset{*}{,} A_\mu] \neq 0$
- $\therefore$ $F_{\mu\nu} \neq \partial_\mu A_\nu - \partial_\nu A_\mu$ even if $[A_\mu, A_\nu] = 0$, because $[A_\mu \overset{*}{,} A_\nu] \neq 0$

marquee signature:

self couplings of neutral gauge bosons γ and Z
in leading order (not suppressed by loop factors)!

∴ form and strength of couplings among gauge bosons determined by couplings to matter!

- only one independent coupling for each non abelian gauge theory
- also in noncommutative extensions of QED:



$$\stackrel{!}{=} 0 \Rightarrow \boxed{g_M = g_{TGC}}$$

- incompatible with hypercharge quantum numbers in the $SU(3)_C \times SU(2)_T \times U(1)_Y$ standard model:

$$Y(L_e, e_R, \nu_{e,R}, L_{u,d}, u_R, d_R) = (-1, -2, 0, 1/3, 4/3, -2/3)$$

- also: $SU(N)$ can not be realized, only $U(N)$ closes:

$$[A_\mu \stackrel{*}{,} A_\nu]_- = \frac{1}{2}[A_\mu^a \stackrel{*}{,} A_\nu^b]_+ [T^a, T^b]_- + \frac{1}{2}[A_\mu^a \stackrel{*}{,} A_\nu^b]_- [T^a, T^b]_+$$

introduce noncommutative objects as nonlinear functions of commutative objects (and derivatives)

$$\hat{A}_\mu(x) = \hat{A}_\mu(A_{\nu_1}(x), \partial_{\nu_1} A_{\nu_2}(x), \partial_{\nu_1}\partial_{\nu_2} A_{\nu_3}(x), \ldots, \theta)$$
$$\hat{\eta}(x) = \hat{\eta}(\eta(x), \partial_{\nu_1}\eta(x), \ldots, A_{\nu_1}(x), \partial_{\nu_1} A_{\nu_2}(x), \ldots, \theta)$$
$$\hat{\psi}(x) = \hat{\psi}(\psi(x), \partial_{\nu_1}\psi(x), \ldots, A_{\nu_1}(x), \partial_{\nu_1} A_{\nu_2}(x), \ldots, \theta)$$

realize noncommutative gauge transformations as commutative gauge transformations:

$$\hat{A} \to \hat{A}'(A, \theta) = e^{ig\hat{\eta}*} \hat{A}_\mu(A, \theta) e^{-ig\hat{\eta}*} + \frac{i}{g} e^{ig\hat{\eta}*} \left(\partial_\mu e^{-ig\hat{\eta}*}\right) \stackrel{!}{=} \hat{A}(A', \theta)$$

$$\hat{\psi} \to \hat{\psi}'(\psi, A, \theta) = e^{ig\hat{\eta}*}\hat{\psi} \stackrel{!}{=} \hat{\psi}(\psi', A', \theta)$$

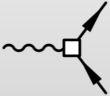
solution (not unique) as power series in θ:

$$\hat{A}_\mu(x) = A_\mu(x) + \frac{1}{4}\theta^{\rho\sigma}\left[A_\sigma(x), \partial_\rho A_\mu(x) + F_{\rho\mu}(x)\right]_+ + \mathcal{O}(\theta^2)$$
$$\hat{\psi}(x) = \psi(x) + \frac{1}{2}\theta^{\rho\sigma} A_\sigma(x)\partial_\rho\psi(x) + \frac{i}{8}\theta^{\rho\sigma}\left[A_\rho(x), A_\sigma(x)\right]_- \psi(x) + \mathcal{O}(\theta^2)$$
$$\hat{\eta}(x) = \eta(x) + \frac{1}{4}\theta^{\rho\sigma}\left[A_\sigma(x), \partial_\rho\eta(x)\right]_+ + \mathcal{O}(\theta^2)$$

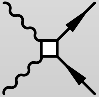
New interaction vertices among gauge and matter fields from expanding Moyal-Weyl-$*$-products and Seiberg-Witten-Maps

$$g(\hat{\bar{\psi}} * \hat{\not{A}} * \hat{\psi})(x) = g\bar{\psi}(x)\not{A}(x)\psi(x) + \mathcal{O}(\theta)$$

e. g. at $\mathcal{O}(\theta)$ with all momenta outgoing



$$= ig \cdot \frac{i}{2}\left[(k\theta)_\mu \not{p} + (\theta p)_\mu \not{k} - (k\theta p)\gamma_\mu\right]$$

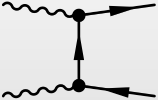
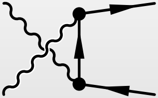


$$= ig^2 \cdot \frac{i}{2}\left[(\theta(k_1 - k_2))_{\mu_1}\gamma_{\mu_2} - (\theta(k_1 - k_2))_{\mu_2}\gamma_{\mu_1} - \theta_{\mu_1\mu_2}(\not{k}_1 - \not{k}_2)\right]$$

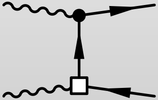
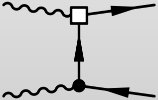
Ward Identity satisfied by

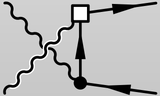
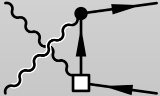
alone, TGVs not necessary (but allowed and only constrained from matching to SM at $\theta^{\mu\nu} \to 0$)!

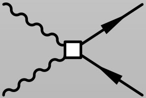
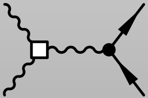
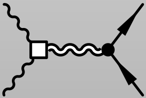
$\gamma(k_1)\gamma(k_2) \to f(p_1)\bar{f}(p_2)$ in the standard model (γ certainly polarized):

$A_t^{\mathrm{SM}} =$  , $A_u^{\mathrm{SM}} =$  .

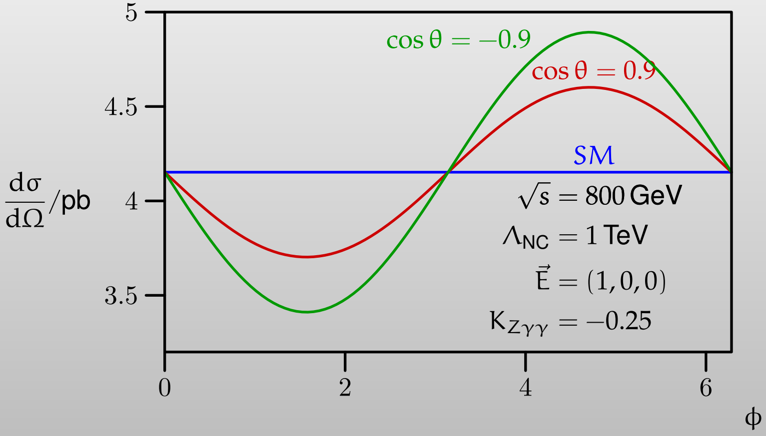
NCSM [T. O., Reuter, PRD70]:

$A_{t,1}^{\mathrm{NC}} =$  , $A_{t,2}^{\mathrm{NC}} =$  ,

$A_{u,1}^{\mathrm{NC}} =$  , $A_{u,2}^{\mathrm{NC}} =$  .

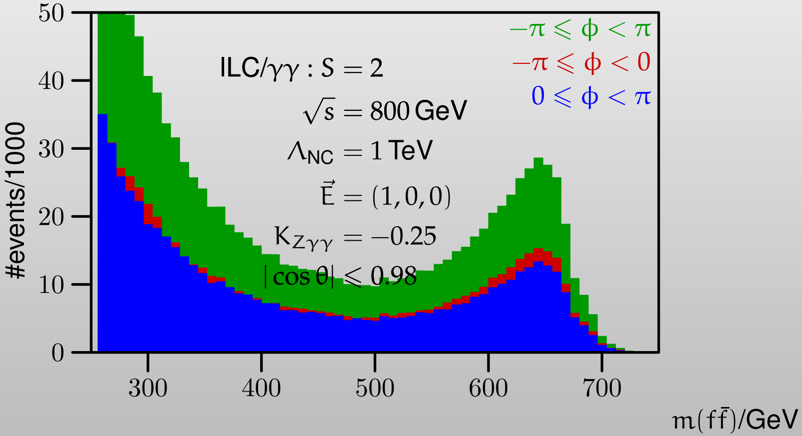
$A_c^{\mathrm{NC}} =$  , $A_{s,\gamma}^{\mathrm{NC}} =$  , $A_{s,Z}^{\mathrm{NC}} =$  .

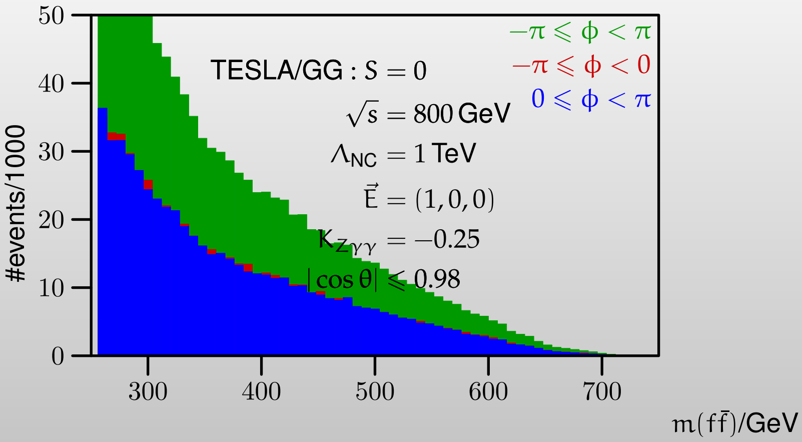
Differential cross section depends in the NCSM on the azithumal angle ϕ:



$\theta^{\mu\nu}$ defines directions $\vec{E}$ and $\vec{B}$ $\Longrightarrow$ no rotational invariance!

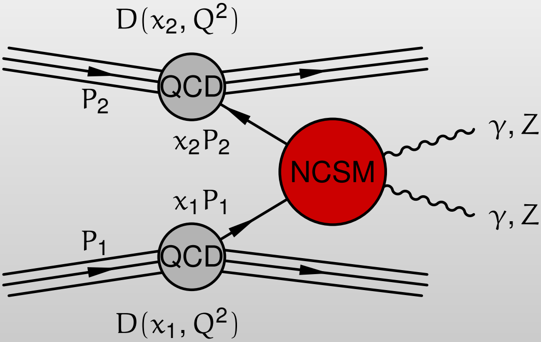
Number of events in the hemispheres $\phi < 0$ and $\phi > 0$ for $\sqrt{s} = 800\,\text{GeV}$





☹ no signal in the Higgs friendly $S = 0$ mode …

☺ In the much nearer future: LHC [Alboteanu, T. O., Rückl, PRD74]

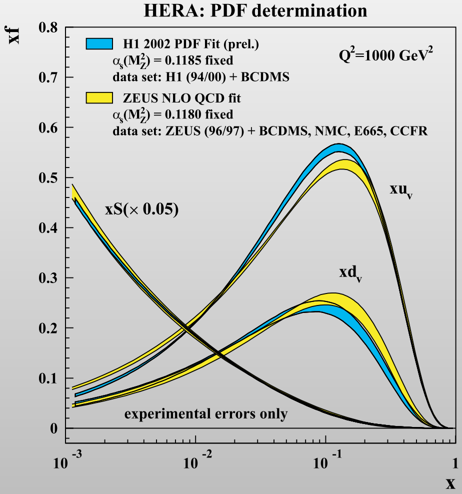
- 

- complications:
  - no polarization
  - symmetric initial state
  - broad energy spectrum
  - parton CMS strongly boosted

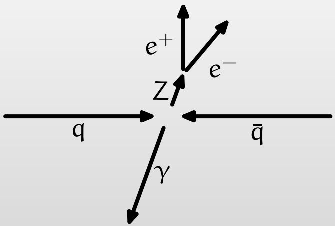
- symmetric PP initial state

$$\left.\begin{matrix} q\bar{q} \\ \bar{q}q \end{matrix}\right\} \to Z\gamma$$

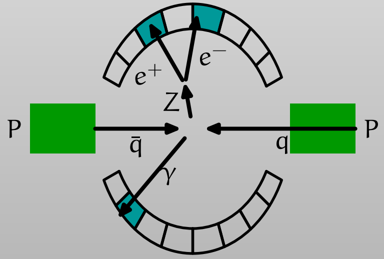
- cuts on $\cos\theta^*_\gamma$ useless without separation of quarks and antiquarks
- $\because \langle x_q \rangle > \langle x_{\bar{q}} \rangle$
- events with large longitudinal momenta favor $q\bar{q} \to Z\gamma$
- cut $\cos\theta_Z > 0 \wedge \cos\theta_\gamma > 0$?

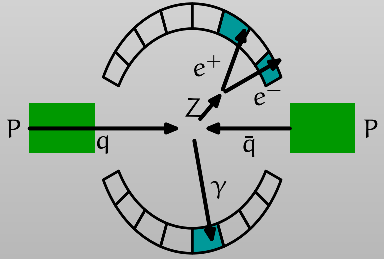


- CMS of quarks and antiquarks:

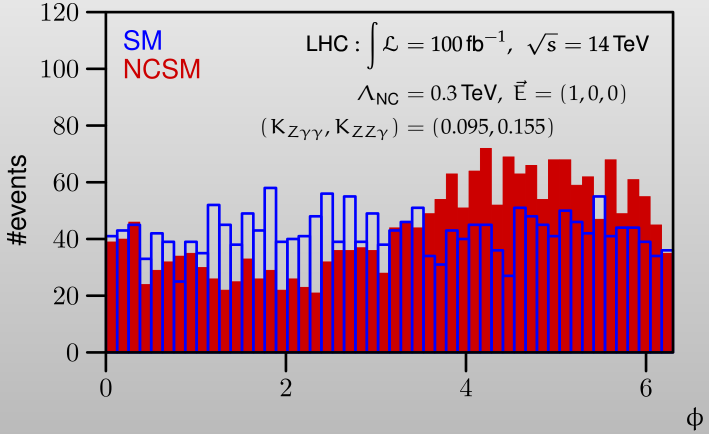


- lab system:





standard acceptance cuts and $85\,\text{GeV} < m_{\ell^+\ell^-} < 97\,\text{GeV}$, $200\,\text{GeV} < m_{\ell^+\ell^-\gamma} < 1\,\text{TeV}$, $0 < \cos\theta^*_\gamma < 0.9$,
$\cos\theta_Z > 0$ and $\cos\theta_\gamma > 0$ (favoring $\bar{q}q$ over $q\bar{q}$!)

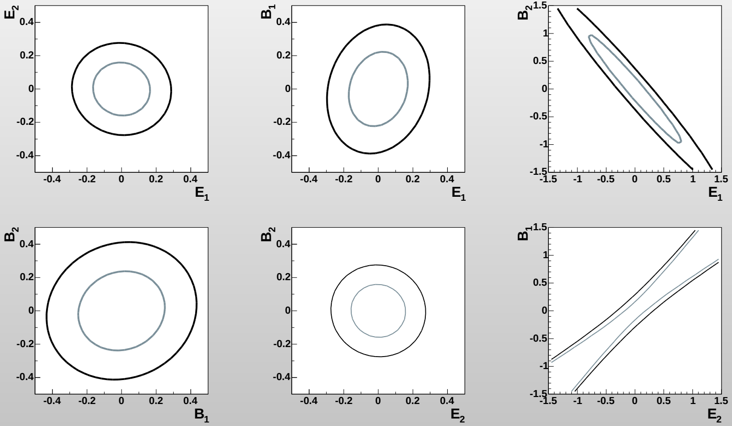


- $f\bar{f} \to \gamma\gamma$ depends in the NCSM only on E_1 and E_2 [T. O., Reuter, PRD70]
- $f\bar{f} \to Z\gamma$ richer due to axial couplings
- dependence on $\vec{E}$ in the CMS of the quarks much stronger than on $\vec{B}$ (except for $\cos\theta^*_\gamma \approx 0$)
- Lorentz boosts along beam axis x_3

$$\begin{aligned} E_1 &\to \gamma(E_1 - \beta B_2) & B_1 &\to \gamma(B_1 + \beta E_2) \\ E_2 &\to \gamma(E_2 + \beta B_1) & B_2 &\to \gamma(B_2 - \beta E_1) \\ E_3 &\to E_3 & B_3 &\to B_3 \end{aligned}$$

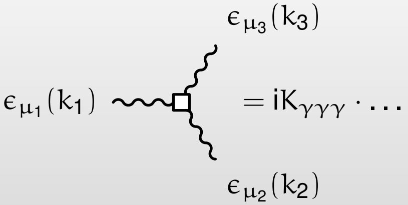
  with $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$
- measurements of (E_1, B_2) and (E_2, B_1) correlated
- correlation determined by $\langle\beta\rangle$

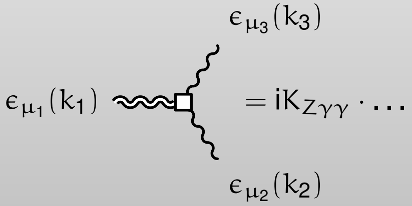
likelihood fits for $\Lambda_{NC} = 500\,$GeV [Alboteanu, T. O., Rückl, PRD74]

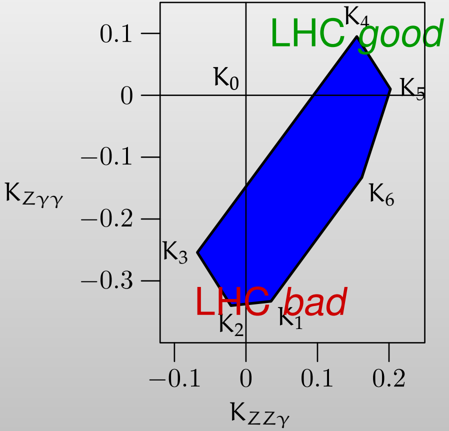


- only the expected kinematical correlations of (E_1, B_2) and (E_2, B_1)
- ☺ $\Lambda_{NC} = 1$ TeV can be easily probed at the LHC

boson couplings depend on the representation of enveloping algebra:





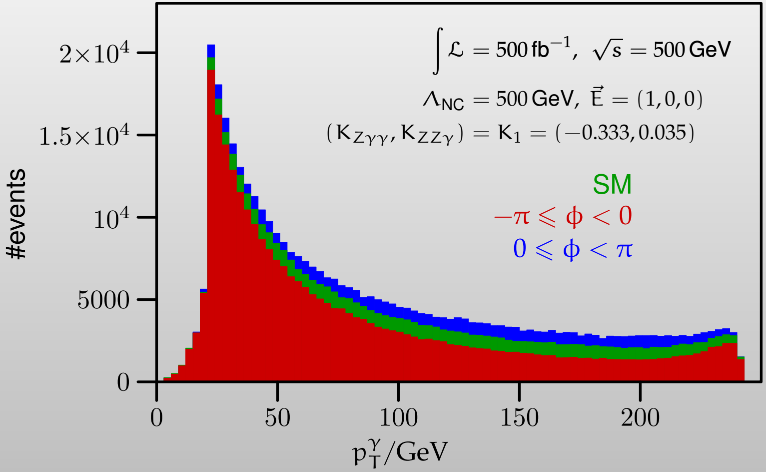


- dependence of limits on $K = (K_{Z\gamma\gamma}, K_{ZZ\gamma})$ at LHC:
  - best bound for $K_4 = (0.095, 0.155)$: $\Lambda_{NC} \gtrsim 1.2\,\text{TeV}$
  - cancellations for K_1 and K_2: no sensible limits

- polarized scattering cross sections contain more observables, can be more sensitive (cf. photon collider)
- ☺ old trick from the LEP2 days: angular distribution of $W \to f\bar{f}'$ decays can be used to measure $W^\pm$-polarization
- ☹ in the real world
  - $W^- \to \ell^-\bar\nu_\ell$ looses the neutrino momentum
  - $W^\pm \to q\bar{q}' \to jj$ looses charge information
- Monte Carlo Simulation required [Speckner, diploma thesis 2006]
  - (as above) using the automated adaptive phase space generator WHiZard [Kilian] and noncommutative extensions of the automated matrix element generator O'Mega [Ohl, Reuter et al.]
  - ☺ polarization analysis helps
  - ☹ similar cuts and luminosities lead to slightly less sensitive bounds

$$\Lambda_{NC} \gtrsim 1\,\text{TeV}$$

- p_T distribution of photons with $\cos\theta > 0$ [Alboteanu, 2007]

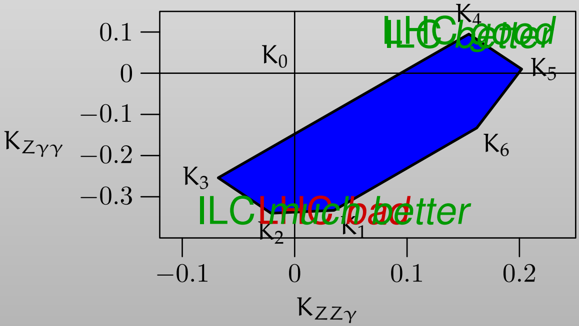


☺ much clearer signal at ILC compared to LHC

- χ^2-analysis

$(K_{Z\gamma\gamma}, K_{ZZ\gamma})$	$\lvert\vec{E}\rvert^2 = 1, \vec{B} = 0$	$\vec{E} = 0, \lvert\vec{B}\rvert^2 = 1$
$K_0 \equiv (0, 0)$	$\Lambda_{NC} \gtrsim 2$ TeV	$\Lambda_{NC} \gtrsim 0.4$ TeV
$K_1 \equiv (-0.333, 0.035)$	$\Lambda_{NC} \gtrsim 5.9$ TeV	$\Lambda_{NC} \gtrsim 0.9$ TeV
$K_5 \equiv (0.095, 0.155)$	$\Lambda_{NC} \gtrsim 2.6$ TeV	$\Lambda_{NC} \gtrsim$??? TeV

- ILC and LHC are complementary



- Problem at LHC
  - partonic CMS energy $\sqrt{\hat{s}}$ distribution peaks at low energy ...
  - ... and has long tail to high energies

☹ there will be events with

$$\hat{s} > \Lambda_{\mathrm{NC}}^2$$

- pragmatical ad-hoc solution: cut

$$\hat{s} \leqslant \hat{s}_{\mathrm{max}} \approx \Lambda_{\mathrm{NC}}^2$$

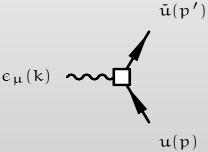
☹ waste of the most interesting events

- can we go beyond $\mathcal{O}(\theta)$ instead?

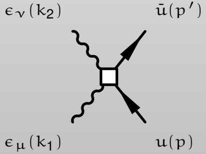
☺ no fundamental obstacle: gauge equivalence equations can be solved in higher orders

- open questions
  - technically feasible?
  - new ambiguities beyond the gauge boson self couplings?

- solve gauge equivalence relations using computer algebra (FORM) and derive Feynman rules [Alboteanu, 2007]
- e. g. fermionic couplings:



$$= ig \cdot \begin{cases} \frac{i}{2}\left[k\theta^\mu \not{p}(1-4\xi^1_\Psi) + 2\,k\theta^\mu \not{k}(\xi^1_A - \xi^1_\Psi) - p\theta^\mu \not{k} - (k\theta p)\gamma_\mu\right] \\ +\frac{1}{8}(k\theta p)\left[k\theta^\mu \not{p}(1-16\xi^2_\Psi) + 4\,k\theta^\mu \not{k}(\xi^1_A - \xi^2_\Psi) \right. \\ \left. -p\theta^\mu \not{k} - (k\theta p)\gamma_\mu\right] \end{cases}$$



$$= ig^2 \cdot \begin{cases} \frac{i}{2}\left[k_2\theta^\mu\gamma^\nu - k_1\theta^\mu\gamma^\nu(1-4\xi^1_\Psi) - \theta^{\mu\nu}\not{k}_1 \right. \\ \left. + (\mu \leftrightarrow \nu, k_1 \leftrightarrow k_2)\right] \\ + f(\theta^2, p, k_1, k_2, \xi^1_\lambda, \xi^1_\Psi, \xi^1_A, \xi^2_A) \end{cases}$$

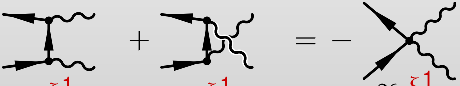
- finds more ambiguities than early work [Möller, 2004]

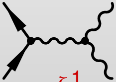
- $\mathcal{O}(\theta)$: ambiguities correspond to field redefinitions, e. g.

$$\hat{A}^{(1)}_{0\xi} \to \hat{A}^{(1)}_{0\xi} - 2i\xi^1_A\ \theta^{\mu\nu}\ D_\xi(F_{\mu\nu})$$

∴ ambiguities must cancel in the on-shell amplitude

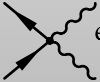
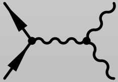
☺ indeed:



⇒ NB:  not needed!

- $\mathcal{O}(\theta^2)$: e. g.

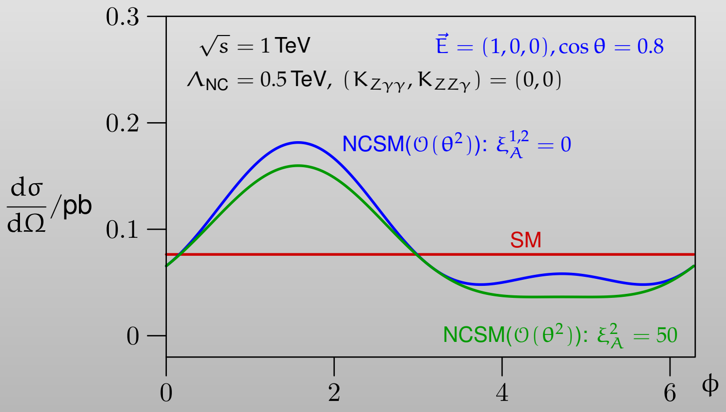
$$\hat{A}^{(2)}_{0\xi} \to \hat{A}^{(2)}_{0\xi} + i\xi^2_A\ \theta^{\mu\nu}\theta^{\kappa\lambda}\partial_\mu\partial_\kappa A_\nu(\partial_\lambda A_\xi - \partial_\xi A_\lambda)$$

only in  ⇒  required for cancellation

☹ freedom of choice of representation of enveloping algebra destroys cancellation of (some) ambiguities!

$$q\bar{q} \to Z\gamma$$

- SWM: only special solution (ambiguities = 0)
- SWM: one homogenous solution added: $\xi_A^2 \neq 0$



- Can θ-expanded theories be extendend to the region $s > \Lambda_{\text{NC}}^2$ (cf. “transplanckian” regime of quantum gravity ...)?
- obviously: can't be answered with finite orders in the θ-expansions
- obstacles:
  - closed all-order expressions for Seiberg-Witten-Maps are not known (yet?)
  - even if they exist, they must be non-polynomial in the fields
- observation:
  - $\because$ in each fixed order in the loop expansion of a particular scattering amplitude, the degree of contributing vertices is bounded
  - $\therefore$ expand Seiberg-Witten-Maps in the number of fields
  - ☺ recursive solution available [Barnich, Brandt, Grigoriev, NPB677 (2004)]
- first step:
  - test tree-level-unitarity

- NB: nothing spectacular can come from the Moyal phases for real momenta

$$\left|e^{ip\wedge q}\right| = 1$$

(except for spoiling symmetries and corresponding cancellations)

- what about Seiberg-Witten-Maps?
- e. g. [Rauh, 2006, Zeiner 2007]

$$A_\lambda^{[2]} = \frac{1}{2}\theta^{\mu\nu}\left[2(\partial_\mu A_\lambda) *_{\text{sin}} A_\nu - (\partial_\lambda A_\mu) *_{\text{si}} A_\nu\right] - \frac{1}{4}\theta^{\mu\nu}\theta^{\rho\sigma}(\partial_\lambda\partial_\mu A_\rho)\frac{*_{\text{sin}} - *_{\text{si}}}{\wedge}(\partial_\sigma A_\nu)$$

with

$$A \wedge B = \frac{\theta^{\mu\nu}}{2}\partial_\mu A \partial_\nu B, \qquad A *_f B = A\frac{f(\wedge)}{\wedge}B$$

- Ambiguity: sine integral si can be replaced by other regular function!

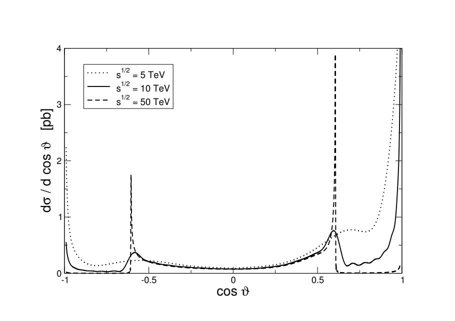
- amplitudes should not rise faster than corresponding QED amplitudes for $s \to \infty$
- Seiberg-Witten-Maps contribute to amplitudes like

$$\mathcal{A} \propto s \frac{\sin(p\theta q)}{p\theta q}$$

☺ bounded almost everywhere, i. e. for $p\theta q \neq 0$

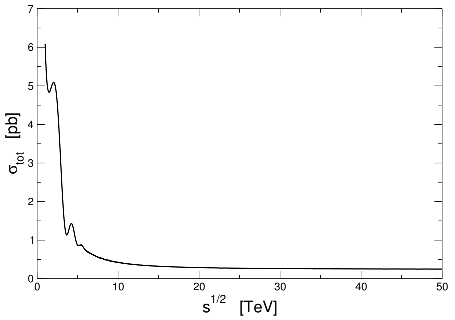
☹ diverges linearly for exceptional points, i. e. for $p\theta q \neq 0$!

$$e^+e^- \to \gamma\gamma$$



- scattering amplitude for fixed energies and angles unphysical: always small experimental uncertainties
- smearing reduces the power of growth
- performing all possible and required smearings restores tree-level-unitarity (NB: in the example we looked at!)

$$e^+e^- \to \gamma\gamma$$



- NCSM can be probed at LHC, ILC and a Photon Collider up to several TeV
- higher orders in the θ-expansions are advised for hadron colliders and be calculated, but introduce additional ambiguities
- all-order-θ tree amplitudes can be calculated in simple models and appear so satisfy tree-level-unitarity just barely

Thanks to my noncommutative students and colleagues at Würzburg:

- Ana Alboteanu: NCSM phenomenology at LHC, NCSM at $\mathcal{O}(\theta^2)$
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- Reinhold Rückl: CEO
- Christian Speckner: NCSM phenomenology at LHC
- Jörg Zeiner: all order U(1)-NCYM

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Revisit this talk on the web:

- `http://theorie.physik.uni-wuerzburg.de/` `/~ohl/talks/ncsm2.pdf`