

Bayrischzell Workshop 2007

"Non-Commutativity and Physics"

**θ -expanded noncommutative gauge
field theory: Renormalizability and
Phenomenology**

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Introduction

Example of noncommutativity (NC): Heisenberg algebra

$$[\hat{x}^\mu, p^\nu] = i\hbar\delta^{\mu\nu}, \quad [p^\mu, p^\nu] = 0$$

Constructing models on noncommutative space-time

Motivations: String Theory

Quantum Gravity

Lorentz invariance breaking

Heuristic

* The star product: $[x^\mu, x^\nu] = i\hbar\theta^{\mu\nu} = x^\mu \star x^\nu - x^\nu \star x^\mu$

$$(f \star g)(x) = e^{-\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}} f(x)g(y)|_{y \rightarrow x}$$

* Noncommutative space is flat Minkowski space:

$$x^\mu \rightarrow \hat{x}^\mu \implies [\hat{x}^\mu, \hat{x}^\nu] = i\hbar\theta^{\mu\nu}, \quad [\theta^{\mu\nu}, \hat{x}^\rho] = 0,$$

θ - constant, antisymmetric and real 4×4 matrix

$\hbar = 1/\Lambda_{\text{NC}}^2$ - NC deformation parameter

* Symmetry extended to enveloping algebra

* Seiberg-Witten map (SW)

There are 2 essential points in which NCGFT differ from standard gauge theories:

* The breakdown of Lorentz invariance with respect to a fixed $\neq 0$ background field $\theta^{\mu\nu}$ (which fixes preferred directions)

* The appearance of new interactions and the modification of standard ones. For example, triple-neutral-gauge boson, 2 fermion-2 gauge bosons, photon-neutrino, etc.

Both properties have a common origin and appear in a number of phenomena

AT VERY HIGH ENERGIES AND/OR VERY SHORT DISTANCES.

CONSTRUCTION

VIA SEIBERG-WITTEN MAP

- [N. Seiberg and E. Witten; String theory and non-commutative geometry, JHEP **9909**, 032 (1999)]
- [J. Madore, S. Schraml, P. Schupp and J. Wess; Gauge theory on noncommutative spaces, Eur. Phys. J. **C16** (2000) 161]
- [B. Jurčo, S. Schraml, P. Schupp and J. Wess; Enveloping algebra valued gauge transformations for non-Abelian gauge groups on non-commutative spaces, Eur. Phys. J. C **17**, 521 (2000)]
- [X. Calmet, B. Jurčo, P. Schupp, J. Wess and M. Wohlgenannt; The standard model on non-commutative space-time, EPJ **C23** (2002) 363]
- [W. Behr, N. G. Deshpande, G. Duplanić, P. Schupp, J.T. and J. Wess; The $Z \rightarrow \gamma \gamma$, $g g$ decays in the non-commutative standard model, Eur. Phys. J. C **29**, 441 (2003)]
- [G. Duplanić, P. Schupp and J. Trampetić; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. **C32** (2003) 141]
- [B. Melić, K. Passek-Kumerički, J.T., P. Schupp and M. Wohlgenannt; The Standard Model on Non-Commutative Space-Time: Electroweak Currents and Higgs Sector, EPJ **C24** (2005) 483 *ibid.* 499]
- [F. Brandt, C.P. Martin and F. Ruiz Ruiz; Anomaly freedom in Seiberg-Witten noncommutative gauge theories JHEP **07** (2003) 068]
- [M. Buric, D. Latas and V. Radovanovic, Renormalizability of noncommutative $SU(N)$ gauge theory; JHEP **0602** (2006) 046]
- [M. Buric, V. Radovanovic and J.T., The one-loop renormalization of the gauge sector in the noncommutative standard model; JHEP**03** (2007) 030]
- [D. Latas, V. Radovanovic and J.T., Non-commutative $SU(N)$ gauge theories and asymptotic freedom; [hep-th/0703018](#).]

- * Models based on the Seiberg-Witten mapping
- * Expansion in power series in $\theta \rightarrow$ new vertices
- * Any gauge groups
- * Arbitrary matter representation
- * No charge quantization problem
- * No UV/IR mixing due to θ expansion
- * Unitarity is OK for: $\theta^{ij} \neq 0, \theta^{0i} = 0$;
carefull canonical quantization produces always unitary theory: (*Bahns, Fredenhagen, Doplicher, Piatelli: Time in S matrix treated in form of slices*)
- * By covariant generalization of $\theta^{0i} = 0$ to:

$$\theta_{\mu\nu}\theta^{\mu\nu} = -\theta^2 = \frac{2}{\Lambda_{\text{NC}}^4} (\vec{B}_\theta^2 - \vec{E}_\theta^2) > 0$$

known as *perturbative unitarity condition* one avoids potential difficulties with unitarity in noncommutative gauge field theories

- * Construction of covariant NCSM Yukawa couplings OK
- * One-loop renormalizable gauge sector at the first order in noncommutative parameter θ
- * Models 1 & 2: mNCSM & nmNCSM constructed as an effective, anomaly free and partly renormalizable theories
- * Model 3: SU(N) GFT constructed as an renormalizable theory via renormalization of $\theta \rightarrow$ RGE for noncommutative deformation parameter h

NC gauge transformation

Consider infinitesimal NC local gauge transformation $\hat{\delta}$ of a fundamental matter field that carries a representation ρ_Ψ

$$\hat{\delta}\hat{\Psi} = i\rho_\Psi(\hat{\Lambda}) \star \hat{\Psi}$$

In Abelian case ρ_Ψ fixed by the hypercharge.

Covariant coordinates in NC theory introduced in analogy to covariant derivatives in ordinary theory

$$\hat{x}^\mu = x^\mu + \theta^{\mu\nu} \hat{A}_\nu$$

Locality

A $\star$ – product of ordinary functions f, g , determined by a Poisson tensor $\theta^{\mu\nu}(x)$, is local function of f, g with finite number of derivatives at each order in θ :

$$f \star g = f \cdot g + \frac{i}{2} \theta^{\mu\nu}(x) \partial_\mu f \cdot \partial_\nu g + \mathcal{O}(\theta^2)$$

Gauge equivalence, and consistency conditions

Ordinary gauge transformations $\delta A_\mu = \partial_\mu \Lambda + i[\Lambda, A_\mu]$ and $\delta \Psi = i\Lambda \cdot \Psi$ induce non-commutative gauge transformations of the fields $\hat{A}, \hat{\Psi}$ with gauge parameter $\hat{\Lambda}$

$$\delta \hat{A}_\mu = \hat{\delta} \hat{A}_\mu \quad \delta \hat{\Psi} = \hat{\delta} \hat{\Psi}$$

Consistency require that any pair of non-commutative gauge parameters $\hat{\Lambda}, \hat{\Lambda}'$ satisfy

$$[\hat{\Lambda}, \hat{\Lambda}'] + i\delta_{\hat{\Lambda}} \hat{\Lambda}' - i\delta_{\hat{\Lambda}'} \hat{\Lambda} = [\hat{\Lambda}, \hat{\Lambda}'].$$

Enveloping algebra-valued gauge transformation

The commutator

$$[\hat{\Lambda}, \hat{\Lambda}'] = \frac{1}{2} \{ \Lambda_a(x) \star \Lambda'_b(x) \} [T^a, T^b] + \frac{1}{2} [\Lambda_a(x), \Lambda'_b(x)] \{ T^a, T^b \}$$

of two Lie algebra-valued **NC** gauge parameters $\hat{\Lambda} = \Lambda_a(x)T^a$ and $\hat{\Lambda}' = \Lambda'_a(x)T^a$ **does not close in Lie**. For **NC SU(N) & Lie algebra** traceless condition incompatible with commutator. Extension to enveloping algebra-valued **NC** gauge parameters and fields.

$$\hat{\Lambda} = \Lambda_a^0(x)T^a + \Lambda_{ab}^1(x) : T^a T^b : + \Lambda_{abc}^2(x) : T^a T^b T^c : + \dots$$

Closing condition for gauge transformation algebra are homogenous differential equations which are solved by iteration, order by order in θ . This solution is known as Seiberg–Witten map:

$$\hat{\Lambda} = \Lambda + \frac{1}{4} \theta^{\mu\nu} \{ V_\nu, \partial_\mu \Lambda \} + \dots$$

$$\hat{\psi} = \psi - \frac{1}{2} \theta^{\alpha\beta} \left(V_\alpha \partial_\beta - \frac{i}{4} [V_\alpha, V_\beta] \right) \psi + \dots$$

$$\hat{V}_\mu = V_\mu + \frac{1}{4} \theta^{\alpha\beta} \{ \partial_\alpha V_\mu + F_{\alpha\mu}, V_\beta \} + \dots$$

GAUGE SECTOR FRAMEWORK PROPOSAL

1: Commutative GFT, that are renormalizable are extended to the NC space with deformed gauge transformations. These deformations are not unique. For instance deformed action S_g depends on the choice of representation. This derives from the fact that $\hat{F}^{\mu\nu}$ is enveloping algebra but not Lee algebra valued. The trace Tr in S_g is over all representations:

$$S_g = -\frac{1}{2} \text{Tr} \int d^4x \hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu}.$$

2: Seiberg-Witten map up to 1st order in θ :

$$\begin{aligned}\hat{V}_\mu(x) &= V_\mu(x) - \frac{\hbar}{4} \theta^{\rho\sigma} \{V_\rho(x), \partial_\sigma V_\mu(x) + F_{\sigma\mu}(x)\}, \\ \hat{F}_{\mu\nu} &= F_{\mu\nu} + \frac{\hbar}{4} \theta^{\rho\sigma} \left(2\{F_{\rho\mu}, F_{\sigma\nu}\} - \{V_\rho, (\partial_\sigma + D_\sigma)F_{\mu\nu}\} \right), \\ \hat{F}_{\mu\nu} &= \partial_\mu \hat{V}_\nu - \partial_\nu \hat{V}_\mu - i[\hat{V}_\mu \star \hat{V}_\nu]\end{aligned}$$

Points 1: and 2: leads to:

$$\begin{aligned}S_g^1 &= S_{cl} = S_{SM}^0 + S^\theta = -\frac{1}{2} \text{Tr} \int d^4x F_{\mu\nu} F^{\mu\nu} \\ &\quad + \hbar \theta^{\rho\sigma} \text{Tr} \int d^4x \left[\left(\frac{1}{4} F_{\rho\sigma} F_{\mu\nu} - F_{\rho\mu} F_{\sigma\nu} \right) F^{\mu\nu} \right]\end{aligned}$$

Note: Solution for the SW map is NOT UNIQUE.
All expressions $\hat{V}'_\mu, \hat{F}'_{\mu\nu}$ of the form

$$\hat{V}'_\mu = \hat{V}_\mu + X_\mu, \quad \hat{F}'_{\mu\nu} = \hat{F}_{\mu\nu} + D_\mu X_\nu - D_\nu X_\mu,$$

are solutions to the closing condition to linear order, if X_μ is a gauge covariant expression linear in θ , otherwise arbitrary. This transformation is a redefinition of the fields V_μ and $F_{\mu\nu}$.

3: Clearly we do not know the meaning of ‘minimal coupling concept’ for some NCGFT in the NC space. However, renormalization is the principle that help us to find such acceptable couplings. We learned that the renormalizability condition of some specific NCGFT requires introduction of the higher order NC gauge interaction by expanding general NC action in terms of NC field strengths. This lead us to the extension of ‘minimal’ action S_g to higher order

$$S_{\star} = \text{Tr} \int d^4x \left[-\frac{1}{2} \hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu} + h\theta^{\mu\nu} \left(b\hat{F}_{\mu\nu} \star \hat{F}_{\rho\sigma} + c\hat{F}_{\mu\rho} \star \hat{F}_{\nu\sigma} \right) \star \hat{F}^{\rho\sigma} \right],$$

with b, c being free parametes determining renormalizable deformation.

4: SW map for NC field strength up to the first order in $h\theta^{\mu\nu}$ and requirement $c \neq 1$

$$\begin{aligned} S_{\star} &= \text{Tr} \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + h\theta^{\mu\nu} \left(\frac{1}{4} F_{\mu\nu} F_{\rho\sigma} - F_{\mu\rho} F_{\nu\sigma} \right) F^{\rho\sigma} + h\theta^{\mu\nu} (bF_{\mu\nu} F_{\rho\sigma} + cF_{\mu\rho} F_{\nu\sigma}) F^{\rho\sigma} \right], \\ &= \text{Tr} \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + h\theta^{\mu\nu} \left(\left(\frac{1}{4} + b \right) F_{\mu\nu} F_{\rho\sigma} + (c - 1) F_{\mu\rho} F_{\nu\sigma} \right) F^{\rho\sigma} \right], \end{aligned}$$

5: Free choice $c = 0$ and redefinition $1 + 4b = a$:

$$S = \text{Tr} \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + h\theta^{\mu\nu} \left(\frac{a}{4} F_{\mu\nu} F_{\rho\sigma} - F_{\mu\rho} F_{\nu\sigma} \right) F^{\rho\sigma} \right]$$

Proposed framework 1,...,5 gives starting action

$$S_{\star}^a = -\frac{1}{2}\text{Tr} \int d^4x \left(1 - \frac{a-1}{2} h \theta^{\mu\nu} \hat{F}_{\mu\nu} \right) \star \hat{F}_{\rho\sigma} \star \hat{F}^{\rho\sigma},$$

which, via SW map for $\hat{F}^{\mu\nu}$, produces directly:

$$S_{gauge} = S_{cl} = S_{SM}^0 + S^{\theta} = -\frac{1}{2}\text{Tr} \int d^4x F_{\mu\nu} F^{\mu\nu} \\ + h \theta^{\mu\nu} \text{Tr} \int d^4x \left(\frac{a}{4} F_{\mu\nu} F_{\rho\sigma} - F_{\mu\rho} F_{\nu\sigma} \right) F^{\rho\sigma}.$$

REQUIREMENT OF RENORMALIZABILITY

fixes the freedom parameter $a \Rightarrow$

PRINCIPLE OF RENORMALIZATION

DETERMINES

NC RENORMALIZABLE DEFORMATION

Trace of three generators in the above action
lead to dependence of the gauge group
representation!

The choice of the trace corresponds to the
choice of the representation of the gauge
group

Choosing however vector field in the adjoint representation, i.e. using a sum of three traces over the SM gauge group we have:

$\Rightarrow$ Model 1: mNCSM

Choosing a trace over all massive particle multiplets with different quantum numbers in the model that have covariant derivative acting on them we found:

$\Rightarrow$ Model 2: nmNCSM

Choosing however vector field in the adjoint representation $SU(N)$ we have:

$\Rightarrow$ Model 3: NC $SU(N)$ GFT

Gauge sector Model 1: mNCSM

Short review of mNCSM gauge sector

The mNCSM gauge action is given by

$$S_{\text{gauge}}^{\text{mNCSM}} = -\frac{1}{2} \int d^4x \left(\frac{1}{g'^2} \text{Tr}_1 + \frac{1}{g^2} \text{Tr}_2 + \frac{1}{g_s^2} \text{Tr}_3 \right) \hat{F}_{\mu\nu} \star \hat{F}^{\mu\nu}.$$

In the definition of Tr_1 :

$$Y = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The fundamental representations for $SU(2)$ and $SU(3)$ generators in Tr_2 and Tr_3 , respectively. In terms of physical fields, the action then reads

$$S_{\text{gauge}}^{\text{mNCSM}} = -\frac{1}{2} \int d^4x \left(\frac{1}{2} \mathcal{A}_{\mu\nu} \mathcal{A}^{\mu\nu} + \text{Tr} \mathcal{B}_{\mu\nu} \mathcal{B}^{\mu\nu} + \text{Tr} G_{\mu\nu} G^{\mu\nu} \right) \\ + \frac{1}{4} g_s d^{abc} \theta^{\rho\sigma} \int d^4x \left(\frac{a}{4} G_{\rho\sigma}^a G_{\mu\nu}^b - G_{\rho\mu}^a G_{\sigma\nu}^b \right) G^{\mu\nu,c},$$

where $\mathcal{A}_{\mu\nu}$, $\mathcal{B}_{\mu\nu}(= B_{\mu\nu}^a T_L^a)$ and $G_{\mu\nu}(= G_{\mu\nu}^a T_S^a)$ denote the $U(1)$, $SU(2)_L$ and $SU(3)_c$ field strengths, respectively:

$$\begin{aligned} \mathcal{A}_{\mu\nu} &= \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu, \\ \mathcal{B}_{\mu\nu}^a &= \partial_\mu B_\nu^a - \partial_\nu B_\mu^a + g \epsilon^{abc} B_\mu^b B_\nu^c, \\ G_{\mu\nu}^a &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c. \end{aligned}$$

For adjoint representation $\Rightarrow$

* NO NEW NEUTRAL EW TGB INTERACTIONS

Gauge sector Model 2: nmNCSM

The action $S_{\text{gauge}}^{\text{nmNCSM}}$ up to linear order in θ :

$$S_{\text{gauge}}^{\text{nmNCSM}} = S_{cl} = S_{\text{SM}}^0 + S^\theta = -\frac{1}{2} \int d^4x \text{Tr}_{\mathbf{G}^2} \frac{1}{F_{\mu\nu} F^{\mu\nu}} + \theta^{\rho\sigma} \int d^4x \text{Tr}_{\mathbf{G}^2} \left[\left(\frac{a}{4} F_{\rho\sigma} F_{\mu\nu} - F_{\rho\mu} F_{\sigma\nu} \right) F^{\mu\nu} \right]$$

where $\text{Tr}_{\mathbf{G}^2}$ is trace over all massive particle multiplets with different quantum numbers in the model that have covariant derivative acting on them; 5 multiplets for each generation of fermions and 1 Higgs multiplet (Table). Here $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu]$ is SM field strenght, i.e. V_μ is the SM gauge potential:

$$V^\mu = g' \mathcal{A}^\mu(x) Y + g \sum_{a=1}^3 B_a^\mu(x) T_L^a + g_s \sum_{b=1}^8 G_b^\mu(x) T_S^b$$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_Q$	T_3
$e_R^{(i)}$	1	1	-1	-1	0
$L_L^{(i)} = \begin{pmatrix} \nu_L^{(i)} \\ e_L^{(i)} \end{pmatrix}$	1	2	-1/2	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$u_R^{(i)}$	3	1	2/3	2/3	0
$d_R^{(i)}$	3	1	-1/3	-1/3	0
$Q_L^{(i)} = \begin{pmatrix} u_L^{(i)} \\ d_L^{(i)} \end{pmatrix}$	3	2	1/6	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	2	1/2	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
W^+, W^-, Z	1	3	0	$(\pm 1, 0)$	$(\pm 1, 0)$
A	1	1	0	0	0
G^b	8	1	0	0	0

The SM fields. Here $i \in \{1, 2, 3\}$ denotes the generation index. The electric charge is given by the Gell-Mann-Nishijima relation $Q = (T_3 + Y)$. The physical electroweak fields A , W^+ , W^- and Z are expressed through the unphysical $U(1)_Y$ and $SU(2)$ fields $\mathcal{A}$ and B_a ($a \in \{1, 2, 3\}$). The gluons G^b ($b \in \{1, 2, \dots, 8\}$) are in the octet representation of $SU(3)_C$.

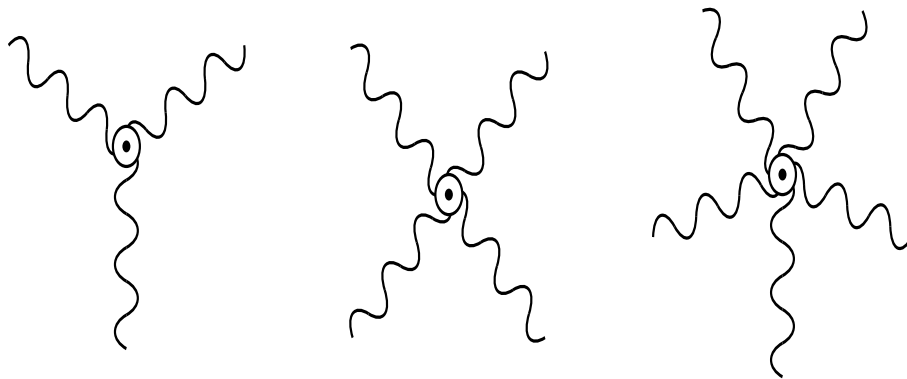
For SM gauge group we denote field strength as:

$$F^{\mu\nu} = g' f^{\mu\nu} \mathcal{R}(Y) + g \sum_{a=1}^3 B_a^{\mu\nu} \mathcal{R}(T_L^a) + g_s \sum_{b=1}^8 G_b^{\mu\nu} \mathcal{R}(T_S^b)$$

Lagrangian linear correction in θ has trace of products of 3 field strengths. Written generically that is:

$$\begin{aligned} F^3 &\sim g'^3 f^3 \text{Tr} \mathcal{R}(Y)^3 \text{Tr} I \text{Tr} I \neq 0 \\ &+ g^3 B^3 \text{Tr} \mathcal{R}(T^i)^3 \text{Tr} I \sim d^{ijk} \text{ for SU}(2) \\ &+ g_s^3 G^3 \text{Tr} I \text{Tr} \mathcal{R}(T^a)^3 \text{Tr} I \sim d^{abc} \text{ for SU}(3) \\ &+ g_s^3 f^2 B \text{Tr} T^i \text{Tr} I = 0 \\ &+ g' g^2 f B^2 \text{Tr} (T^i)^2 \text{Tr} I \neq 0 \\ &+ g'^2 g_s f G^2 \text{Tr} I \text{Tr} (T^a)^2 = 0 \\ &+ g' g_s^2 f G^2 \text{Tr} I \text{Tr} (T^a)^2 \text{Tr} I \neq 0 \\ &+ g g_s^2 B G^2 \text{Tr} T^i \text{Tr} (T^a)^2 = 0 \\ &+ g^2 g_s B^2 G \text{Tr} (T^i)^2 \text{Tr} T^a = 0 \end{aligned}$$

Nonzero are only 3 terms containing 3, 4 and 5 fields linear in θ



The NC couplings $\rightarrow$ additional vertices.

The lines are gauge fields $\mathcal{A}_\mu$, B_μ^i and G_μ^a

Matching the SM action at zeroth order in θ , three consistency conditions are imposed

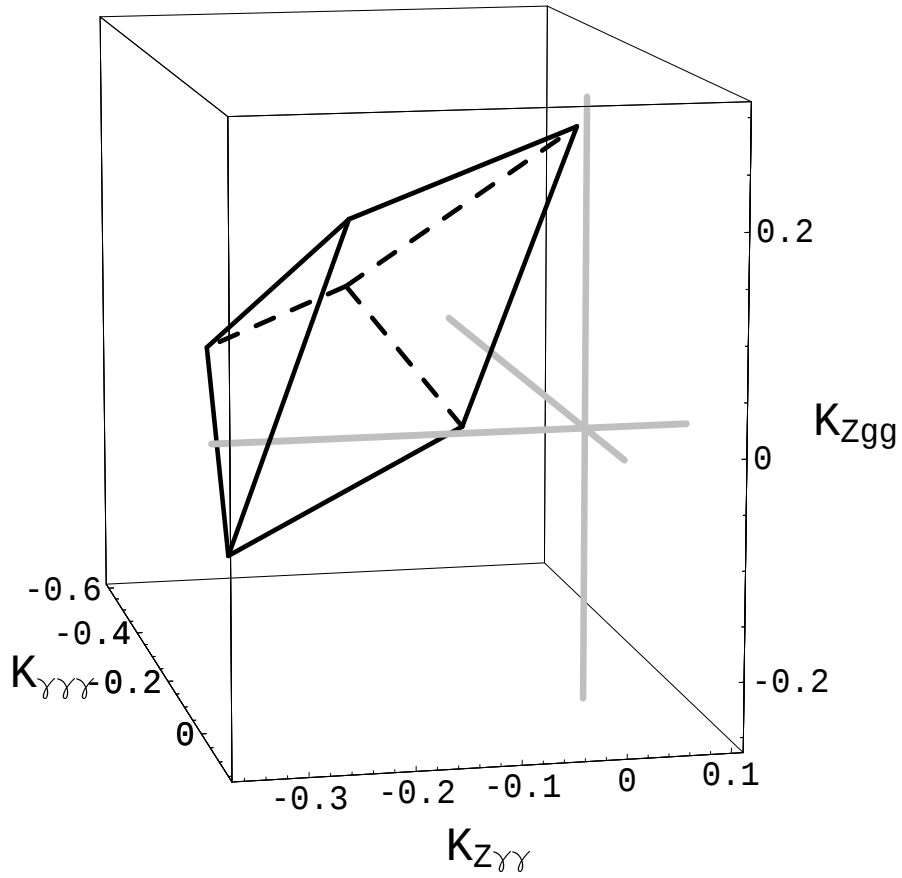
$$\begin{aligned}\frac{1}{g'^2} &= \frac{2}{g_1^2} + \frac{1}{g_2^2} + \frac{8}{3g_3^2} + \frac{2}{3g_4^2} + \frac{1}{3g_5^2} + \frac{1}{g_6^2}, \\ \frac{1}{g^2} &= \frac{1}{g_2^2} + \frac{3}{g_5^2} + \frac{1}{g_6^2}, \\ \frac{1}{g_s^2} &= \frac{1}{g_3^2} + \frac{1}{g_4^2} + \frac{2}{g_5^2}.\end{aligned}$$

giving final expression for TGB action

$$\begin{aligned}S_{gauge} &= S_{cl} = S_{SM}^0 + S^\theta = -\frac{1}{4} \int d^4x f_{\mu\nu} f^{\mu\nu} - \frac{1}{2} \int d^4x \text{Tr} (B_{\mu\nu} B^{\mu\nu}) \\ &- \frac{1}{2} \int d^4x \text{Tr} (G_{\mu\nu} G^{\mu\nu}) \\ &+ g'^2 \kappa_1 \theta^{\rho\tau} \int d^4x \left(\frac{a}{4} f_{\rho\tau} f_{\mu\nu} - f_{\mu\rho} f_{\nu\tau} \right) f^{\mu\nu} \\ &+ g' g^2 \kappa_2 \theta^{\rho\tau} \int d^4x \sum_{a=1}^3 \left[\left(\frac{a}{4} f_{\rho\tau} B_{\mu\nu}^a - f_{\mu\rho} B_{\nu\tau}^a \right) B^{\mu\nu,a} + c.p. \right] \\ &+ g' g_s^2 \kappa_3 \theta^{\rho\tau} \int d^4x \sum_{b=1}^8 \left[\left(\frac{a}{4} f_{\rho\tau} G_{\mu\nu}^b - f_{\mu\rho} G_{\nu\tau}^b \right) G^{\mu\nu,b} + c.p. \right]\end{aligned}$$

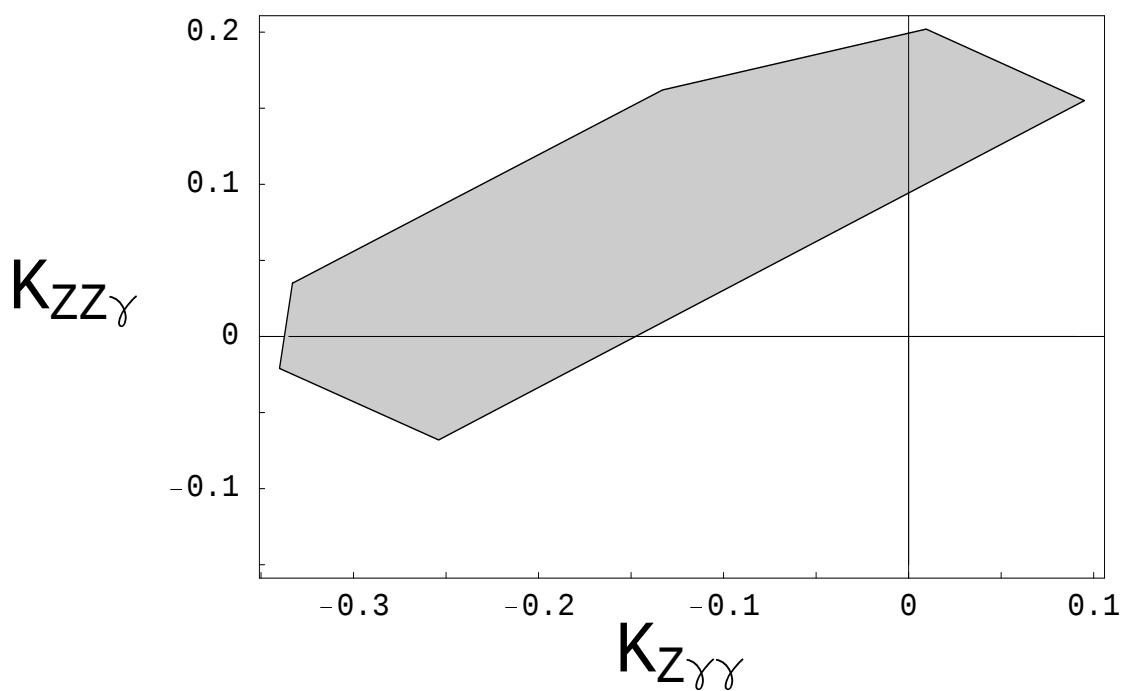
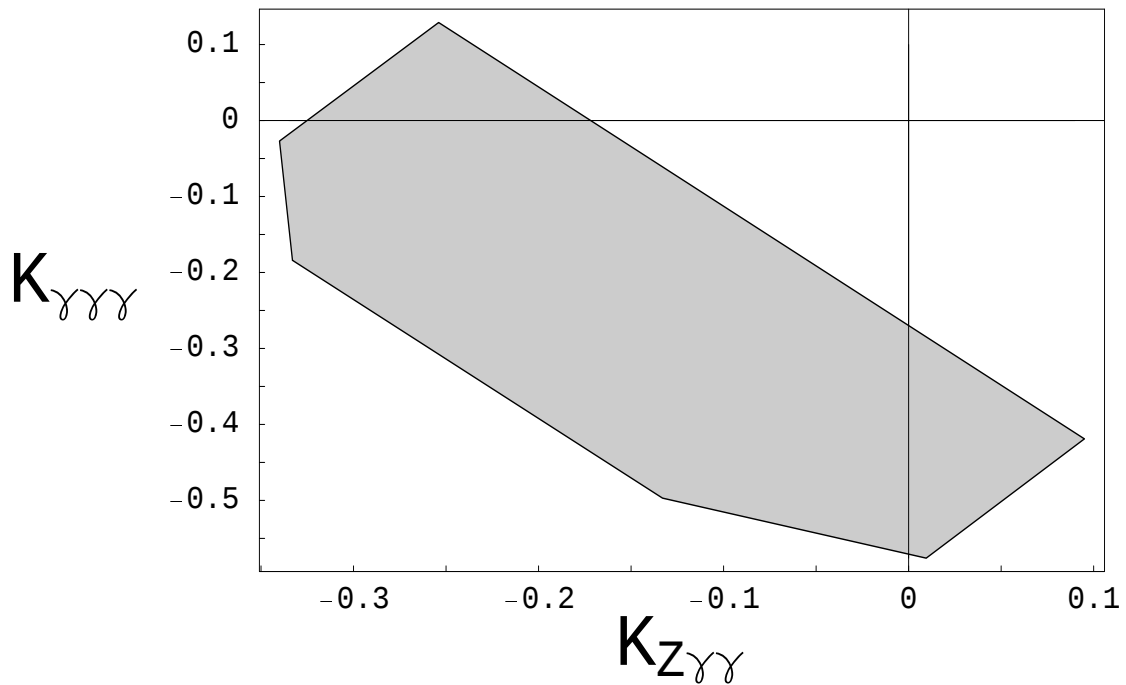
Above three consistency conditions together with the requirement that $1/g_i^2 > 0$ define a 3D pentahedron in the six-dimensional moduli space spanned by $1/g_1^2, \dots, 1/g_6^2$

$$\begin{aligned}
\frac{2K_{\gamma\gamma\gamma}}{gg'} &= -\frac{1}{g_1^2} - \frac{1}{g_2^2} + \frac{8}{9g_3^2} - \frac{1}{9g_4^2} + \frac{7}{9g_5^2} + \frac{1}{g_6^2}, \\
\frac{2K_{Z\gamma\gamma}}{g'^2} &= -\frac{1}{g_1^2} - \left(1 - \left(\frac{g}{g'}\right)^2\right) \frac{1}{2g_2^2} + \frac{8}{9g_3^2} - \frac{1}{9g_4^2} \\
&\quad + \left(5 - 9\left(\frac{g}{g'}\right)^2\right) \frac{1}{18g_5^2} + \left(1 - \left(\frac{g}{g'}\right)^2\right) \frac{1}{2g_6^2}, \\
\frac{2K_{Zgg}}{g_s^2} &= \left(1 + \left(\frac{g'}{g}\right)^2\right) \left(\frac{1}{3g_3^2} - \frac{1}{6g_4^2} + \frac{1}{6g_5^2}\right).
\end{aligned}$$



$K_{\gamma\gamma\gamma}$	$K_{Z\gamma\gamma}$	K_{Zgg}	$K_{ZZ\gamma}$	K_{ZZZ}	$K_{\gamma gg}$
-0.184	-0.333	0.054	0.035	-0.213	-0.098
-0.027	-0.340	-0.108	-0.021	-0.337	0.197
0.129	-0.254	0.217	-0.068	-0.362	-0.396
-0.576	0.010	-0.108	0.202	0.437	0.197
-0.497	-0.133	0.054	0.162	0.228	-0.098
-0.419	0.095	0.217	0.155	0.410	-0.396

[G. Duplančić, P. Schupp and J. Trampetić; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C32 (2003) 141]



The interactions $\mathcal{L}^\theta$ in terms of physical fields (A, Z, W, G)

$$\mathcal{L}_{\gamma\gamma\gamma}^\theta = \frac{e}{4} \sin 2\theta_W \mathbf{K}_{\gamma\gamma\gamma} \theta^{\rho\tau} A^{\mu\nu} (\mathbf{a} A_{\mu\nu} A_{\rho\tau} - 4 A_{\mu\rho} A_{\nu\tau})$$

$$\mathbf{K}_{\gamma\gamma\gamma} = \frac{1}{2} g g' (\kappa_1 + 3\kappa_2)$$

$$\mathcal{L}_{Z\gamma\gamma}^\theta = \frac{e}{4} \sin 2\theta_W \mathbf{K}_{Z\gamma\gamma} \theta^{\rho\tau} [2 Z^{\mu\nu} (2 A_{\mu\rho} A_{\nu\tau} - \mathbf{a} A_{\mu\nu} A_{\rho\tau}) + 8 Z_{\mu\rho} A^{\mu\nu} A_{\nu\tau} - \mathbf{a} Z_{\rho\tau} A_{\mu\nu} A^{\mu\nu}]$$

$$\mathbf{K}_{Z\gamma\gamma} = \frac{1}{2} \left[g'^2 \kappa_1 + (g'^2 - 2g^2) \kappa_2 \right]$$

$$\mathcal{L}_{WW\gamma}^\theta = \frac{e}{4} \sin 2\theta_W \mathbf{K}_{WW\gamma} \theta^{\rho\tau} \{ A^{\mu\nu} [2 (W^+_{\mu\rho} W^-_{\nu\tau} + W^-_{\mu\rho} W^+_{\nu\tau}) - \mathbf{a} (W^+_{\mu\nu} W^-_{\rho\tau} + W^-_{\mu\nu} W^+_{\rho\tau})] + 4 A_{\mu\rho} (W^{+\mu\nu} W^-_{\nu\tau} + W^{-\mu\nu} W^+_{\nu\tau}) - \mathbf{a} A_{\rho\tau} W^+_{\mu\nu} W^{-\mu\nu} \}$$

$$\mathbf{K}_{WW\gamma} = -\frac{g}{g'} \left[g'^2 + g^2 \right] \kappa_2$$

$$\mathcal{L}_{WWZ}^\theta = \mathcal{L}_{WW\gamma} (A \leftrightarrow Z)$$

$$\mathbf{K}_{WWZ} = -\frac{g'}{g} \mathbf{K}_{WW\gamma}$$

$$\mathcal{L}_{ZZ\gamma}^\theta = \mathcal{L}_{Z\gamma\gamma} (A \leftrightarrow Z)$$

$$\mathbf{K}_{ZZ\gamma} = \frac{-1}{2gg'} \left[g'^4 \kappa_1 + g^2 (g^2 - 2g'^2) \kappa_2 \right]$$

$$\mathcal{L}_{ZZZ}^\theta = \mathcal{L}_{\gamma\gamma\gamma} (A \rightarrow Z)$$

$$\mathbf{K}_{ZZZ} = \frac{-1}{2g^2} \left[g'^4 \kappa_1 + 3g^4 \kappa_2 \right]$$

$$\mathcal{L}_{Zgg}^\theta = \mathcal{L}_{Z\gamma\gamma} (A \rightarrow G^b)$$

$$\mathbf{K}_{Zgg} = \frac{g_s^2}{2} \left[1 + \left(\frac{g'}{g} \right)^2 \right] \kappa_3$$

$$\mathcal{L}_{\gamma gg}^\theta = \mathcal{L}_{Zgg} (Z \rightarrow A)$$

$$\mathbf{K}_{\gamma gg} = \frac{-g_s^2}{2} \left[\frac{g}{g'} + \frac{g'}{g} \right] \kappa_3; \quad A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \dots$$

Renormalization Model 2: nmNCSM

- ★ Advantage of the background field method is the guarantee of covariance, because by doing the path integral the local symmetry of the quantum field Φ_V is fixed, while the gauge symmetry of the background field ϕ_V is manifestly preserved.
- ★ Quantization is performed by the functional integration over the quantum vector field Φ_V in the saddle-point approximation around classical (background) configuration. Our case $\phi_V = \text{constant}$.
- ★ The main contribution to the functional integral is given by the Gaussian integral.
- ★ Split the vector potential into the classical background plus the quantum-fluctuation parts, that is: We replace, $\phi_V \rightarrow \phi_V + \Phi_V$, and then compute the terms quadratic in the quantum fields.
- ★ Interactions are of the polynomial type.
- ★ Proper quantization requires the presence of the gauge fixing term $S_{\text{gf}}[\phi]$. Adding to the SM part in the usual way, FFP ghost appears in the effective action. Result of functional integration

$$\begin{aligned}\Gamma[\phi] &= S_{\text{cl}}[\phi] + S_{\text{gf}}[\phi] + \Gamma^{(1)}[\phi], \\ S_{\text{gf}}[\phi] &= -\frac{1}{2} \int d^4x (D_\mu \phi_V^\mu)^2,\end{aligned}$$

produce the standard result of the commutative part of our action:

$$\Gamma^{(1)}[\phi] = \frac{i}{2} \log \det S^{(2)}[\phi] = \frac{i}{2} \text{Tr} \log S^{(2)}[\phi].$$

The $S^{(2)}[\phi]$ is the 2nd-functional derivative of the classical action,

$$S^{(2)}[\phi] = \frac{\delta^2 S_{cl}}{\delta\phi_{V_1} \delta\phi_{V_2}}.$$

After making the splitting

$$\mathcal{A}_\mu \rightarrow \mathcal{A}_\mu + \mathbf{A}_\mu, \quad B_\mu^i \rightarrow B_\mu^i + \mathbf{B}_\mu^i, \quad G_\mu^a \rightarrow G_\mu^a + \mathbf{G}_\mu^a,$$

we obtain for the quadratic part of the action :

$$\frac{1}{2} (\mathbf{A}_\alpha \mathbf{B}_\alpha^i \mathbf{G}_\alpha^a) \begin{pmatrix} g^{\alpha\beta} \square + M^{\alpha\beta} & * & * \\ * & g^{\alpha\beta} \delta^{ij} \square + V^{\alpha\beta;ij} & 0 \\ * & 0 & g^{\alpha\beta} \delta^{ab} \square + W^{\alpha\beta ab} \end{pmatrix} \begin{pmatrix} \mathbf{A}_\beta \\ \mathbf{B}_\beta^j \\ \mathbf{G}_\beta^b \end{pmatrix}.$$

$\square$ - propagator of any field

$*$ - terms which will not contribute θ^1 : they give only higher-order corrections.

$$M^{\alpha\beta} = \overleftarrow{\partial}_\mu M^{\mu\alpha, \nu\beta}(x) \overrightarrow{\partial}_\nu$$

$$\begin{aligned} M^{\mu\rho, \nu\sigma} &= \frac{1}{2} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) \theta^{\alpha\beta} f_{\alpha\beta} \\ &+ g^{\mu\nu} (\theta^{\alpha\rho} f^\sigma{}_\alpha + \theta^{\alpha\sigma} f^\rho{}_\alpha) + g^{\rho\sigma} (\theta^{\alpha\mu} f^\nu{}_\alpha + \theta^{\alpha\nu} f^\mu{}_\alpha) \\ &- g^{\mu\sigma} (\theta^{\alpha\rho} f^\nu{}_\alpha + \theta^{\alpha\nu} f^\rho{}_\alpha) - g^{\nu\rho} (\theta^{\alpha\sigma} f^\mu{}_\alpha + \theta^{\alpha\mu} f^\sigma{}_\alpha) \\ &+ \theta^{\mu\rho} f^{\nu\sigma} + \theta^{\nu\sigma} f^{\mu\rho} - \theta^{\rho\sigma} f^{\mu\nu} - \theta^{\mu\nu} f^{\rho\sigma} - \theta^{\nu\rho} f^{\mu\sigma} - \theta^{\mu\sigma} f^{\nu\rho} \end{aligned}$$

The structure of $V^{\alpha\beta;ij}$ is as follows:

$$V^{\alpha\beta;ij} = (N_1 + N_2 + T_1 + T_2 + T_3)^{\alpha\beta;ij}.$$

The operators N_1 and N_2 come from the commutative 3-vertex and 4-vertex interactions:

$$\begin{aligned} (N_1)_{\alpha\beta}^{ij} &= -2ig_{\alpha\beta} (B_\mu)^{ij} \partial^\mu - i(\partial^\mu B_\mu)^{ij} g_{\alpha\beta}, \\ (N_2)_{\alpha\beta}^{ij} &= -(B_\mu B^\mu)^{ij} g_{\alpha\beta} - 2i(B_{\alpha\beta})^{ij}, \end{aligned}$$

the notation $(X_\mu)^{ij} = -if^{ijk}X_\mu^k$. The operators T_1 , T_2 and T_3 describe the θ^1 , noncommutative vertices.

$$\begin{aligned}
(T_1)_{\alpha\beta}^{ij} &= g'g^2\kappa_2\delta^{ij}\left[\mathbf{a}(\overleftarrow{\partial}_\mu\theta^{\rho\sigma}f_{\rho\sigma}g_{\alpha\beta}\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\beta\theta^{\rho\sigma}f_{\rho\sigma}\overrightarrow{\partial}_\alpha) \right. \\
&\quad - 2(\overleftarrow{\partial}_\beta\theta_{\rho\alpha}f^{\mu\rho}\overrightarrow{\partial}_\mu - \overleftarrow{\partial}^\nu\theta_\alpha f_{\beta\rho}\overrightarrow{\partial}_\nu - \overleftarrow{\partial}_\sigma\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}\overrightarrow{\partial}^\mu + \overleftarrow{\partial}_\sigma\theta^{\rho\sigma}f_{\beta\rho}\overrightarrow{\partial}_\alpha \\
&\quad + \overleftarrow{\partial}_\mu\theta_{\rho\beta}f^{\mu\rho}\overrightarrow{\partial}_\alpha - \overleftarrow{\partial}^\nu\theta_\beta f_{\alpha\rho}\overrightarrow{\partial}_\nu - \overleftarrow{\partial}^\mu\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}\overrightarrow{\partial}_\sigma \\
&\quad + \overleftarrow{\partial}_\beta\theta^{\rho\sigma}f_{\alpha\rho}\overrightarrow{\partial}_\sigma) + 2\mathbf{a}(\overleftarrow{\partial}_\rho\theta^\rho_\alpha f_{\mu\beta}\overrightarrow{\partial}^\mu + \overleftarrow{\partial}^\mu\theta^\rho_\beta f_{\mu\alpha}\overrightarrow{\partial}_\rho) \\
&\quad \left. - 2(\overleftarrow{\partial}_\mu\theta_{\alpha\beta}f^{\mu\nu}\overrightarrow{\partial}_\nu - \overleftarrow{\partial}^\mu\theta_{\alpha\sigma}f_{\mu\beta}\overrightarrow{\partial}^\sigma - \overleftarrow{\partial}^\sigma\theta_{\beta\sigma}f_{\mu\alpha}\overrightarrow{\partial}^\mu + \overleftarrow{\partial}_\rho\theta^{\rho\sigma}f_{\alpha\beta}\overrightarrow{\partial}_\sigma)\right], \\
(T_2)_{\alpha\beta}^{ij} &= g'g^2i\kappa_2\left[\mathbf{a}(-\overleftarrow{\partial}_\mu\theta^{\rho\sigma}g_{\alpha\beta}f_{\rho\sigma}(B^\mu)^{ij} - \theta^{\rho\sigma}f_{\rho\sigma}g_{\alpha\beta}(B^\mu)^{ji}\overrightarrow{\partial}_\mu \right. \\
&\quad + \overleftarrow{\partial}_\beta\theta^{\rho\sigma}f_{\rho\sigma}(B_\alpha)^{ij} + \theta^{\rho\sigma}f_{\rho\sigma}(B_\beta)^{ji}\overrightarrow{\partial}_\alpha + \theta_{\rho\sigma}f^{\rho\sigma}(B_{\alpha\beta})^{ij}) \\
&\quad - 2(-\overleftarrow{\partial}_\beta\theta_{\rho\alpha}f^{\mu\rho}(B_\mu)^{ij} - \theta_{\rho\beta}f^{\mu\rho}(B_\mu)^{ji}\overrightarrow{\partial}_\alpha + \overleftarrow{\partial}_\nu\theta_{\rho\alpha}f_\beta{}^\rho(B^\nu)^{ij} \\
&\quad + \theta_{\rho\beta}f_\alpha{}^\rho(B^\nu)^{ji}\overrightarrow{\partial}_\nu + \overleftarrow{\partial}_\sigma\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B^\mu)^{ij} + \theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B^\mu)^{ji}\overrightarrow{\partial}_\sigma \\
&\quad - \overleftarrow{\partial}_\sigma\theta^{\rho\sigma}f_{\beta\rho}(B_\alpha)^{ij} - \theta^{\rho\sigma}f_{\alpha\rho}(B_\beta)^{ji}\overrightarrow{\partial}_\sigma - \overleftarrow{\partial}_\mu\theta_{\rho\beta}f^{\mu\rho}(B_\alpha)^{ij} - \theta_{\rho\alpha}f^{\mu\rho}(B_\beta)^{ji}\overrightarrow{\partial}_\mu \\
&\quad + \overleftarrow{\partial}_\mu\theta^{\rho\sigma}g_{\alpha\beta}f_\rho{}^\mu(B_\sigma)^{ij} + \theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B_\sigma)^{ji}\overrightarrow{\partial}^\mu + \overleftarrow{\partial}_\mu\theta^\rho_\beta f_{\alpha\rho}(B^\mu)^{ij} \\
&\quad + \theta_{\rho\alpha}f_\beta{}^\rho(B^\mu)^{ji}\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\beta\theta^{\rho\sigma}f_{\alpha\rho}(B_\sigma)^{ij} - \theta^{\rho\sigma}f_{\beta\rho}(B_\sigma)^{ji}\overrightarrow{\partial}_\alpha + \theta^{\rho\sigma}f_{\alpha\rho}(B_{\beta\sigma})^{ij} \\
&\quad + \theta_{\rho\beta}f^{\mu\rho}(B_{\mu\alpha})^{ij} + \theta^{\rho\sigma}f_{\beta\rho}(B_{\alpha\sigma})^{ji} \\
&\quad + \theta_{\rho\alpha}f^{\mu\rho}(B^\mu)^{ji}) - 2\mathbf{a}(\overleftarrow{\partial}^\rho\theta_{\rho\alpha}f_{\mu\beta}(B^\mu)^{ij} + \theta_{\rho\beta}f_{\mu\alpha}(B^\mu)^{ji}\overrightarrow{\partial}^\rho \\
&\quad + \overleftarrow{\partial}^\mu\theta_{\rho\beta}f_{\mu\alpha}(B^\rho)^{ij} + \theta_{\rho\alpha}f_{\mu\beta}(B^\rho)^{ji}\overrightarrow{\partial}^\mu - \frac{1}{2}\theta_{\rho\sigma}f_{\alpha\beta}(B^{\rho\sigma})^{ij} \\
&\quad - \frac{1}{2}\theta_{\alpha\beta}f_{\rho\sigma}(B^{\rho\sigma})^{ij}) - 2(-\overleftarrow{\partial}^\mu\theta_{\alpha\beta}f_{\mu\nu}(B^\nu)^{ij} - \theta_{\beta\alpha}f_{\mu\nu}(B^\nu)^{ji}\overrightarrow{\partial}^\mu \\
&\quad + \overleftarrow{\partial}^\mu\theta_{\alpha\sigma}f_{\mu\beta}(B^\sigma)^{ij} + \theta_{\beta\sigma}f_{\mu\alpha}(B^\sigma)^{ji}\overrightarrow{\partial}^\mu + \overleftarrow{\partial}^\rho\theta_{\rho\beta}f_{\alpha\nu}(B^\nu)^{ij} + \theta_{\rho\alpha}f_{\beta\nu}(B^\nu)^{ji}\overrightarrow{\partial}^\rho \\
&\quad - \overleftarrow{\partial}_\rho\theta_{\beta\sigma}f_{\alpha\beta}(B_\sigma)^{ij} - \theta^{\rho\sigma}f_{\beta\alpha}(B_\sigma)^{ji}\overrightarrow{\partial}_\rho + \theta_{\beta\sigma}f_{\alpha\nu}(B^{\nu\sigma})^{ij} + \theta_{\alpha\sigma}f_{\beta\nu}(B^{\nu\sigma})^{ji})\right], \\
(T_3)_{\alpha\beta}^{ij} &= g'g^2\kappa_2\left[\mathbf{a}(\theta^{\rho\sigma}f_{\rho\sigma}(B_\mu B^\mu)^{ij}g_{\alpha\beta} - \theta^{\rho\sigma}f_{\rho\sigma}(B_\beta B_\alpha)^{ij}) \right. \\
&\quad - 2(\theta_{\rho\alpha}f^{\mu\rho}(B_\beta B_\mu)^{ij} - \theta^\rho_\alpha f_{\beta\rho}(B_\nu B^\nu)^{ij} - \theta^{\rho\sigma}f_{\mu\rho}(B_\sigma B^\mu)^{ij}g_{\alpha\beta} \\
&\quad + \theta^{\rho\sigma}f_{\beta\rho}(B_\sigma B_\alpha)^{ij} + (\alpha \leftrightarrow \beta \quad i \leftrightarrow j)) \\
&\quad + 2\mathbf{a}(\theta_{\rho\alpha}f_{\mu\beta}(B^\rho B^\mu)^{ij} + 2\theta_{\rho\beta}f_{\mu\alpha}(B^\rho B^\mu)^{ji}) \\
&\quad - 2((\theta_{\alpha\beta}f^{\mu\nu}(B_\mu B_\nu)^{ij} \\
&\quad - \theta_{\alpha\sigma}f_{\mu\beta}(B^\mu B^\sigma)^{ij} - \theta_{\beta\sigma}f_{\mu\alpha}(B^\mu B^\sigma)^{ji} + \theta^{\rho\sigma}f_{\alpha\beta}(B_\rho B_\sigma)^{ij})\left. \right].
\end{aligned}$$

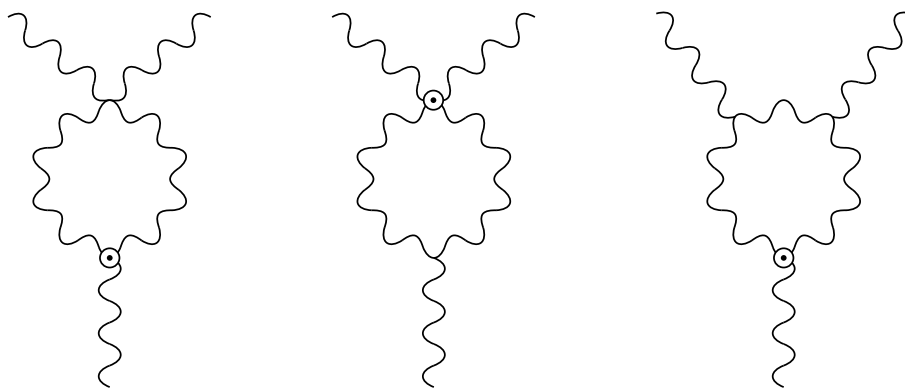
The matrix $W^{\alpha\beta,ab}$ analogous to $V^{\alpha\beta,ij}$ up to the change $B_\mu^i \leftrightarrow G_\mu^a$.

The one-loop effective action is

$$\begin{aligned}\Gamma_{\theta,2}^{(1)} &= \frac{i}{2} \text{Tr} \log \left(\mathcal{I} + \square^{-1} (N_1 + N_2 + T_1 + T_2 + T_3) \right) \\ &= \frac{i}{2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \text{Tr} \left(\square^{-1} N_1 + \square^{-1} N_2 + \square^{-1} T_1 + \square^{-1} T_2 + \square^{-1} T_3 \right)^n.\end{aligned}$$

the divergences in θ -linear order are all of the form $\theta f B^2$. Need to extract and compute only terms that contain three external fields.

$$\Gamma_{\theta,2}^{(1)} = \frac{i}{2} \text{Tr} [(\square^{-1} N_1)^2 \square^{-1} T_1 - \square^{-1} N_1 \square^{-1} T_2 - \square^{-1} N_2 \square^{-1} T_1].$$



1-loop divergent corrections to the θ -3-vertex also contains the contributions to the θ -4-vertex and θ -5-vertex.

Computed divergences due to the $U(1)_Y - SU(2)_L$ part of the noncommutative action, S_2^θ using background field method; divergent part calculated in momentum representation by dimensional regularization.

$$\begin{aligned} \text{Tr} (\Box^{-1} N_1 \Box^{-1} T_2) &= \frac{4i}{3(4\pi)^2 \epsilon} g' g^2 \kappa_2 \\ &\times \left[(6 - 2a)(\theta^{\rho\sigma} f_{\alpha\rho} + \theta_{\rho\alpha} f^{\sigma\rho})(B^{\alpha i} \partial_\mu \partial_\sigma B^{\mu i} - B^{\alpha i} \Box B_\sigma^i) \right. \\ &+ \left. (3a - 4)\theta^{\rho\sigma} f_{\rho\sigma}(B^{\nu i} \partial_\mu \partial_\nu B^{\mu i} - B_\mu^i \Box B^{\mu i}) \right], \end{aligned}$$

$$\begin{aligned} \text{Tr} (\Box^{-1} N_2 \Box^{-1} T_1) &= \frac{4i}{3(4\pi)^2 \epsilon} g' g^2 \kappa_2 \\ &\times \left[(2a - 6)(\theta^{\rho\sigma} f_{\alpha\rho} + \theta_{\rho\alpha} f^{\sigma\rho})(B^{\nu i} \partial_\sigma \partial^\alpha B_\nu^i + \partial_\sigma B^{\mu i} \partial^\alpha B_\mu^i) \right. \\ &+ \left. \theta^{\rho\sigma} f_{\rho\sigma}(18 - 11a)(\partial_\nu B^{\nu i} \partial_\mu B^{\mu i} + B_\mu^i \Box B^{\mu i}) \right], \end{aligned}$$

$$\begin{aligned} \text{Tr} (\Box^{-1} N_1^2 \Box^{-1} T_1) &= \frac{4i}{3(4\pi)^2 \epsilon} g' g^2 \kappa_2 \left[\theta^{\rho\sigma} f_{\rho\sigma} \left((22 - 14a) B_\mu^i \Box B^{\mu i} \right. \right. \\ &+ (15 - 10a) \partial_\nu B^{\mu i} \partial^\nu B_\mu^i \\ &+ (3a - 4) B^{\mu i} \partial_\mu \partial_\nu B^{\nu i} + (3 - a) \partial_\mu B^{\nu i} \partial_\nu B^{\mu i} \\ &+ (\theta^{\rho\sigma} f_{\alpha\rho} + \theta_{\rho\alpha} f^{\sigma\rho}) \left((2a - 6)(B_\sigma^i \Box B^{\alpha i} - B_\sigma^i \partial^\alpha \partial_\mu B^{\mu i} \right. \\ &+ B^{\mu i} \partial_\sigma \partial^\alpha B_\mu^i - \partial_\sigma B^{\mu i} \partial_\mu B^{\alpha i}) + (a - 3) \partial_\mu B^{\alpha i} \partial^\mu B_\sigma^i \\ &+ \left. \left. (3a - 9) \partial_\sigma B^{\mu i} \partial^\alpha B_\mu^i \right) \right]. \end{aligned}$$

The result for $U(1)_Y - SU(3)_C$ is analogous and follows immediately. Finally

$$\begin{aligned} \Gamma_{\text{div}}^{(1)} &= \frac{11}{3(4\pi)^2 \epsilon} \int d^4 x B_{\mu\nu}^i B^{\mu\nu i} + \frac{11}{2(4\pi)^2 \epsilon} \int d^4 x G_{\mu\nu}^a G^{\mu\nu a} \\ &+ \frac{4}{3(4\pi)^2 \epsilon} g' g^2 \kappa_2 (3 - a) \theta^{\mu\nu} \int d^4 x \left(\frac{1}{4} f_{\mu\nu} B_{\rho\sigma}^i B^{\rho\sigma i} - f_{\mu\rho} B_{\nu\sigma}^i B^{\rho\sigma i} \right) \\ &+ \frac{6}{3(4\pi)^2 \epsilon} g' g_S^2 \kappa_3 (3 - a) \theta^{\mu\nu} \int d^4 x \left(\frac{1}{4} f_{\mu\nu} G_{\rho\sigma}^a G^{\rho\sigma a} - f_{\mu\rho} G_{\nu\sigma}^a G^{\rho\sigma a} \right). \end{aligned}$$

Renormalization via Counterterms & $a = 3$

$$\begin{aligned}
 \mathcal{L} + \mathcal{L}_{ct} = & -\frac{1}{4}f_{0\mu\nu}f_0^{\mu\nu} - \frac{1}{4}B_{0\mu\nu}^i B_0^{\mu\nu i} - \frac{1}{4}G_{0\mu\nu}^a G_0^{\mu\nu a} \\
 & + g'^3 \kappa_1 \theta^{\mu\nu} \left(\frac{3}{4}f_{0\mu\nu}f_{0\rho\sigma}f_0^{\rho\sigma} - f_{0\mu\rho}f_{0\nu\sigma}f_0^{\rho\sigma} \right) \\
 & + g'_0 g_0^2 \kappa_2 \theta^{\mu\nu} \left(\frac{3}{4}f_{0\mu\nu}B_{0\rho\sigma}^i B_0^{\rho\sigma i} - f_{0\mu\rho}B_{0\nu\sigma}^i B_0^{\rho\sigma i} + c.p. \right) \\
 & + g'_0 (g_S)_0^2 \kappa_3 \theta^{\mu\nu} \left(\frac{3}{4}f_{0\mu\nu}G_{0\rho\sigma}^a G_0^{\rho\sigma a} - f_{0\mu\rho}G_{0\nu\sigma}^a G_0^{\rho\sigma a} + c.p. \right),
 \end{aligned}$$

Bare quantities are:

$$\begin{aligned}
 \mathcal{A}_0^\mu &= \mathcal{A}^\mu, \quad g'_0 = g', \\
 B_0^{\mu i} &= B^{\mu i} \sqrt{1 + \frac{44g^2}{3(4\pi)^2\epsilon}}, \quad g_0 = \frac{g \mu^{\epsilon/2}}{\sqrt{1 + \frac{44g^2}{3(4\pi)^2\epsilon}}}, \\
 G_0^{\mu a} &= G^{\mu a} \sqrt{1 + \frac{22g_S^2}{(4\pi)^2\epsilon}}, \quad (g_S)_0 = \frac{g_S \mu^{\epsilon/2}}{\sqrt{1 + \frac{22g_S^2}{(4\pi)^2\epsilon}}}.
 \end{aligned}$$

κ_1 , κ_2 and κ_3 unchanged under renormalization

$$\kappa_1 = (\kappa_1)_0, \quad \kappa_2 = (\kappa_2)_0, \quad \kappa_3 = (\kappa_3)_0,$$

replacement:

$$\begin{aligned}
 \frac{1}{g_1^2} &= \left(\frac{1}{g_1^2}\right)_0 + \frac{33}{18(4\pi)^2\epsilon}, & \frac{1}{g_2^2} &= \left(\frac{1}{g_2^2}\right)_0 + \frac{-11}{18(4\pi)^2\epsilon}, & \frac{1}{g_3^2} &= \left(\frac{1}{g_3^2}\right)_0 + \frac{-11}{18(4\pi)^2\epsilon}, \\
 \frac{1}{g_4^2} &= \left(\frac{1}{g_4^2}\right)_0 + \frac{-143}{18(4\pi)^2\epsilon}, & \frac{1}{g_5^2} &= \left(\frac{1}{g_5^2}\right)_0 + \frac{-121}{18(4\pi)^2\epsilon}, & \frac{1}{g_6^2} &= \left(\frac{1}{g_6^2}\right)_0 + \frac{110}{18(4\pi)^2\epsilon}.
 \end{aligned}$$

NC parameter θ need not be renormalized

because $\mathcal{L}^\theta$ is free from divergences.

Gauge sector Model 3: NC SU(N) GFT

$$S_{cl} = S_{\text{NCYM}} = \int d^4x \left(-\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{4} h \theta^{\mu\nu} d^{abc} \left(\frac{a}{4} F_{\mu\nu}^a F_{\rho\sigma}^b - F_{\mu\rho}^a F_{\nu\sigma}^b \right) F^{c\rho\sigma} \right),$$

Here earlier introduced noncommutativity deformation parameter h becomes very important.

Renormalization:

Second functional derivative $S^2[\phi]$ of S_{cl}

$$S^2 = \square + N_1 + N_2 + T_2 + T_3 + T_4 ,$$

N_1, N_2 - commutative vertices

T_2, T_3, T_4 non-commutative vertices

The 1-loop effective action computed by using BFM

$$\begin{aligned} \Gamma_{\theta,2}^{(1)} &= \frac{i}{2} \text{Tr} \log (\mathcal{I} + \square^{-1} (N_1 + N_2 + T_2 + T_3 + T_4)) \\ &= \frac{i}{2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \text{Tr} \left(\square^{-1} N_1 + \square^{-1} N_2 + \square^{-1} T_2 + \square^{-1} T_3 + \square^{-1} T_4 \right)^n . \end{aligned}$$

Vertices are

$$\begin{aligned} (N_1)^{ab\alpha\beta} &= -2i(A^\mu)^{ab} g^{\alpha\beta} \partial_\mu, \\ (N_2)^{ab\alpha\beta} &= -2f^{abc} F^{c\alpha\beta} - (A^\mu A_\mu)^{ab} g^{\alpha\beta}, \end{aligned}$$

Noncommutative vertices are

$$(T_2)^{ab\alpha\beta} = \frac{\hbar}{8} d^{abc} \left\{ \left[\left(\mathbf{a} \theta^{\rho\sigma} F_{\rho\sigma}^c g^{\alpha\nu} g^{\beta\mu} - 2(\mathbf{a} - 1) \theta^{\alpha\mu} F^{c\beta\nu} + 4\theta^\alpha{}_\rho F^{c\beta\rho} g^{\mu\nu} \right. \right. \right. \\ \left. \left. \left. + 4\theta^\mu{}_\rho F^{c\nu\rho} g^{\alpha\beta} \right) - (\beta \leftrightarrow \nu) \right] + [\alpha \leftrightarrow \beta] \right\} \partial_\mu \partial_\nu ,$$

$$(T_3)^{ab\alpha\beta} = \frac{i\hbar}{4} \left\{ d^{acd} \left[-2\mathbf{a} \theta^{\alpha\mu} (A_\nu)^{bc} F^{d\beta\nu} - 2\mathbf{a} \theta^{\beta\nu} (A_\nu)^{bc} F^{d\alpha\mu} \right. \right. \\ \left. \left. - \mathbf{a} \theta^{\rho\sigma} (A^\mu)^{bc} F_{\rho\sigma}^d g^{\alpha\beta} + \mathbf{a} \theta^{\rho\sigma} (A^\alpha)^{bc} F_{\rho\sigma}^d g^{\beta\mu} \right. \right. \\ \left. \left. - 2\theta^\alpha{}_\rho (A_\nu)^{bc} F^{d\nu\rho} g^{\beta\mu} + 2\theta^\alpha{}_\nu (A^\mu)^{bc} F^{d\beta\nu} \right. \right. \\ \left. \left. + 2\theta^{\mu\rho} (A^\nu)^{bc} F_{\nu\rho}^d - 2\theta^\mu{}_\rho (A^\alpha)^{bc} F^{d\beta\rho} - 2\theta^\beta{}_\rho (A^\alpha)^{bc} F^{d\mu\rho} \right. \right. \\ \left. \left. + 2\theta^\beta{}_\rho (A^\mu)^{bc} F^{d\alpha\rho} + 2\theta^\nu{}_\rho (A_\nu)^{bc} F^{d\mu\rho} g^{\alpha\beta} \right. \right. \\ \left. \left. + 2\theta^{\alpha\beta} (A_\nu)^{bc} F^{d\mu\nu} + 2\theta^{\alpha\nu} (A_\nu)^{bc} F^{d\beta\mu} + 2\theta^{\beta\mu} (A_\nu)^{bc} F^{d\alpha\nu} \right. \right. \\ \left. \left. - 2\theta^\nu{}_\rho (A_\nu)^{bc} F^{d\alpha\rho} g^{\beta\mu} + 2\theta^{\mu\nu} (A_\nu)^{bc} F^{d\alpha\beta} \right] \right. \\ \left. - [a \leftrightarrow b, \alpha \leftrightarrow \beta] \right\} \partial_\mu ,$$

$$(T_4)^{ab\alpha\beta} = \frac{\hbar}{8} d^{cde} \left[\left(-4\mathbf{a} \theta^{\alpha\rho} (A_\rho)^{ac} (A_\mu)^{bd} F^{e\beta\mu} - \mathbf{a} \theta^{\rho\sigma} (A^\mu)^{ac} (A_\mu)^{bd} F_{\rho\sigma}^e g^{\alpha\beta} \right. \right. \\ \left. \left. + \mathbf{a} \theta^{\rho\sigma} (A^\beta)^{ac} (A^\alpha)^{bd} F_{\rho\sigma}^e - 4\theta^{\alpha\rho} (A^\beta)^{ad} (A^\mu)^{bc} F_{\mu\rho}^e \right. \right. \\ \left. \left. + 4\theta^\alpha{}_\rho (A^\mu)^{ad} (A_\mu)^{bc} F^{e\beta\rho} + 4\theta^\nu{}_\rho (A_\nu)^{ad} (A_\mu)^{bc} F^{e\mu\rho} g^{\alpha\beta} \right. \right. \\ \left. \left. + 2\theta^{\alpha\beta} (A_\mu)^{ad} (A_\nu)^{bc} F^{e\mu\nu} + 4\theta^{\alpha\rho} (A_\mu)^{ad} (A_\rho)^{bc} F^{e\beta\mu} \right. \right. \\ \left. \left. + 2\theta^{\rho\sigma} (A_\rho)^{ad} (A_\sigma)^{bc} F^{e\alpha\beta} \right) + (a \leftrightarrow b, \alpha \leftrightarrow \beta) \right. \\ \left. + f^{abc} (2\mathbf{a} \theta^{\rho\sigma} F_{\rho\sigma}^d F^{e\alpha\beta} + \mathbf{a} \theta^{\alpha\beta} F_{\rho\sigma}^d F^{e\rho\sigma} \right. \\ \left. + 4\theta_{\rho\sigma} F^{d\alpha\rho} F^{e\beta\sigma} + 8\theta^{\alpha\rho} F^{d\beta\mu} F_{\mu\rho}^e \right] .$$

The divergent parts are calculated in the momentum representation via dimensional regularization

$$\begin{aligned}
D_1^{\text{div}} &= \frac{i}{2} \text{Tr} \left(\left(\square^{-1} N_1 \right)^2 \left(\square^{-1} T_4 \right) \right)^{\text{div}} \\
&= \frac{h}{(4\pi)^2 \epsilon} d^{abc} \\
&\times \int d^4 x \left[\frac{a-3}{4} (\theta^{\alpha\rho} F_{\alpha\nu}^a + \theta_{\alpha\nu} F^{a\alpha\rho}) (V^\nu V^\mu V_\mu V_\rho)^{bc} \right. \\
&\quad \left. + \frac{3a-4}{4} \theta^{\rho\sigma} F_{\rho\sigma}^a (V^\mu V^\nu V_\nu V_\mu)^{bc} \right],
\end{aligned}$$

$$\begin{aligned}
D_2^{\text{div}} &= -\frac{i}{2} \text{Tr} \left(\left(\square^{-1} N_1 \right)^3 \left(\square^{-1} T_3 \right) \right)^{\text{div}} \\
&= \frac{h}{(4\pi)^2 \epsilon} d^{abc} \\
&\times \int d^4 x \left[\frac{7-3a}{6} \theta^{\rho\sigma} F_{\rho\sigma}^a (V_\mu V^\mu V_\nu V^\nu + V_\mu V_\nu V^\mu V^\nu + V_\mu V_\nu V^\nu V^\mu)^{bc} \right. \\
&\quad + \frac{3-2a}{6} (\theta^{\alpha\rho} F_{\alpha\sigma}^a + \theta_{\alpha\sigma} F^{a\alpha\rho}) (V_\rho V^\sigma V^\mu V_\mu + V_\rho V_\mu V^\sigma V^\mu \\
&\quad \left. + V_\rho V^\mu V_\mu V^\sigma)^{bc} \right],
\end{aligned}$$

$$\begin{aligned}
D_3^{\text{div}} &= \frac{i}{2} \text{Tr} \left(\left(\square^{-1} N_1 \right)^4 \left(\square^{-1} T_2 \right) \right)^{\text{div}} \\
&= \frac{h}{(4\pi)^2 \epsilon} d^{abc} \\
&\times \int d^4 x \left[\frac{7a-11}{12} \theta^{\rho\sigma} F_{\rho\sigma}^a (V_\mu V^\mu V_\nu V^\nu + V_\mu V_\nu V^\mu V^\nu + V_\mu V_\nu V^\nu V^\mu)^{bc} \right. \\
&\quad + \frac{a-3}{12} (\theta^{\alpha\rho} F_{\alpha\sigma}^a + \theta_{\alpha\sigma} F^{a\alpha\rho}) (2V^\mu V_\mu V_\rho V^\sigma + 2V^\mu V_\rho V_\mu V^\sigma \\
&\quad \left. + V^\mu V_\rho V^\sigma V_\mu + V_\rho V^\mu V_\mu V^\sigma)^{bc} \right],
\end{aligned}$$

$$\begin{aligned}
D_4^{\text{div}} &= -\frac{i}{2} \text{Tr} \left(\left(\square^{-1} N_2 \right) \left(\square^{-1} T_4 \right) \right)^{\text{div}} \\
&= \frac{h}{(4\pi)^2 \epsilon} d^{abc} \int d^4 x \left[\frac{4-3a}{4} \theta^{\rho\sigma} F_{\rho\sigma}^a (V_\mu V_\nu V^\nu V^\mu)^{bc} \right. \\
&\quad + \frac{2-a}{2} (\theta^{\alpha\rho} F_{\alpha\mu}^a + \theta_{\alpha\mu} F^{a\alpha\rho}) (V^\mu V^\nu V_\nu V_\rho)^{bc} \\
&\quad + i(2(a+1)\theta_{\alpha\nu} F_{\beta\mu}^a (V^\mu F^{\alpha\beta} V^\nu)^{bc} + \theta^{\alpha\beta} F_{\mu\nu}^a (V^\mu F_{\alpha\beta} V^\nu)^{bc}) \\
&\quad + 2i\theta^{\alpha\beta} F_{\beta\mu}^a \left(V_\nu F^{\mu\nu} V_\alpha - V^\mu F^{\alpha\beta} V^\nu \right)^{bc} \\
&\quad - i\theta^{\alpha\beta} F_{\mu\nu}^a (V_\alpha F^{\mu\nu} V_\beta)^{bc} - 2N\theta^{\beta\mu} F_{\mu\nu}^a F^{b\alpha\nu} V_{\alpha\beta}^c \\
&\quad \left. + N\theta^{\mu\nu} F_{\alpha\mu}^a F_{\beta\nu}^b F^{c\alpha\beta} - a\frac{3}{4}N\theta^{\mu\nu} F_{\mu\nu}^a F_{\alpha\beta}^b F^{c\alpha\beta} \right],
\end{aligned}$$

$$\begin{aligned}
D_5^{\text{div}} &= \frac{i}{2} \text{Tr} \left(\left(\square^{-1} N_2 \right)^2 \left(\square^{-1} T_2 \right) \right)^{\text{div}} \\
&= \frac{h}{(4\pi)^2 \epsilon} d^{abc} \\
&\times \int d^4 x \left[\frac{a-3}{2} (\theta_{\alpha\rho} F^{a\alpha\mu} + \theta^{\alpha\mu} F_{\alpha\rho}^a) (F_{\mu\nu} F^{\nu\rho})^{bc} \right. \\
&\quad \left. + \theta^{\rho\sigma} F_{\rho\sigma}^a \left(\frac{3a-4}{4} (F_{\mu\nu} F^{\mu\nu})^{bc} + \frac{3a-7}{4} (V^\mu V_\mu V^\nu V_\nu)^{bc} \right) \right],
\end{aligned}$$

$$\begin{aligned}
D_6^{\text{div}} &= \frac{i}{2} \text{Tr} \left(\left[\left(\square^{-1} N_1 \right) \left(\square^{-1} N_2 \right) + \left(\square^{-1} N_2 \right) \left(\square^{-1} N_1 \right) \right] \left(\square^{-1} T_3 \right) \right)^{\text{div}} \\
&= \frac{h}{(4\pi)^2 \epsilon} d^{abc} \\
&\times \int d^4 x \left[\frac{a-3}{2} \theta^{\alpha\sigma} F_{\rho\sigma}^a (V_\alpha V_\beta V^\beta V^\rho)^{bc} - \frac{a}{4} \theta^{\rho\sigma} F_{\rho\sigma}^a (F_{\alpha\beta} F^{\alpha\beta})^{bc} \right. \\
&\quad - \theta_{\alpha\sigma} F_{\beta\rho}^a (a+1) \left(2i(V^\alpha F^{\rho\sigma} V^\beta)^{bc} - (F^{\rho\sigma} F^{\alpha\beta})^{bc} \right) \\
&\quad + \frac{a}{2} \theta^{\beta\sigma} F_{\rho\sigma}^a (V_\alpha V^\alpha V_\beta V^\rho + V_\alpha V^\alpha V^\rho V_\beta)^{bc} \\
&\quad \left. + \theta^{\rho\sigma} F_{\rho\sigma}^a \left(\frac{3a-4}{4} V_\alpha V_\beta V^\beta V^\alpha + \frac{5a-4}{4} V_\alpha V^\alpha V_\beta V^\beta - 2V_\alpha V_\beta V^\alpha V^\beta \right)^{bc} \right] + \dots,
\end{aligned}$$

$$\begin{aligned}
D_7^{\text{div}} &= -\frac{i}{2} \text{Tr} \left(\sum \left(\square^{-1} N_1 \right)^2 \left(\square^{-1} N_2 \right) \left(\square^{-1} T_2 \right) \right)^{\text{div}} \\
&= \frac{h}{(4\pi)^2 \epsilon} d^{abc} \\
&\times \int d^4 x \left[\frac{18 - 11a}{12} \theta^{\alpha\beta} F_{\alpha\beta}^a (2V^\mu V_\mu V^\nu V_\nu + V^\mu V^\nu V_\nu V_\mu)^{bc} \right. \\
&+ \left. \frac{3 - a}{6} (\theta^{\alpha\mu} F_{\beta\mu}^a + \theta_{\beta\mu} F^{a\alpha\mu}) (V_\alpha V^\beta V^\nu V_\nu + V^\nu V_\nu V^\beta V_\alpha + V_\alpha V^\nu V_\nu V^\beta)^{bc} \right].
\end{aligned}$$

$$\sum_{i=1}^7 D_i^{\text{div}} = \frac{N}{(4\pi)^2 \epsilon} h \theta^{\mu\nu} d^{abc} \int d^4 x \left(-\frac{25a - 3}{48} F_{\mu\nu}^a F_{\rho\sigma}^b + \frac{a + 21}{12} F_{\mu\rho}^a F_{\nu\sigma}^b \right) F^{c\rho\sigma},$$

Renormalization of the theory:

To cancel divergences, counter terms should be added to the starting action, which produces the bare Lagrangian

$$\begin{aligned}
\mathcal{L}_0 &= -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{11Ng^2}{6(4\pi)^2 \epsilon} F_{\mu\nu}^a F^{a\mu\nu} \\
&+ \frac{1}{4} g \mu^{\epsilon/2} h \theta^{\mu\nu} d^{abc} \left(\frac{a}{4} F_{\mu\nu}^a F_{\rho\sigma}^b - F_{\mu\rho}^a F_{\nu\sigma}^b \right) F^{c\rho\sigma} \\
&- \frac{Ng^3 \mu^{\epsilon/2}}{(4\pi)^2 \epsilon} h \theta^{\mu\nu} d^{abc} \left(\frac{3 - 25a}{48} F_{\mu\nu}^a F_{\rho\sigma}^b + \frac{21 + a}{12} F_{\mu\rho}^a F_{\nu\sigma}^b \right) F^{c\rho\sigma} \\
&= -\frac{1}{4} F_0^a{}_{\mu\nu} F_0^{a\mu\nu} + \frac{1}{4} g \mu^{\epsilon/2} h \theta^{\mu\nu} d^{abc} \\
&\times \left[\frac{a}{4} \left(1 - \frac{3 - 25a}{3a} \frac{Ng^2}{(4\pi)^2 \epsilon} \right) F_{\mu\nu}^a F_{\rho\sigma}^b \right. \\
&\quad \left. - \left(1 + \frac{21 + a}{3} \frac{Ng^2}{(4\pi)^2 \epsilon} \right) F_{\mu\rho}^a F_{\nu\sigma}^b \right] F^{c\rho\sigma}.
\end{aligned}$$

To obtain the same structure as in starting Lagrangian we have to impose the condition

$$\left(-\frac{25a-3}{48}\right) : \left(\frac{a+21}{12}\right) = \frac{a}{4} : (-1).$$

Solutions, $a = 1$ and $a = 3$.

The case $a = 1$ correspondes to Model 1 : mNCSM; the deformation parameter h need not to be renormalized. Renormalization obtained through the renormalizations of gauge fields the coupling constant.

The case $a = 3$ is different since the NC deformation parameter h has to be renormalized.

The bare gauge field, the coupling constant and the NC deformation parameter are :

$$\begin{aligned} V_0^\mu &= V^\mu \sqrt{1 + \frac{22Ng^2}{3(4\pi)^2\epsilon}}, \\ g_0 &= \frac{g\mu^{\epsilon/2}}{\sqrt{1 + \frac{22Ng^2}{3(4\pi)^2\epsilon}}}, \\ h_0 &= \frac{h}{1 - \frac{2Ng^2}{3(4\pi)^2\epsilon}}, \end{aligned}$$

Ultraviolet asymptotic behaviour of NC SU(N) GFT via RGE

Gauge coupling constant g in our theory depends on energy i.e., the renormalization point μ , satisfying the same beta function as in QCD

$$\beta_g = \mu \frac{\partial}{\partial \mu} g(\mu) = -\frac{11Ng^3(\mu)}{3(4\pi)^2},$$

our theory is UV stable, i.e. asymptotically free:

$$\alpha_s(\mu) = \frac{g^2(\mu)}{4\pi} = \frac{6\pi}{11N} \frac{1}{\ln \frac{\mu}{\Lambda}}.$$

Λ is an integration constant not predicted by the theory; free parameter to be determined from experiment: hadronic production in e^+e^- annihilation at the Z resonance has given $\alpha_s(m_Z) = 0.12$ corresponding to $\Lambda = \Lambda_{\text{QCD}} \simeq 250$ MeV.

$$\beta_h = \mu \frac{\partial}{\partial \mu} h(\mu) = -\frac{11Ng^2(\mu)}{24\pi^2} h(\mu).$$

Both β functions are negative $\rightarrow$ decrease with increasing energy μ . Solution to β_h :

$$h(\mu) = \frac{h_0}{\ln \frac{\mu}{\Lambda}}, \quad \Rightarrow$$

running deformation parameter h . h_0 is an additional integration constant, physical interpretation later.

By increase of μ the $\hbar$ decreases, $\Rightarrow$ modification of Heisenberg uncertainty relations at high energy

$$[x, p] = i\hbar(1 + \beta p^2),$$

$$\Delta x = \frac{\hbar}{2} \left(\frac{1}{\Delta p} + \beta \Delta p \right).$$

Large momenta $\rightarrow$ distance Δx grows linearly: So large energies do not necessarily correspond to small distances. Running $\hbar$ does not imply that noncommutativity vanishes at small distances. Related to UV/IR correspondence. By assuming

$$\hbar(\mu) = \frac{1}{\Lambda_{\text{NC}}^2(\mu)}$$

Λ_{NC} becomes a function of energy μ too giving:

$$\mu \frac{d}{d\mu} \Lambda_{\text{NC}}(\mu) = \frac{11N}{3(4\pi^2)} g^2(\mu) \Lambda_{\text{NC}}(\mu),$$

$$\Lambda_{\text{NC}}(\mu) = \Lambda_\theta \sqrt{\ln \frac{\mu}{\Lambda}}.$$

Λ_{NC} becomes the running scale of non-commutativity.

* Physical interpretation of $\hbar_0$ and/or Λ_θ is not quite clear; they have to be proportional to the scale of noncommutativity Λ_{NC} .

* Assume that in a first approximation

$$\hbar_0 = 1/\Lambda_\theta^2 = 1/\Lambda_{\text{NC}}^2.$$

* Considering typical QCD energies, $\mu = m_Z$, factor $\sqrt{\ln(m_Z/\Lambda_{\text{QCD}})} \simeq 2.4$

FORBIDDEN DECAYS

GAUGE SECTOR: $Z \rightarrow \gamma\gamma$ decay

[W. Behr, N. G. Deshpande, G. Duplanić, P. Schupp, J.T. and J. Wess;
The $Z \rightarrow \gamma\gamma$, $g g$ decays in the non-commutative standard model, Eur.
Phys. J. C **29**, 441 (2003)]

[G. Duplanić, P. Schupp and J. Trampetić; Comment on triple gauge
boson interactions in the non-commutative electroweak sector, Eur. Phys.
J. C **32** (2003) 141]

[M. Buric, V. Radovanovic and J.T., The one-loop renormalization of the
gauge sector in the noncommutative standard model; JHEP **03** (2007) 030]

[M. Buric, D. Latas, V. Radovanovic and J.T., Improved $Z \rightarrow \gamma\gamma$ decay in
the renormalizable NCSM; hep-ph/0611299, to appear in PRD.]

From $\mathcal{L}_{Z\gamma\gamma} \Rightarrow$ the gauge-invariant amplitude $\mathcal{A}_{Z \rightarrow \gamma\gamma}$

$$\begin{aligned} \mathcal{A}^\theta(Z \rightarrow \gamma\gamma) &= -2e \sin 2\theta_W \mathbf{K}_{Z\gamma\gamma} \Theta_3^{\mu\nu\rho}(a; k_1, -k_2, -k_3) \\ &\times \epsilon_\mu(k_1) \epsilon_\nu(k_2) \epsilon_\rho(k_3); \end{aligned}$$

$$k_1 + k_2 + k_3 = 0;$$

$$\begin{aligned} \Theta_3^{\mu\nu\rho}(a; k_1, k_2, k_3) &= -(k_1 \theta k_2) \\ &\times [(k_1 - k_2)^\rho g^{\mu\nu} + (k_2 - k_3)^\mu g^{\nu\rho} + (k_3 - k_1)^\nu g^{\rho\mu}] \\ &- \theta^{\mu\nu} [k_1^\rho (k_2 k_3) - k_2^\rho (k_1 k_3)] \\ &- \theta^{\nu\rho} [k_2^\mu (k_3 k_1) - k_3^\mu (k_2 k_1)] \\ &- \theta^{\rho\mu} [k_3^\nu (k_1 k_2) - k_1^\nu (k_3 k_2)] \\ &+ (\theta k_2)^\mu [g^{\nu\rho} k_3^2 - k_3^\nu k_3^\rho] + (\theta k_3)^\mu [g^{\nu\rho} k_2^2 - k_2^\nu k_2^\rho] \\ &+ (\theta k_3)^\nu [g^{\mu\rho} k_1^2 - k_1^\mu k_1^\rho] + (\theta k_1)^\nu [g^{\mu\rho} k_3^2 - k_3^\mu k_3^\rho] \\ &+ (\theta k_1)^\rho [g^{\mu\nu} k_2^2 - k_2^\mu k_2^\nu] + (\theta k_2)^\rho [g^{\mu\nu} k_1^2 - k_1^\mu k_1^\nu] \\ &+ \theta^{\mu\alpha} (a k_1 + k_2 + k_3)_\alpha [g^{\nu\rho} (k_3 k_2) - k_3^\nu k_2^\rho] \\ &+ \theta^{\nu\alpha} (k_1 + a k_2 + k_3)_\alpha [g^{\mu\rho} (k_3 k_1) - k_3^\mu k_1^\rho] \\ &+ \theta^{\rho\alpha} (k_1 + k_2 + a k_3)_\alpha [g^{\mu\nu} (k_2 k_1) - k_2^\mu k_1^\nu] . \end{aligned}$$

Z-boson at rest

$$\begin{aligned}\Gamma_{Z\rightarrow\gamma\gamma} &= \frac{\alpha}{12} \frac{M_Z^5}{\Lambda_{\text{NC}}^4} \sin^2 2\theta_W \mathcal{K}_{Z\gamma\gamma}^2 \\ &\times \frac{1}{6} \left[(13a^2 - 50a + 51) \vec{E}_\theta^2 + (a^2 + 2a + 3) \vec{B}_\theta^2 \right], \\ \vec{E}_\theta &= (\theta^{01}, \theta^{02}, \theta^{03}), \quad \vec{B}_\theta = (\theta^{23}, \theta^{13}, \theta^{12})\end{aligned}$$

For $a = 1$

$$\Gamma_{Z\rightarrow\gamma\gamma} = \frac{\alpha}{12} \frac{M_Z^5}{\Lambda_{\text{NC}}^4} \sin^2 2\theta_W \mathcal{K}_{Z\gamma\gamma}^2 \left[\frac{7}{3} \vec{E}_\theta^2 + \vec{B}_\theta^2 \right]$$

Z-boson at rest and polarized along the 3-axis

$$\begin{aligned}\Gamma_{Z\rightarrow\gamma\gamma}^3 &= \frac{\alpha}{4} \frac{M_Z^5}{\Lambda_{\text{NC}}^4} \sin^2 2\theta_W \mathcal{K}_{Z\gamma\gamma}^2 \\ &\times \left[\frac{2}{5} \left((E_\theta^1)^2 + (E_\theta^2)^2 \right) + \frac{23}{15} (E_\theta^3)^2 + (B_\theta^3)^2 \right]\end{aligned}$$

For $a = 3$

Z-boson at rest

$$\begin{aligned}\Gamma_{Z\rightarrow\gamma\gamma} &= \frac{\alpha}{4} \frac{M_Z^5}{\Lambda_{\text{NC}}^4} \sin^2 2\theta_W \mathcal{K}_{Z\gamma\gamma}^2 (\vec{E}_\theta^2 + \vec{B}_\theta^2) \\ \Gamma_{Z\rightarrow\gamma\gamma}^3 &= \frac{\alpha}{60} \frac{M_Z^5}{\Lambda_{\text{NC}}^4} \sin^2 2\theta_W \mathcal{K}_{Z\gamma\gamma}^2 \left[\vec{E}_\theta^2 + \vec{B}_\theta^2 \right. \\ &\quad \left. + 42 \left((E_\theta^3)^2 + (B_\theta^3)^2 \right) \right]\end{aligned}$$

Experiments

Decay mode: $Z \rightarrow \gamma\gamma$, old measurements:

$$BR = \frac{\Gamma(Z \rightarrow \gamma\gamma)}{\Gamma_{tot}(Z)} \left\{ \begin{array}{lll} < 5.2 \times 10^{-5} & \text{L3} & 1995 \\ < 5.5 \times 10^{-5} & \text{DELPHI} & 1994 \\ < 1.4 \times 10^{-4} & \text{OPAL} & 1991 \end{array} \right.$$

$e^+e^- \rightarrow \gamma\gamma$ near Z resonance is an ideal process to test QED. The present statistic enables comparison of data with the QED up to $\mathcal{O}(\alpha^3)$.

Deviation of the experimentally measured cross sections from the QED prediction $\longrightarrow$ evidence for $Z \rightarrow \gamma\gamma$ (SM forbidden) and $Z \rightarrow \pi^0\gamma / \eta\gamma$.

The forbidden decay $Z \rightarrow \gamma\gamma$ and the real decays $Z \rightarrow \pi^0\gamma / \eta\gamma$ would have the same experimental signature as the SM forbidden process

$$e^+e^- \rightarrow Z^* \rightarrow \gamma\gamma$$

Rare decays at high energies, the two photons from π^0 or η decays are very close seen in EM calorimeter as a **single** high energy photon:

$$e^+e^- \rightarrow Z^* \rightarrow (\pi^0, \eta)\gamma \rightarrow (\gamma\gamma)\gamma$$

Theoretical estimates $Br(Z \rightarrow \pi^0\gamma / \eta\gamma) \sim 10^{-10}$.
(Arnellos et al. Nucl.Phys.B 196 (1982) 378)

$Z \rightarrow \gamma\gamma$ LHC experimental possibilities:

CMS Physics Technical Design Report:

10^7 events of $Z \rightarrow e^+e^-$ for 10 fb^{-1} in 2 years of LHC

Assuming $BR(Z \rightarrow \gamma\gamma) \sim 10^{-8}$ and using $BR(Z \rightarrow e^+e^-) = 0.03 \Rightarrow \sim 3$ events of $Z \rightarrow \gamma\gamma$ with 10 fb^{-1}

Background sources (CMS Note 2006/112, Fig.3):

1. Study for $Higgs \rightarrow \gamma\gamma$ shows that, when e^- from $Z \rightarrow e^+e^-$ radiates very high energy Bremsstrahlung photon into pixel detector, for similar energies of e^- and γ , there is a huge probability of misidentification of e^- with γ !

2. Irreducible di-photon background may kill signal.

After 10 years of LHC running $\text{Int.L} \sim 1000 \text{ fb}^{-1}$ and assuming $BR(Z \rightarrow \gamma\gamma) \sim 10^{-8}$

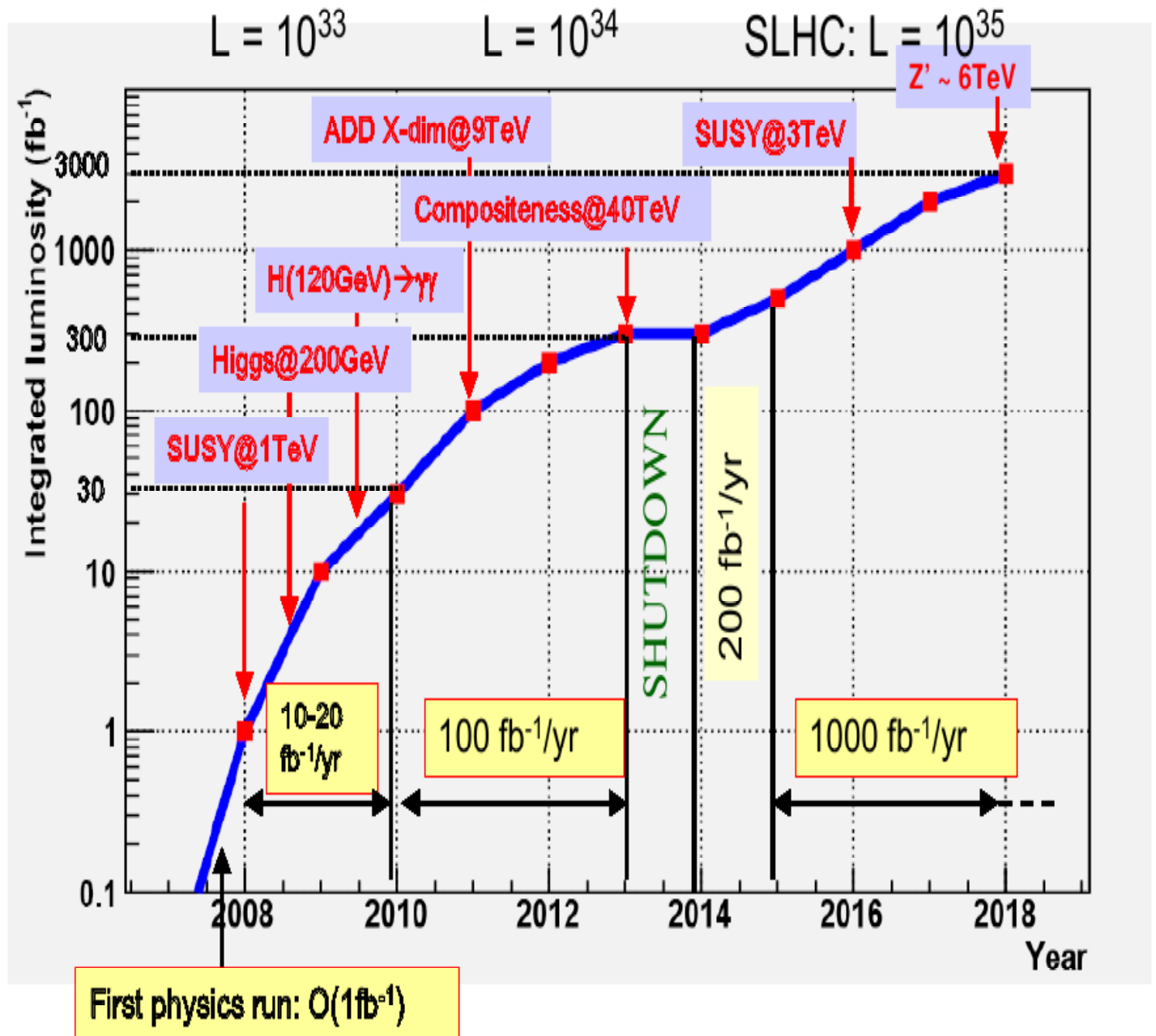
$\Rightarrow \sim 300$ events of $Z \rightarrow \gamma\gamma$ decays, OR

$\Rightarrow \sim 3$ events with $BR(Z \rightarrow \gamma\gamma) \sim 10^{-10}$

$\Rightarrow$ NC scale $\Lambda_{\text{NC}} \gtrsim 3.0 \text{ TeV}$



Probable/possible LHC luminosity profile - need for L-upgrade in a longer term



for the 2008 run likely to get from 100pb^{-1} to 1fb^{-1}

Discussion: $Z \rightarrow \gamma\gamma$

The range of the scale of non-commutativity:

$$1 \geq \Lambda_{\text{NC}}/\text{TeV} \geq 0.25$$

For $|K_{Z\gamma\gamma}| \simeq 0.05$ & NC strenght ($\vec{B}_\theta^2 + \vec{E}_\theta^2 \simeq 1$)

$$10^{-8} \lesssim BR^{\text{nmNCSM}}(Z \rightarrow \gamma\gamma) \lesssim 10^{-5}$$

– From L3 Collaboration (1995) experiment:

$$BR(Z \rightarrow \gamma\gamma) < 5.2 \times 10^{-5}$$

$$a = 3, \quad (\vec{E}_\theta^2 + \vec{B}_\theta^2 \simeq 1) \Rightarrow \Lambda_{\text{NC}} \gtrsim 110 \text{ GeV}$$

– From LHC/CMS Collaboration simulation we should have:

1.) In 2008 with integrated luminosity $10 \text{ fb}^{-1}/\text{yr}$:

$$3 \text{ events \& } BR(Z \rightarrow \gamma\gamma) \lesssim 10^{-8}$$

2.) In 2016 with integrated luminosity $1000 \text{ fb}^{-1}/\text{yr}$:

$$300 \text{ events \& } BR(Z \rightarrow \gamma\gamma) \lesssim 10^{-8}$$

$$a = 3, \quad (\vec{E}_\theta^2 + \vec{B}_\theta^2 \simeq 1) \Rightarrow \Lambda_{\text{NC}} \gtrsim 1.0 \text{ TeV}$$

$$3 \text{ events \& } BR(Z \rightarrow \gamma\gamma) \lesssim 10^{-10}$$

$$a = 3, \quad (\vec{E}_\theta^2 + \vec{B}_\theta^2 \simeq 1) \Rightarrow \Lambda_{\text{NC}} \gtrsim 3.0 \text{ TeV}$$

SUMMARY

★ Principle of renormalizability implemented on our θ -expanded NCGFT led us to well defined deformations via introduction of higher order NC gauge action class for mNCSM, nmNCSM and NC SU(N) models. This extension was parametrized by generically free parameter a .

★ Divergences cancel differently than in commutative GFT and this depends on the representations.

★ Model 1: mNCSM gauge sector is renormalizable for $a = 1$. No renormalization of h .

★ Model 2: nmNCSM gauge sector is renormalizable and FINITE for $a = 3$. No renormalization of h .

★ Model 3: NC SU(N) theory is renormalizable only for $a = 1, 3$.

– Case $a = 1$, requires no renormalization of h .

– Case $a = 3$, requires renormalization of NC deformation parameter h , which becomes *the running deformation parameter and vanishes for large μ* .

– Λ_{NC} runs too and it is very smooth, $\Rightarrow$ small change when μ increases $\Rightarrow$ large degree of stability of NC SU(N) theory within a wide range of μ .

CONCLUSION

★ Renormalization principle is fixing the freedom parameter $a = 1, 3$ for our θ -expanded NC GFT :

$$S_{\star}^a = -\frac{1}{2} \text{Tr} \int d^4x \left(1 - \frac{a-1}{2} h \theta^{\mu\nu} \hat{F}_{\mu\nu} \right) \star \hat{F}_{\rho\sigma} \star \hat{F}^{\rho\sigma}.$$

This way principle of renormalization determines NC renormalizable deformation.

★ The solution $a = 3$, while shifting the model to the higher order, hints into the discovery of the key role of the higher NC gauge interaction in 1-loop renormalizability of classes of NCGFT at θ^1 .

★ Hence, the nmNCSM gauge sector, which produces SM forbidden $Z \rightarrow \gamma\gamma$ decay, is renormalizable and FINITE $\rightarrow$ no renormalization of h needed.

★ Hence, in the case of NC SU(N) the noncommutativity deformation parameter h had to be renormalized and it is asymptotically free, opposite to the previous expectations.

★ We believe that this could be of paramount importance in modifying matter sector so that it becomes renormalizable.

★ Phenomenological results as $Z \rightarrow \gamma\gamma$ are ROBUST due to the 1-loop renormalizability and finiteness of the nmNCSM gauge sector.