Translation-invariant models for non-commutative gauge fields

Talk presented by Daniel N. Blaschke

Institute for Theoretical Physics,
Vienna University of Technology

Collaborators: F. Gieres, H. Grosse, E. Kronberger, M. Schweda, R. Sedmik, M. Wohlgenannt

May 17, 2008

Weyl-Moyal correspondence

- assume non-commuting space-time coordinates:

  $$[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}\theta^{\mu\nu}, \qquad \Rightarrow \text{leads to uncertainty relation:}$$

  $$\Delta x^\mu \Delta x^\nu \geq \frac{1}{2}|\theta^{\mu\nu}| \sim (\lambda_p)^2$$

- definition of the Groenewold-Moyal $\star$-product:

  $$f(x) \star g(x) = e^{\frac{\mathrm{i}}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu} f(x) g(y)\Big|_{x=y}$$

  $$\neq g(x) \star f(x)$$

- invariance under cyclic permutations of the integral

  $$\int d^4x f(x) \star g(x) \star h(x) = \int d^4x h(x) \star f(x) \star g(x)$$

  $$\Longrightarrow \int d^4x f(x) \star g(x) = \int d^4x f(x) g(x)$$

QFT on θ-deformed space-time

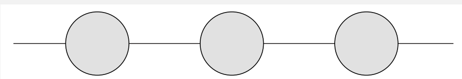
For a field theory this means:

- interaction vertices gain phases, whereas propagators remain unchanged
- some Feynman integrals ("non-planar diagrams") have phases, e.g.

  $$\int d^4k \frac{e^{\mathrm{i}k\tilde{p}}}{k^2 + \mathrm{i}\epsilon} \propto \frac{1}{\tilde{p}^2} \quad \text{with} \quad \tilde{p}^\mu = \theta^{\mu\nu} p_\nu$$

- phases act as UV-regulators,

  $\Rightarrow$ origin of the *UV/IR mixing* problem



Recent successes

So far there are two types of scalar field theories where the UV/IR mixing problem could be solved:

- the Grosse-Wulkenhaar model (2003), where the ϕ^4 theory was supplemented by a (translation-invariance breaking) oscillator term ($\approx \tilde{x}^2\phi^2$)
- and recently a translation-invariant model by Gurau, Magnen, Rivasseau and Tanasa (2008), where the oscillator term is replaced by a $\frac{1}{\tilde{p}^2}$ term.

$\Rightarrow$ *Both models could be proved to be renormalizable to all orders.*

IN GAUGE THEORIES ...

...matters are more complicated. Recent ansatzes:

- Slavnov model (2003): relies on a constraint $\rightarrow$ reduces degrees of freedom
- models involving oscillator terms in analogy to the scalar model: break translation invariance
  (de Goursac et. al., Grosse+Wohlgenannt, D.N.B. et. al., 2007)
- proofs of renormalizability still missing for gauge theories

Inspired by the scalar model of Gurau et. al., whose propagator has "damping" behaviour for vanishing momentum:

$$G^{\phi\phi}(k) = \frac{1}{k^2 + m^2 + \frac{a}{\theta^2 k^2}} \approx \frac{\theta^2 k^2}{a} \quad \text{for } k \to 0\,,$$

a new gauge field model is put forward:

Translation-invariant models for NCGFT
Talk presented by Daniel N. Blaschke
Introduction
NC gauge fields
Conclusion and Outlook

A NEW GAUGE FIELD MODEL

$$\int d^4x \phi(x) \frac{1}{\theta^2 \square} \phi(x) \quad \Rightarrow \quad \int d^4x \frac{1}{4} F^{\mu\nu} \star \frac{1}{D^2 \widetilde{D}^2} \star F_{\mu\nu}\,,$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g \left[A_\mu \stackrel{\star}{,} A_\nu\right]\,,$$
$$\widetilde{D}^2 = \widetilde{D}^\mu \star \widetilde{D}_\mu\,, \qquad \text{with } \widetilde{D}_\mu = \theta_{\mu\nu} D^\nu\,,$$

Expression $\frac{1}{D^2} F \equiv Y$ transforms covariantly ($sY = \mathrm{i}g \left[c \stackrel{\star}{,} Y\right]$) and is to be understood as formal power series in the gauge field A_μ:

$$F = D^2 \star \frac{1}{D^2} \star F = D^2 Y =$$
$$= \square Y - \mathrm{i}g\partial^\mu \left[A_\mu \stackrel{\star}{,} Y\right] - \mathrm{i}g \left[A^\mu \stackrel{\star}{,} \partial_\mu Y\right] + (\mathrm{i}g)^2 \left[A^\mu \stackrel{\star}{,} \left[A_\mu \stackrel{\star}{,} Y\right]\right]\,.$$

Translation-invariant models for NCGFT
Talk presented by Daniel N. Blaschke
Introduction
NC gauge fields
Conclusion and Outlook

COMPLETE ACTION INCLUDING GAUGE FIXING

- recursion formula leads to

$$Y^0 = \frac{1}{\square} F\,,$$
$$Y^1 = \frac{1}{\square} F + \frac{\mathrm{i}g}{\square} \left\{ \partial^\mu \left[A_\mu \stackrel{\star}{,} Y^0\right] + \left[A^\mu \stackrel{\star}{,} \partial_\mu Y^0\right] - \mathrm{i}g \left[A^\mu \stackrel{\star}{,} \left[A_\mu \stackrel{\star}{,} Y^0\right]\right] \right\}$$

- New action (where stars have been suppressed):

$$\Gamma^{(0)} = S_{\text{inv}} + S_{\text{gf}}\,,$$
$$S_{\text{inv}} = \int d^4x \left[\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{4} F^{\mu\nu} \frac{1}{D^2 \widetilde{D}^2} F_{\mu\nu} \right]\,,$$
$$S_{\text{gf}} = s \int d^4x\, \bar{c} \left[\left(1 + \frac{1}{\square\widetilde{\square}}\right) \partial^\mu A_\mu - \frac{\alpha}{2} B \right]$$
$$= \int d^4x \left[B \star \left(1 + \frac{1}{\square\widetilde{\square}}\right) \partial^\mu A_\mu - \frac{\alpha}{2} B^2 - \bar{c} \left(1 + \frac{1}{\square\widetilde{\square}}\right) \partial^\mu D_\mu c \right].$$

Translation-invariant models for NCGFT
Talk presented by Daniel N. Blaschke
Introduction
NC gauge fields
Conclusion and Outlook

BRST SYMMETRY AND PROPAGATORS

- this action is invariant under the BRST transformations

$$sA_\mu = D_\mu c \equiv \partial_\mu c - \mathrm{i}g \left[A_\mu \stackrel{\star}{,} c\right]\,, \qquad s\bar{c} = B\,,$$
$$sc = \mathrm{i}gc \star c\,, \qquad sB = 0\,,$$
$$s^2 \varphi = 0 \qquad \text{for } \varphi \in \{A_\mu, c, \bar{c}, B\}\,.$$

- the bilinear action leads to the improved propagators

$$G^A_{\mu\nu}(k) = \frac{1}{k^2 + \frac{1}{\tilde{k}^2}} \left(-\delta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} - \alpha \frac{k_\mu k_\nu}{k^2 + \frac{1}{\tilde{k}^2}} \right)\,,$$
$$G^{\bar{c}c}(k) = \frac{1}{k^2 + \frac{1}{\tilde{k}^2}} \qquad \text{with } \tilde{k}^\mu = \theta^{\mu\nu} k_\nu\,.$$

LOOP INTEGRALS

simplest non-planar one-loop integral:

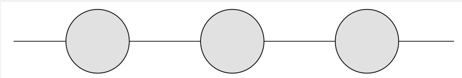
$$\int d^4k \frac{e^{\pm ik\tilde{p}}}{k^2+\frac{1}{\tilde{k}^2}} = \frac{1}{2}\int d^4k \left[\frac{e^{\pm ik\tilde{p}}}{(k^2+\frac{\mathrm{i}}{\theta})} + \frac{e^{\pm ik\tilde{p}}}{(k^2-\frac{\mathrm{i}}{\theta})}\right] \approx \frac{1}{\tilde{p}^2}\,.$$

for $\tilde{p}^2 \to 0$.

- In the sum of 1-loop graphs due to gauge symmetry:

$$\approx \frac{\tilde{p}_\mu \tilde{p}_\nu}{(\tilde{p}^2)^2}$$

- IR damping due to propagators



MORE CONVENIENT FORMULATION

Translation-invariant models for NCGFT
Talk presented by Daniel N. Blaschke
Introduction
NC gauge fields
Conclusion and Outlook

$$S_{\text{new}} = \int d^4x \left[B^{\mu\nu} \star F_{\mu\nu} - B^{\mu\nu} \star \widetilde{D}^2 D^2 \star B_{\mu\nu}\right]$$

is gauge invariant if new field $B_{\mu\nu}$ transforms covariantly, i.e.

$$sB_{\mu\nu} = \mathrm{i}g\left[c \stackrel{\star}{,} B_{\mu\nu}\right]$$

and has only a finite number of new vertices.
Re-inserting the e.o.m.

$$\frac{\delta S_{\text{new}}}{\delta B^{\rho\sigma}} = F_{\rho\sigma} - 2\widetilde{D}^2 D^2 \star B_{\rho\sigma} = 0$$

leads again to

$$\int d^4x \frac{1}{4} F^{\mu\nu} \star \frac{1}{D^2 \widetilde{D}^2} \star F_{\mu\nu}\,,$$

ADDITIONAL FEYNMAN RULES

Translation-invariant models for NCGFT
Talk presented by Daniel N. Blaschke
Introduction
NC gauge fields
Conclusion and Outlook

- two additional propagators:

$$G^{AB}_{\rho,\sigma\tau}(k) = \frac{\mathrm{i}\,(k_\sigma \delta_{\rho\tau} - k_\tau \delta_{\rho\sigma})}{2k^2\tilde{k}^2\left(k^2+\frac{1}{\tilde{k}^2}\right)}\,,$$

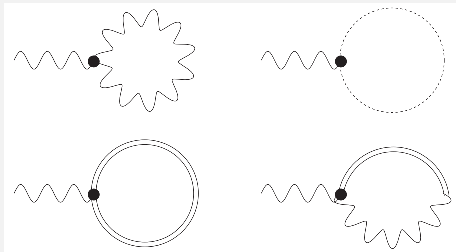
$$G^{BB}_{\rho\sigma,\tau\epsilon}(k) = \frac{1}{4k^2\tilde{k}^2}\Bigg[\delta_{\rho\tau}\delta_{\sigma\epsilon} - \delta_{\rho\epsilon}\delta_{\sigma\tau} + \\ + \frac{k_\sigma k_\tau \delta_{\rho\epsilon} + k_\rho k_\epsilon \delta_{\sigma\tau} - k_\sigma k_\epsilon \delta_{\rho\tau} - k_\rho k_\tau \delta_{\sigma\epsilon}}{k^2\tilde{k}^2\left(k^2+\frac{1}{\tilde{k}^2}\right)}\Bigg]\,.$$

- and 5 new vertices, namely a BAA-vertex, a BBA-vertex, a $2B2A$-vertex, a $2B3A$-vertex and a $2B4A$-vertex.

VANISHING TADPOLE GRAPHS

Translation-invariant models for NCGFT
Talk presented by Daniel N. Blaschke
Introduction
NC gauge fields
Conclusion and Outlook

There are only 4 possible one-loop tadpole graphs with external gauge boson lines in this model:



From the Feynman rules one sees that all 4 come with a factor

$$\delta^4(p+k-k)\sin\left(\frac{k\theta p}{2}\right) \to 0\,,$$

Hence, all four graphs vanish.

Conclusion and Outlook

- the model seems to be a promising candidate for a renormalizable non-commutative gauge model
- gauge field and ghost propagators damp IR divergences
- it is translation-invariant (tadpoles vanish)
- one-loop calculations are in progress

References

Translation-invariant models for NCGFT
Talk presented by Daniel N. Blaschke
Introduction
NC gauge fields
Conclusion and Outlook

D. N. Blaschke, F. Gieres, E. Kronberger, M. Schweda and M. Wohlgenannt, to appear in J. Phys. A, [arXiv:0804.1914].

R. Gurau, J. Magnen, V. Rivasseau and A. Tanasa, [arXiv:0802.0791].

H. Grosse and F. Vignes-Tourneret, [arXiv:0803.1035].

F. Aigner, M. Hillbrand, J. Knapp, G. Milovanovic, V. Putz, R. Schoefbeck and M. Schweda, *Czech. J. Phys.* **54** (2004) 711–719, [arXiv:hep-th/0302038].

Thank you for your attention!