The Correspondence between Noncommutative Field Theory and Gravity

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Outline

NC geometry

Quantum mechanics implies NC geometry at short distances.

(Doplicher+Fredenhagen+Roberts, CMP **172** (1995) 187)

Quantum mechanics: $\hbar$-deformation

- NC phase space: $[x^\mu, p_\nu] = i\hbar\delta^\mu{}_\nu$
- Uncertainty principle: $\Delta x \Delta p \sim \hbar$
- Equivalence principle: NC geometry at short distances

NC field theory: θ-deformation

- NC spacetime: $[y^\mu, y^\nu]_\star = i\theta^{\mu\nu}$
- Translation = inner automorphism: $[y^\mu, f(y)]_\star = i\theta^{\mu\nu}\partial_\nu f(y)$
- ∞-dimensional symmetry: symplectomorphism = NC gauge symmetry
  (Cornalba, ATMP **4** (2000) 271; Jurčo+Schupp, EPJ **C4** (2000) 367;
  Jurčo+Schupp+Wess, NPB **584** (2000) 784)

- Translation = NC gauge symmetry: $e^{ik\cdot y} \star f(y) \star e^{-ik\cdot y} = f(y + \theta \cdot k)$
- No local gauge invariant observables

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Large N duality

(HSY, hep-th/0612231; arXiv:0704.0929)

NC field theory = matrix model or large N field theory.

Large N gauge theory ('t Hooft, NPB **72** (1974) 461)

- Large N gauge theory = dual description of gravity in higher dimensions
- 1/N expansion = genus expansion of (closed) string amplitudes
- BFSS, IKKT matrix models and AdS/CFT duality

NC field = master field in large N gauge theory

- NC $\mathbf{R}^2$: $[a, a^\dagger] = 1, \quad \mathcal{H} = \{|n\rangle; n = 0, 1, \cdots\}$
- Complete operator basis:
  $\mathcal{A}_\theta = \{|m\rangle\langle n|; n, m = 0, 1, \cdots\} \ni \phi(x, y) = \sum_{n,m} M_{mn}|m\rangle\langle n|$
- Scalar field on NC $\mathbf{R}^2$ (or Σ_g) $\Leftrightarrow$ $N \times N$ matrix at $N \to \infty$

- NC gauge symmetry $U_{\rm cpt}(\mathcal{H}) \supset U(N \to \infty)$
- 1/N expansion = NC deformation in terms of θ

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NC field theory is a theory of gravity ?

YES !!!

What is the origin of diffeomorphism symmetry, the underlying local symmetry of gravity ?

Symplectic geometry

- Symplectic manifold (M, ω): M a smooth manifold and $\omega \in \Lambda^2(M)$ a closed 2-form i.e., $d\omega = 0$
- Darboux theorem: Locally, $(M, \omega) \cong (\mathbf{C}^n, \sum dq^i \wedge dp_i)$

Riemannian geometry

- Riemannian manifold (M, g): M a smooth manifold and $g : TM \otimes TM \rightarrow \mathbf{R}$ a symmetric bilinear form
- Equivalence principle: Locally, $(M, g) \cong (\mathbf{R}^n, \sum dx^\mu \otimes dx^\mu)$

- $[\omega'] = [\omega] \in H^2(M)$ such that $\omega' = \omega + dA$, $\exists\, \phi : M \rightarrow M \in Diff(M)$

$$\frac{\partial x^\alpha}{\partial y^\mu}\frac{\partial x^\beta}{\partial y^\nu}\omega'_{\alpha\beta}(x) = \omega_{\mu\nu}(y) \qquad (1)$$

DBI O(θ)

- $g_{\mu\nu}(x) = e^a_\mu e^b_\nu \delta_{ab}$ where $e^a_\mu \in GL(n, \mathbf{R})$

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DBI action as a generalized geometry

(HSY, hep-th/0611174; hep-th/0612231)

- DBI action

$$S = \frac{2\pi}{g_s(2\pi\kappa)^{\frac{p+1}{2}}} \int d^{p+1}x \sqrt{\det(g + \kappa(B+F))} + \mathcal{O}(\sqrt{\kappa}\partial F, \cdots)$$

- Λ-symmetry: $B \to B - d\Lambda, \quad A \to A + \Lambda$
  $\xrightarrow{\Lambda = d\lambda}$ $U(1)$ gauge symmetry: $B \to B, \quad A \to A + d\lambda$

- S depends only on the symplectic structure $\Omega \equiv B + F$ on M
- D-brane data = derived category: $(M, g, \Omega) = (M, g + \kappa\Omega)$
- D-manifold = generalized complex geometry:
  symplectic geometry $(|\kappa B g^{-1}| \gg 1)$ $\leftrightharpoons$ complex geometry $(|\kappa B g^{-1}| \ll 1)$
  (Hitchin, QJM Oxford **54** (2003) 281; Gualtieri, math.DG/0401221)
- $[\Omega = B + F] = [\omega = B] \in H^2(M)$ since $F = dA$
  $\exists\, \phi \in \mathit{Diff}(M)$ such that $\phi^*(\Omega) = \omega$ Darboux

- A remarkable identity (Cornalba, ATMP **4** (2000) 271)

$$\int d^{p+1}x \sqrt{\det(g + \kappa(B + F(x)))} = \int d^{p+1}y \sqrt{\det(\kappa B + h(y))} \tag{2}$$

where fluctuations of gauge fields now appear as an induced metric on the brane given by

$$h_{\mu\nu}(y) = \frac{\partial x^\alpha}{\partial y^\mu} \frac{\partial x^\beta}{\partial y^\nu} g_{\alpha\beta}$$

- Covariant coordinates $x^\mu(y) \equiv y^\mu + \theta^{\mu\nu}\widehat{A}_\nu(y)$ are dynamical fields: RHS of Eq.(2) $=$ NC (semi-classical) gauge theory
- Eq.(2) $=$ Seiberg-Witten equivalence (Seiberg+Witten, JHEP **09** (1999) 032)
- LHS of Eq.(2): deformation of symplectic structure Ω
  $=$ RHS of Eq.(2): deformation of complex structure J
- Homological mirror symetry (Kontsevich, alg-geom/94110118)
  category of A-brane (Fukaya category)
  $=$ category of B-brane (category of coherent sheaves)

Self-dual Einstein gravity from self-dual NC electromagnetism

Self-dual NC electromagnetism = twistor space describing curved self-dual spacetime
(HSY, hep-th/0608013 & hep-th/0611174)

NC instanton = gravitational instanton (HSY+Salizzoni, PRL **96** (2006) 201602)

- NC $U(1)$ instanton: $\widehat{F}_{\mu\nu}(y) = \pm\frac{1}{2}\varepsilon_{\mu\nu\lambda\sigma}\widehat{F}_{\lambda\sigma}(y)$
- Gravitational instanton: $R_{abcd} = \pm\frac{1}{2}\varepsilon_{abef}R^{ef}{}_{cd}$
- Seiberg-Witten map: $\mathbf{F}_{\mu\nu}(x) = \pm\frac{1}{2}\varepsilon_{\mu\nu\lambda\sigma}\mathbf{F}_{\lambda\sigma}(x)$
  where $\mathbf{F}_{\mu\nu}(x) = \left(\mathrm{g}^{-1}F\right)_{\mu\nu}(x), \quad \mathrm{g}_{\mu\nu} = \delta_{\mu\nu} + (F\theta)_{\mu\nu}$
- $\mathrm{g}_{\mu\nu} := \frac{1}{2}(\delta_{\mu\nu} + \widetilde{\mathrm{g}}_{\mu\nu}) \Rightarrow \widetilde{\mathrm{g}}_{\mu\nu}$ = hyper-Kähler metric

- E.g., Nekrasov-Schwarz instanton = Eguchi-Hanson space
  (Salizzoni+Torrielli+HSY, PLB **634** (2006) 427; HSY+Salizzoni, PRL **96** (2006) 201602)
- ALE & ALF spaces and real heavens are solutions of NC electromagnetism
- $\exists$ a twistor space associated to every hyper-Kähler manifold
  (Hitchin+Karlhede+Lindtröm+Roček, CMP **108** (1987) 535)
- $U(1)$ instantons act as a deformation of complex structure of twistor space

NCFT/Gravity correspondence as a large N duality

large N

(HSY, arXiv:0704.0929)

- NC $U(1)$ gauge theory on $\mathbf{R}^d_C \times \mathbf{R}^{2n}_{NC} = U(N \to \infty)$ Yang-Mills theory on $\mathbf{R}^d_C$

$$S = \frac{1}{4g_{YM}^2} \int d^D X \sqrt{\det \mathcal{G}} \mathcal{G}^{MP} \mathcal{G}^{NQ} (F_{MN} + \Phi_{MN}) \star (F_{PQ} + \Phi_{PQ}) \qquad (3)$$

with $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]_\star$

$$= \frac{(2\pi\kappa)^{\frac{4-d}{2}}}{2\pi g_s} \int d^d z \sqrt{\det g_{\mu\nu}} \mathrm{Tr} \Big(\frac{1}{4} g^{\mu\lambda} g^{\nu\sigma} F_{\mu\nu} F_{\lambda\sigma} + \frac{1}{2} g^{\mu\nu} g_{ab} D_\mu \Phi^a D_\nu \Phi^b - \frac{1}{4} g_{ac} g_{bd} [\Phi^a, \Phi^b][\Phi^c, \Phi^d] \Big), \qquad (4)$$

where we defined adjoint scalar fields $\Phi^a \equiv x^a/\kappa$ of mass dimension

- NC field $\cong$ (generalized) vector field: Derivation Einstein

$\mathrm{ad}_{\Xi^M}[f] \equiv [\Xi^M, f]_\star = -i\theta^{ab} \frac{\partial \Xi^M}{\partial y^b} \frac{\partial f}{\partial y^a} + \cdots \equiv V^a_M(z,y)\partial_a f(z,y) + \mathcal{O}(\theta^3)$

where $\Xi^M = D_\mu = \partial_\mu - iA_\mu$ or Φ^a

NCFT/Gravity correspondence as a large N duality

▸ large N

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Ward geometry (Ward, CQG **7** (1990) L217)

- Consider gauge-Higgs system (A_μ, Φ^a) on $\mathbf{R}^d_C$ with the gauge group $G = SDiff(N)$ and with a volume form $\nu = d^d z \wedge \omega$
- Let y^a be local coordinates on N, then locally,

$$A_\mu(z) = A^a_\mu(z,y)\frac{\partial}{\partial y^a}, \qquad \Phi_a(z) = \Phi^b_a(z,y)\frac{\partial}{\partial y^b}$$

- $f^{-1}(D_1, \cdots, D_d, \Phi_1, \cdots, \Phi_m)$ forms an orthonormal frame and hence defines a metric on $\mathbf{R}^d_C \times N$ where f is a scalar, a conformal factor, defined by $f^2 = \omega(\Phi_1, \cdots, \Phi_m)$

Emergent geometry from NC gauge theory ▸ AdS

$$ds^2 = f^2 \delta_{\mu\nu} dz^\mu dz^\nu + f^2 \delta_{ab} V^a_c V^b_d (dy^c - \mathbf{A}^c)(dy^d - \mathbf{A}^d) \tag{5}$$

where $\mathbf{A}^a = A^a_\mu dz^\mu$ and $V^a_c \Phi^c_b = \delta^a_b$

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Examples for $D = 4$ (HSY, hep-th/0608013)

- Self-duality equation in NC electromagnetism

$$\mathrm{ad}_{[x^a,x^b]_\star} = \pm\frac{1}{2}\varepsilon_{abcd}\,\mathrm{ad}_{[x^c,x^d]_\star} \Leftrightarrow [\Phi^a,\Phi^b] = \pm\frac{1}{2}\varepsilon_{abcd}[\Phi^c,\Phi^d] \qquad (6)$$

- If $\Phi^a \in LSDiff(N)$,
  Eq.(6) = self-dual Einstein gravity =self-dual NC electromagnetism
  (Ashtekar+Jacobson+Smolin, CMP **115** (1988) 631;
  Mason+Newman, CMP **121** (1989) 659;
  Chakravarty+Mason+Newman, JMP **32** (1991) 1458)

ALE & ALF spaces Gibbons+Hawking, PLB. **78** (1978) 430; Joyce, DMJ **77** (1995) 519

- $\Phi_i = -a_i\frac{\partial}{\partial\tau} + \frac{\partial}{\partial x^i}$ and $\Phi_4 = U\frac{\partial}{\partial\tau}$
- Eq.(6) $\Leftrightarrow$ Abelian Bogomol'nyi equation: $\nabla U + \nabla\times\vec{a} = 0$
- $ds^2 = U^{-1}(d\tau + \vec{a}\cdot d\vec{x})^2 + U d\vec{x}\cdot d\vec{x}$

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Continued

Real heaven Boyer+Finley, JMP **23** (1982) 1126; Ootsuka+Miyagi+Yasui+Zeze, CQG **16** (1999) 1305

- $\Phi_1 = \frac{\partial}{\partial x^1} - \partial_2\psi \frac{\partial}{\partial \tau}$
  $\Phi_2 = \frac{\partial}{\partial x^2} + \partial_1\psi \frac{\partial}{\partial \tau}$
  $\Phi_3 = e^{\psi/2}\left(\sin\left(\frac{\tau}{2}\right)\frac{\partial}{\partial x^3} - \partial_3\psi\cos\left(\frac{\tau}{2}\right)\frac{\partial}{\partial \tau}\right)$
  $\Phi_4 = e^{\psi/2}\left(\cos\left(\frac{\tau}{2}\right)\frac{\partial}{\partial x^3} + \partial_3\psi\sin\left(\frac{\tau}{2}\right)\frac{\partial}{\partial \tau}\right)$
- Eq.(6) $\Leftrightarrow$ 3-dimensional continual Toda equation: $(\partial_1^2 + \partial_2^2)\psi + \partial_3^2 e^{\psi} = 0$
- $ds^2 = (\partial_3\psi)^{-1}(d\tau + a^i dx^i)^2 + (\partial_3\psi)(e^{\psi} dx^i dx^i + dx^3 dx^3)$
  with $a^i = \varepsilon^{ij}\partial_j\psi$

Bubbling geometry (HSY, arXiv:0704.0929)

- Vacuum geometry: $(A_\mu, \Phi^a) = (0, y^a/\kappa)$ ▸ Ward

$$\omega = dy^1 \wedge \cdots \wedge dy^6 \;\Leftrightarrow\; \mathbf{R}^{1,9}$$

$$\omega = \frac{dy^1 \wedge \cdots \wedge dy^6}{\rho^2} \;\Leftrightarrow\; AdS_5 \times \mathbf{S}^5$$

since $AdS_5 \times \mathbf{S}^5$ space is conformally flat, i.e.,

$$ds^2 = \frac{L^2}{\rho^2}(\eta_{\mu\nu} dz^\mu dz^\nu + dy^a dy^a) = \frac{L^2}{\rho^2}(\eta_{\mu\nu} dz^\mu dz^\nu + d\rho^2) + L^2 d\Omega_5^2$$

where $\rho^2 = \sum_{a=1}^6 y^a y^a$

- Turn on $(A_\mu, \Phi^a) \;\Rightarrow\;$ bubbling geometry
- E.g., for (A_μ, Φ^a)= half-BPS configuration with a particular isometry, Ward metric (5) = LLM geometry
(Lin+Lunin+Maldacena, JHEP **10** (2004) 025)
- $AdS_p \times \mathbf{S}^q$ is also similarly realized

Deformation Quantization

(Kontsevich, LMP **66** (2003) 157)

- Einstein gravity emerges at semi-classical limit, i.e., $\mathcal{O}(\theta)$ ▸ DBI
- What do we get after full NC deformations ?
- NC gravity !
  (Aschieri+Blohmann+Dimitrijevic+Meyer+Schupp+Wess, CQG **22** (2005) 3511;
  Aschieri+Dimitrijevic+Meyer+Wess, CQG **23** (2006) 1883;
  Calmet+Kobakhidze, PRD **72** (2005) 045010)
- Darboux theorem in deformation quantization: automorphism of $\mathcal{A}[[\hbar]]$

$$\star \to \star', \quad f(\hbar) \star' g(\hbar) = D(\hbar)\Big(D(\hbar)^{-1}(f(\hbar)) \star D(\hbar)^{-1}(g(\hbar))\Big)$$

for $f(\hbar), g(\hbar) \in \mathcal{A}[[\hbar]]$.

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  Aschieri+Dimitrijevic+Meyer+Wess, CQG **23** (2006) 1883;
  Calmet+Kobakhidze, PRD **72** (2005) 045010)
- Darboux theorem in deformation quantization: automorphism of $\mathcal{A}[[\hbar]]$

$$\star \to \star', \quad f(\hbar) \star' g(\hbar) = D(\hbar)\Big(D(\hbar)^{-1}(f(\hbar)) \star D(\hbar)^{-1}(g(\hbar))\Big)$$

for $f(\hbar), g(\hbar) \in \mathcal{A}[[\hbar]]$.

Deformation Quantization

(Kontsevich, LMP **66** (2003) 157)

- Einstein gravity emerges at semi-classical limit, i.e., $\mathcal{O}(\theta)$ ▸ DBI
- What do we get after full NC deformations ?
- NC gravity !
  (Aschieri+Blohmann+Dimitrijevic+Meyer+Schupp+Wess, CQG **22** (2005) 3511;
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Seiberg-Witten map in general

(Jurčo+Schupp+Wess, NPB **604** (2001) 148; HSY, hep-th/0611174)

- *Theorem 1.1* and *Theorem 2.3* in (Kontsevich, LMP **66** (2003) 157): the set of gauge equivalence classes of star products on M = the set of equivalence classes of Poisson structures modulo $Diff(M)$, and, if we change coordinates, $y^\mu \mapsto x^\mu(y)$, we obtain a gauge equivalent star product
- Arbitrary change of coordinates, $y^\mu \mapsto x^\mu(y)$, in the Moyal $\star$-product with the constant Poisson bi-vector $\theta^{\mu\nu}$ $\cong$ Kontsevich star product defined by a Poisson bi-vector $\alpha(\hbar)$
  (Zotov, MPL **A16** (2001) 615)
- Seiberg-Witten map: $[x^\mu, x^\nu]_\star = i(\theta - \theta\widehat{F}(y)\theta)^{\mu\nu} = 2D(\hbar)^{-1}(\alpha^{\mu\nu})$
  where $\alpha^{\mu\nu}(x) = \frac{i}{2}\left(\frac{1}{B+F}\right)^{\mu\nu}(x)$
- General Seiberg-Witten map $\Leftarrow$ NC deformation of diffeomorphism symmetry $\Rightarrow$ NC gravity as an emergent geometry
- At semi-classical limit $\mathcal{O}(\theta)$, $D(\hbar) \approx 1$

Darboux

Where to go from here ?

- Gravity may be not a fundamental force but a collective phenomenon emerging from NC (or non-Abelian) gauge fields.
- Spacetime seems also to be emergent from gauge field interactions.
- Why 4 ?
  Electromagnetism (only long range force in Nature) should determine the large scale structure of spacetime.
  The number of physical polarizations: $A_\mu = D - 2 = \frac{D(D-3)}{2} = g_{\mu\nu}$
  $\Rightarrow \; D = 1$ or $D = \mathbf{4}$
- An important difference between emergent gravity and Einstein gravity: Uniform condensation of energy does not gravitate but is used to make flat spacetime. Only vacuum fluctuations couple to gravity ! That is, the flat spacetime is very heavy ! This fact may resolve the long standing cosmological constant problem.
  (Padmanabhan, CQG **22** (2005) L107)
- Open problems:
  How to understand fermions in the context of emergent geometry ?
  Black hole solutions ? etc.
- Final word: **QFT is so rich and even contains quantum gravity, and still remains largely unexplored !**

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