Constructing the AdS/CFT Duality for Non-Anticomutative SYM

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work in collaboration with Shou-Huang Dai and Douglas Smith, 0803.0895 [hep-th]

Outline

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1. I. Review and Motivation

2. II. The Non-Anticommutative deformed $\mathcal{N} = 4$ SYM

3. III. Construction of the SUGRA dual

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1. Noncommutative geometry in string theory

- Quantum field theory on noncommutative space, $[x^\mu, x^\nu] = i\theta^{\mu\nu}$, displays a rich spectrum of unusual properties, some of which are believed to be relevant for quantum gravity.
  (...,Heisenberg,Synder,Yang...) (DFR,Wess,Zumino,Groose,Madore, Frolich,Connes,...) (string community)
- Noncommutative geometry arises as low energy theory of D-brane in NSNS B-field . (Douglas, Hull; Chu, Ho; Schomerus; Seiberg, Witten)
- A natural extension of the noncommutative space is to consider a deformed superspace. e.g.

$$\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta},$$
$$\{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\} = \{\theta^\alpha, \bar\theta^{\dot\beta}\} = 0,$$
$$[y^\mu, y^\nu] = i\theta^{\mu\nu}, \quad [y^\mu, \theta^\alpha] = [y^\mu, \bar\theta^{\dot\alpha}] = 0,$$

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2. Non-anticoncommutative (NAC) deformation

- It was realized that string theory in graviphoton background leads to a deformation of the 4-dimensional superspace. In particular, a self-dual graviphoton field strength $C_{\mu\nu}$ induces $\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}$. (Ooguri, Vafa)
- Initially it was thought $[x^\mu, x^\nu] = 0$ and all susy are broken. (Ooguri, Vafa; de Boer, Grassi, van Nieuwenhuizen.)
- Seiberg realized that if we choose

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- The $*$-product is defined as usual

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\begin{aligned}
f(y,\theta,\overline{\theta}) * g(y,\theta,\overline{\theta}) &= f \exp\left(-\frac{C^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\overrightarrow{\partial}}{\partial\theta^\beta}\right) g \\
&= f(\theta)\left(1-\frac{C^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\overrightarrow{\partial}}{\partial\theta^\beta} - \det C\,\frac{\overleftarrow{\partial}}{\partial\theta\theta}\frac{\overrightarrow{\partial}}{\partial\theta\theta}\right) g(\theta)
\end{aligned}
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- Written in the chiral basis $y,\theta,\overline{\theta}$, the supercharges and covariant derivatives take the standard expressions

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Q_\alpha &= \frac{\partial}{\partial\theta^\alpha}, \qquad \overline{Q}_{\dot\alpha} = -\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} + 2i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu}, \\
D_\alpha &= \frac{\partial}{\partial\theta^\alpha} + 2i\sigma^\mu_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\frac{\partial}{\partial y^\mu}, \quad D_{\dot\alpha} = -\frac{\partial}{\partial\overline{\theta}^{\dot\alpha}}.
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- They satisfy

$$\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = 2i\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu},$$

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  with all the remaining anti-commutators equal to zero.
- The $\mathcal{N} = 1/2$ supersymmetry is generated by the unbroken Q's. We have a $\mathcal{N} = (1/2, 0)$ supersupperspace.
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Examples of Non-anticommutative theory

WZ model

- $\mathcal{N} = 1/2$ Wess-Zumino model

$$\mathcal{L} = \int d^4\theta \bar{\Phi} * \Phi + \int d^2\theta \mathcal{W}(\Phi) + \int d^2\bar{\theta}\, \overline{\mathcal{W}}(\bar{\Phi})$$

- For example, for a cubic superpotential

$$\begin{aligned} \mathcal{L} &= \int d^4\theta\ \bar{\Phi}\Phi + \int d^2\theta\ (\frac{1}{2} m\Phi * \Phi + \frac{1}{3} g\Phi * \Phi * \Phi) + c.c. \\ &= \mathcal{L}(C=0) - \frac{1}{3} g \det C F^3 + \text{total derivative.} \end{aligned}$$

- For general superpotential, one have an infinite sum in powers of F. Remarkably, it has been shown quite recently can be written in terms of a simple smearing of the Zumino's Lagrangian and the holomorphic superpotential.
  (Azorkina, Banin, Buchbinder, Pletnev; Alvarez-Gaume, Vazquez-Mozo)

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Gauge theory

- The vector superfield transforms as

$$e^V \rightarrow e^{-i\bar{\Lambda}} * e^V * e^{i\Lambda}.$$

- In the Wess-Zumino gauge, the $\mathcal{N} = 1/2$ SYM is given by the Lagrangian

$$\mathcal{L} = \frac{1}{16g^2}\left(\int d^2\theta \, \mathrm{tr} W^\alpha * W_\alpha + \int d^2\bar{\theta} \, \mathrm{tr} \overline{W}_{\dot{\alpha}} * \overline{W}^{\dot{\alpha}}\right)$$

In terms of the component fields, the Lagrangian reads (up to total derivatives)

$$\begin{aligned} \mathcal{L} \;=\; & \frac{1}{16kg^2}\mathrm{tr}\Big(-4i\bar{\lambda}\bar{\sigma}^\mu \mathcal{D}_\mu \lambda - F^{\mu\nu}F_{\mu\nu} + \frac{i}{2}F^{\mu\nu}F^{\rho\sigma}\epsilon_{\mu\nu\rho\sigma} + 2D^2 \\ & -2iC^{\mu\nu}F_{\mu\nu}\bar{\lambda}\bar{\lambda} + \frac{|C|^2}{2}(\bar{\lambda}\bar{\lambda})^2\Big) \end{aligned}$$

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NAC QFT and their properties

- Although power counting non-renormalizable, nevertheless they are renormalizable.
  (Grisaru, Penati, Romagnoni; Berenstein, Britto, Feng, Lunin, Rey; Jack, Jones, Worthy)
  Also exists nonrenormalization theorem.
- Instanton (Imannpur; Britto, Feng, Lunin, Rey; Billo,Frau,Pesando,Lerda)
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3. AdS/CFT correspondence

- Explicit realization of the *holographic principle*. (t' Hooft; Susskind)
- Original Maldacena correspondence $AdS_5 \times S^5$ is maximally supersymmetric. Many generalizations. In particular, some of these are characterized by a deformed $*$-product.
- *Maldacena-Russo* : The SYM is deformed by a Moyal star-product

$$(\theta\text{-deformation}) : \quad f * g = e^{i\theta(\partial_f^1 \partial_g^2 - \partial_f^2 \partial_g^1)} fg$$

  The dual is a supergravity background which is $AdS_5 \times S^5$ deformed by a (radially varying) NSNS B-field.
- *Lunin-Maldacena* : The product between fields carrying different $U(1)$ charges is deformed

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Two motivations:

- It is natural and of interest to construct a gauge/gravity duality for the supersymmetric gauge theory deformed by non-anticommutative *C-deformation*.
- In the original Maldacena AdS/CFT correspondence, the amount of preserved supersymmetry is maximal. Since holography is believed to be a generic property of quantum gravity, it is interesting to understand how gauge/gravity duality works in a less or non-supersymmetric setting, especially when the supersymmetry is preserved in a non-standard manner.

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Construction of the NAC deformed $\mathcal{N} = 4$ SYM

- The $\mathcal{N} = 4$ SYM can be written in terms of $\mathcal{N} = 1$ language or $\mathcal{N} = 2$ language. One can introduce NAC deformation to each of them.
- For the NAC deformation of the simple superspace, the Lagrangian is easy to write down and it has $\mathcal{N} = (1/2, 0)$ susy.

$$W = \mathrm{tr}\left[\Phi_1 * \Phi_2 * \Phi_3 - \Phi_1 * \Phi_3 * \Phi_2\right]$$

- The action of the NAC deformed $\mathcal{N} = 4$ SYM with $\mathcal{N} = (1, 0)$ susy can be constructed using NAC harmonic superspace.

(Ivanov,Lechtenfeld, Zupnik, Sokatchev; Ito, Sasai)

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NAC deformed extended SUSY

- Deformation of $\mathcal{N} = 2 = (1,1)$ susy is defined by

$$\{\theta^{\alpha i}, \theta^{\beta j}\} = C^{\alpha\beta ij}, \quad i = 1, 2,$$

- The deformation parameter can be decomposed into irreducible parts

$$C^{\alpha\beta ij} = \epsilon^{\alpha\beta}\epsilon^{ij} I + C^{(\alpha\beta)(ij)}.$$

The first term preserves Euclidean $SO(4)$ invariance and $SU(2)$ R-symmetry, and is called the singlet deformation.
The second term is called non-singlet deformation. The deformation breaks $\mathcal{N} = (1,1)$ supersymmetry down to $\mathcal{N} = (1,0)$ generically.
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- The non-singlet deformation can be obtained from string theory in a *constant RR 5-form background* which satisfies the "double self-duality" condition

$$F_{\mu\nu abc} = \frac{1}{2!}\epsilon_{\mu\nu\rho\lambda}F_{\rho\lambda abc},$$
$$F_{\mu\nu abc} = \frac{-i}{3!}\epsilon_{abcdef}F_{\mu\nu def}.$$

where $\mu, \nu = 0, 1, 2, 3$ denote the 4-dimensional indices and $a, b, c, = 4, \cdots, 9$ are the indices of the transverse space.

- The deformation parameter in field theory is related to the string by

$$C^{\alpha\beta AB} := (2\pi\alpha')^{3/2}\mathcal{F}^{\alpha\beta AB},$$

and is kept fixed in the $\alpha' \to 0$ limit. $A, B = 1, 2, 3, 4 \in SU(4)$

$$\mathcal{F}^{\alpha\beta AB} = F^{\mu\nu abc}(\sigma^{\mu\nu})^{\alpha\beta}(\Gamma^{abc})^{AB}$$

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RR-flux config for the NAC deformation of $\mathcal{N} = 2$ susy

- To introduce a deformation to the $\mathcal{N} = (1,1)$ superspace, the RR-5 form $\mathcal{F}^{\alpha\beta AB}$ should be non-vanishing only for a 2×2 sub-block of the indices for A, B. This can be achieved with the following configuration of RR 5-form:

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- This gives

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RR-flux config for the NAC deformation of $\mathcal{N} = 1$ susy

- One can similarly write down the necessary 5-form configuration that will give rises to a $\mathcal{F}^{\alpha\beta AB}$ which is rank 1. (skipped)

Outline

Intersecting brane configuration for NAC deformation of $\mathcal{N} = 2$ susy

- The constant RR 5-form field strength does not generate any energy-momentum tensor in flat Euclidean space:

$$T_{MN} = F_{MM_1M_2M_3M_4} F_N{}^{M_1M_2M_3M_4} - \frac{1}{10} g_{MN} F^2 = 0.$$

However this is no longer the case once one takes into account the backreaction of the N D3-branes, which turns the flat spacetime to $AdS_5 \times S^5$.

- One can obtain the desired components of RR-5form field by considering a configuration of intersecting D3-branes. For the case of NAC deformation of $\mathcal{N} = 2$, one has

$$\begin{array}{c|cccccccccc} D3_1 & (& 0 & 1 & 2 & 3 & & & & &) \\ D3_2 & (& 0 & 1 & & & 4 & 5 & & &) \\ D3_{2'} & (& 0 & 1 & & & & & 7 & 8 &) \\ D3_3 & (& & & 2 & 3 & 4 & 5 & & &) \\ D3_{3'} & (& & & 2 & 3 & & & 7 & 8 &) \end{array} .$$

Here $D3_1$ denotes the original N D3-branes; and we have introduced four additional sets of D3-branes.

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- One can check the supersymmetry and find that there are four perserved susy. This matches with the $\mathcal{N} = (1,0)$ susy.
- The metric of our intersecting branes system is given by

$$\begin{aligned} ds^2 &= \sqrt{\frac{H_3 H_{3'}}{H_1 H_2 H_{2'}}}(dx_0^2 + dx_1^2) + \sqrt{\frac{H_2 H_{2'}}{H_1 H_3 H_{3'}}}(dx_2^2 + dx_3^2) \\ &+ \sqrt{\frac{H_1 H_{2'} H_{3'}}{H_2 H_3}}(dx_4^2 + dx_5^2) + \sqrt{\frac{H_1 H_2 H_3}{H_{2'} H_{3'}}}(dx_7^2 + dx_8^2) \\ &+ \sqrt{H_1 H_2 H_3 H_{2'} H_{3'}}(dx_6^2 + dx_9^2) \end{aligned}$$

and the RR 5-form is

$$F = F_0 + F_1,$$

$$\begin{aligned} F_0 &:= d(\frac{1}{H_1})dx^{0123} + \text{dual}, \\ F_1 &:= d(\frac{1}{H_2})dx^{0145} + d(\frac{1}{H_{2'}})dx^{0178} + d(\frac{1}{H_3})dx^{2345} + d(\frac{1}{H_{3'}})dx^{2378} + \text{dual}. \end{aligned}$$

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$$\left(\partial_i^2 + \partial_m^2 + \frac{1}{H_2^2}\partial_a^2\right)H_1 = 0.$$

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- We obtain the near horizon metric

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Intersecting brane configuration for NAC deformation of $\mathcal{N}=1$ susy

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Conclusions

- We have constructed the supergravity duals for the NAC deformed $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (1/2, 0)$ supersymmetries.
- Checks of the duality by looking at the two point functions (refer to our paper).
- Interesting to analyse further:
  - the fact that the metric is non-dilatonic suggests that the field theory coupling is not renormalized. It will be interesting to check this explicitly.
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