From Fuzzy Physics to ncQFT

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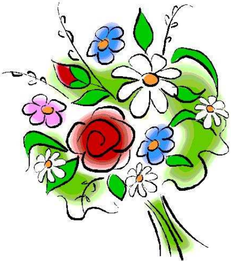
Introduction

- Fuzzy Physics John Madore
- RG flows
- Scalar fields, modified action H G + R Wulkenhaar
- Taming the Landau ghost H G + R Wulkenhaar, V Rivasseau et al
- Fermions: spectral triple H G + R Wulkenhaar
- Further transl. inv. renormalizable models H G + F Vignes-Tourneret
- Sketch of proof
- Conclusions



to John Madore

Congratulations to your 70th birthday

Fuzzy Physics

QFT needs regularization, example $D = 2$, euclidean John Madore: Fuzzy Sphere matrix algebra A_N generators: x_n

$$[x_n^N, x_m^N] = i\frac{R}{\sqrt{N^2-1}}\epsilon_{nmp}x_p^N$$

gives embedding of algebra A_N into A_{N+1}.......A_{comm} regularization of QFT

$$< F >= \frac{1}{N}\int [d\Phi]_N e^{-S_N[\Phi]}F(\Phi)$$

$$S_N[\Phi] = \frac{1}{N}Tr([x_m^N, \Phi][x_m^N, \Phi] + V(\Phi))$$

cutoff $\frac{R}{N}$, ex: CP^2.....,shows IR/UV mixing

RG Flow

Project

merge general relativity with quantum field theory through noncommutative geometry, use RG flow

- 50.... success of ren. pert. th. BPHZ....Connes-Kreimer
- 56.... ghost and triviality Landau....
- 74 RG flow Wilson, Polchinski... summability, safety
- use multi scale analysis V Rivasseau et al
- cure problems of ren. pert. exp. (IR,UV,convergence)
- require (Borel) summability
- take into account qu. gravity effects

Φ^4_Θ

Formulation

ϕ^4 on nc $\mathbb{R}^4$, $[x^\mu, x^\nu] = i\theta^{\mu\nu}$ antisymmetric,
or equivalently star product

$$(a * b)(x) = \int dy \int dk a(x + \frac{\theta k}{2}) b(x + y) e^{iky}$$

ϕ^4 action

$$S = \int dp (p^2 + m^2) \phi_p \phi_{-p} + \lambda \int \prod_{j=1}^{4} (dp_j \phi_{p_j}) \, \delta(\sum_{j=1}^{4} p_j) e^{-i \sum_{i<j} p_i \wedge p_j}$$

Feynman rules

cyclic order of momenta leads to ribbon graphs
Model is not renormalizable
One possible solution: modify action

Theorem

H. G. and R. Wulkenhaar ϕ^4 model modified,
IR/UV mixing: short and long distances related
Theorem: Action

$$S = \int d^4x \Big(\frac{1}{2} \partial_\mu \phi \star \partial^\mu \phi + \frac{\Omega^2}{2} (\tilde{x}_\mu \phi) \star (\tilde{x}^\mu \phi) + \frac{\mu_0^2}{2} \phi \star \phi + \lambda \phi \star \phi \star \phi \star \phi \Big)(x)$$

for $\tilde{x}_\mu := 2(\theta^{-1})_{\mu\nu} x^\nu$
is perturbativly renormalizable to all orders in λ, 3 proofs,
Rivasseau et al: Multiscale analysis in matrix base and in position space
Action has Langmann-Szabo position-momentum duality
$S\big[\phi; \mu_0, \lambda, \Omega\big] \mapsto \Omega^2 S\big[\phi; \frac{\mu_0}{\Omega}, \frac{\lambda}{\Omega^2}, \frac{1}{\Omega}\big]$

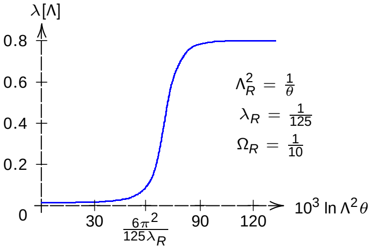
β function

evaluate β function, H. G. and R. Wulkenhaar,

$$\beta_\lambda = \frac{\lambda^2_{\text{phys}}}{48\pi^2}\frac{(1-\Omega^2_{\text{phys}})}{(1+\Omega^2_{\text{phys}})^3} + \mathcal{O}(\lambda^3_{\text{phys}})$$

flow bounded, L. ghost killed!
Due to wave fct. renormalization
$\Omega = 1$ betafunction vanishes
$\Omega^2[\Lambda] \leq 1$

($\lambda[\Lambda]$ diverges in comm. case)



- perturbation theory remains valid at all scales!
- non-perturbative construction of the model seems possible!

Gurau, Magnen, Rivasseau, Tanasa new ren.m: add $\int \Phi^2 \frac{\alpha}{p^2}$

A spectral triple

H. G. and Raimar Wulkenhaar,
Take Dirac operator on Hilbert space $L^2(R^4) \otimes C^{16}$

$$D_8 = (i\Gamma^\mu \partial_\mu + \Omega \Gamma^{\mu+4} \tilde{x}_\mu)$$

$\mu = 1, ...4$, Γ_k generate 8-dim Clifford algebra $\{\Gamma_k \Gamma_l\} = 2\delta_{kl}$

$$D_8^2 = (-\Delta + \Omega^2 ||\tilde{x}||^2) 1 - i\Omega\Theta^{-1}_{\mu\nu}[\Gamma^\mu, \Gamma^{\nu+4}]$$

compute action of Dirac operator on sections of spinor bundle

$$[D_8, f] * \psi = i[\Gamma^\mu + \Omega \Gamma^{\mu+4}](\partial \mu f) * \psi$$

only 4 dim. differential appears
leads to spectral triple
configuration space dimension 4
phase space dim. 8 Clifford alg. dim., KO dim.,..

- nc Gross-Neveu model ren. by F Vignes-Tourneret

Translation invariant rncQFT

H G + F Vignes-Tourneret, action:

$$S[\Phi] = \int d^4x \Phi(x)(-\Delta + m^2)\Phi(x) + \frac{\mu}{\Theta^3}(\int d^4x \Phi(x))^2 + \int d^4x \lambda \Phi^{*4}(x)$$

Main result: This QFT is renormalizable to all orders of pertubation theory

Topology of Ribbon graphs: V vertices, I internal lines, F faces, B broken faces: $2 - 2g = V - I + F$, $2L + N = 4n$

$g = 0$ planar graph, $g > 0$ nonplanar,

$B = 1$ regular graph, $B > 1$ nonregular

Filk: phases for Rosette graphs: $e^{-i\sum_{l<m} I_{l,m} p_l \wedge p_m}$

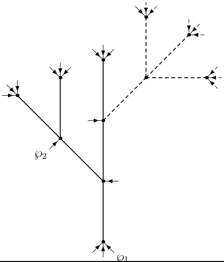
use multiscale analysis

Sketch of Proof

Slice propagator

$$C^k(p,q) = \delta(p+q) \int_{M^{-2k}}^{M^{-2(k-1)}} dt e^{-t(p^2+m^2)}$$

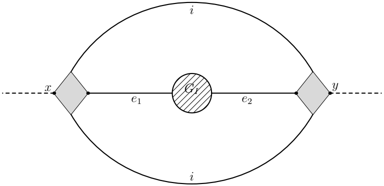
Derive power counting lemma:
The degree of convergence of a Feynman graph is given by $N(G) - 4$ if g=0, and $N(G) + 4$ for $g > 0$, where N is nr of external points, choose tree



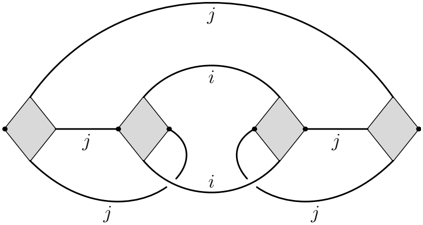
For non planar graphs use trick:
Go over a tree, there exist oscillations between momenta of lines crossing line l:

$$e^{-ip_l \wedge P_l} = \frac{1 - M^{2l}\partial^2_{p_l^2}}{1 + M^{2l}P_l^2} e^{-ip_l \wedge P_l}$$

do partial integration and use bound, gain factor M^{-8k}
Irregular 4 pt fcts may diverge for B up to 4
finite number of counter terms
Ex: Irregular log. divergent 4-point subgraph



renormalization of 2 pt graph regulates 4 pt subdivergence, If momenta of subgraph are neither external nor connected, it leads to nonplanar graphs, e g:



it has oscillations and is convergent
Planar regular and irregular 2 point fcts have to be renormalized
Four point fct: Planar regular and some planar irregular graphs are divergent (appear only as subgraphs of planar two point graphs).

Renormalization of two and four point function

$B = 1$ perform Taylor expansion of propagator around zero external momenta

$$C^{i_k}(P+p) = C^{i_k}(p) + P\partial C^{i_k}(p) + \frac{1}{2}(P\partial)^2 C^{i_k}(p) + ...$$

1^{st} term: quadratic mass ren., 2^{nd} term vanishes, 3^{rd} term: wave fct ren. Two broken faces: Similar expansion of

$$A_G = \int d^4p \Phi(p)\Phi(-p) \int Propagators.Phases$$

Expand fields $\Phi(p) = \Phi(0) + \int_0^1 dsp\nabla\Phi(sp)$
amplitude splits into 3 parts 1^{st} term can be absorbed into nonlocal μ term next terms treated by the trick...

Conclusions

- Graphs couple internal and external momenta IR divergence not renormalizable
- modified actions for bosonic fields yield renorm. models
  Might give a nontrivial scalar Higgs model?
- RG flows save
- fermions give spectral triple
- further models: add nonlocal term
- generalization to gauge models?
- gravity?

to John Many Happy

Recurrencies