THE SEIBERG-WITTEN MAP AND SUPERSYMMETRY

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PLAN

1. Introduction
2. Ordinary dual with deformed susy transformations of U(1) NC SYM
3. Superfield Seiberg-Witten eqs. and their solutions
4. Conclusions and outlook

The Seiberg-Witten map

- In 1999 Seiberg-Witten –in "String Theory and NC Geometry"–, to account for the fact that two regularisations of the same underlying world-sheet field theory led to seemingly different theories, showed that

NC U(1) YANG-MILLS THEORY $\longleftrightarrow$ AN ORDINARY U(1) THEORY

(Point-Splitting Reg.) (Pauli-Villars Reg.)

$$A_\mu[a_\sigma] \quad \longleftrightarrow \quad a_\sigma$$

$$A_\mu + \eta\, \hat{s} A_\mu = A_\mu[a_\sigma + \eta\, s a_\sigma] \quad \eta = \text{Grass. par.} \quad \text{(SW eq.)},$$
$$\hat{s}\Lambda[a_\mu, \lambda] = s\Lambda[a_\mu, \lambda] \quad \text{(Consistency cond.)},$$

$$\hat{s}A_\mu = -D_\mu\Lambda = -\partial_\mu\Lambda - i[A_\mu, \Lambda]_\star \quad s a_\mu = -\partial_\mu\lambda,$$
$$\hat{s}\Lambda[a_\mu, \lambda] = i\Lambda \star \Lambda,\, s\lambda = 0,\, (f \star g)(x) = f(x)\exp(\tfrac{i}{2} h\,\omega^{\mu\nu}\overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu})g(x),\, \omega^{0i} = 0.$$

Most general local, up to $O(h^2)$, solution reads

$$A_\mu = a_\mu - \tfrac{h}{2}\omega^{\alpha\beta} a_\alpha(\partial_\beta a_\mu + f_{\beta\mu}) + h\, a_1\, \partial_\mu \omega^{\alpha\beta} f_{\alpha\beta} + h\, a_2\, \omega_\mu{}^\rho \partial^\nu f_{\nu\rho} + O(h^2),$$
$$\Lambda = \lambda + \tfrac{1}{2}\, h\,\omega^{\alpha\beta} a_\alpha \partial_\beta \lambda + h\, b_1\, \omega^{\alpha\beta} f_{\alpha\beta}\lambda + O(h^2).$$

NC field theory for any gauge group?

Two drawbacks of "standard" –ie, ordinary fields are not needed to define the theory– NC gauge field theory regarding phenomenology:

- Gauge groups products of U(N)´s in the fundamental, anti-fundamental, adjoint and bi-fundamental reps.

  No SU(5), SO(10) etc..,i.e., NO GUTS

- The NC anomaly cancelation condition implies that anomaly free theories have a tendency to be vector-like. See however the models proposed by Chu, Khoze, Travaglini, Levell, Chaichian, Tureanu, Arai, Saxell, Abel, Jaeckel, Ringwald, Mondragon, Zoupanos, Aschieri.....

NC field theory for any gauge group

Between 2000 and 2003 [Madore, Scharml, Schupp, Wess, Jurco, Calmet, Wohlgennant, Calmet, Aschieri] it was put forward and developed a formalism which makes it possible in NC space-time the use of any gauge group in any rep.

- In this formalism the NC gauge fields are defined in terms of ordinary fields –ie, fields having ordinary gauge transformation properties– by means of the Seiberg-Witten map. The NC gauge fields take values in the enveloping algebra of the Lie algebra of the ordinary gauge group–>NC gauge theories in the enveloping-algebra formalism.
- The NC Standard Model was thus formulated and ...
- NC GUTS, as well.

Good and bad features

Some features of NC Gauge theories in the enveloping-algebra formalism:

- Good features
  - Plenty of gauge anomaly free models to choose from –The NC SM model, NC GUTS ...]
    See work of Brandt, Ruiz and myself on gauge anomalies
  - Give rise to new and rich phenomenology (will show up at the LHC?)
    See work of Schupp, Trampetic, Wess, Aschieri, Raffelt, Minkowski, Buric, Latas, Radovanovic, Alboteanu, Ohl, Ruckl..

Good and bad features

- Bad features ???
  - It is not clear whether they may come from String Theory (although they have not been ruled out by it yet).
  - These NC theories have tendency to be nonrenormalizable in the finite-number-of-counterterms sense since they contain power-counting nonrenormalizable vertices. But surprisingly their failing to be renormalizable is not as severe as standard renormalization theory would predict. In fact, against all odds, the gauge sector is one-loop renormalizable, which most surprising according to standard renormalization theory [New hidden symmetry?]. ANYHOW RENORMALIZABILITY IS A PARADIGM NO LONGER.
    See the work of Bilch, Grimstrup,Grosse, Wulkenhaar, Buric, Radovanovic, Latas, Trampetic

SUSY is in demand

- SUSY versions of the NC SM and GUTS much needed! Why?
  - Not altogether unlikely that SUSY is needed to give a correct description of Nature around 1 Tev (Some of us believe that SUSY will be discovered at the LHC).
  - SUSY is a key ingredient of String Theory.
  - The obvious choice if one wants to improve the UV behaviour of a field theory.
- But, it is not clear that the Seiberg-Witten map –being non-linear– mapped a NC SUSY theory into an ordinary SUSY theory. In fact, the map worked out by Seiberg and Witten does not seem to do that.

U(1) NC SYM in the WZ gauge

Let us consider NC U(1) SYM theory in the Wess-Zumino gauge. Its action in components reads:

$$S_{NCSYM} = \frac{1}{2g^2}\int d^4x\,[-\frac{1}{2}F^{\mu\nu}\star F_{\mu\nu} - 2i\,\Lambda^\alpha\star\sigma^\mu_{\alpha\dot\alpha}D_\mu\bar\Lambda^{\dot\alpha} + D\star D]$$

The action is invariant under NC SUSY transformations in the WZ gauge

$$\begin{aligned}\delta_\epsilon A^\mu &= i\epsilon^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\Lambda^{\dot\alpha} + i\bar\epsilon^{\dot\alpha}\bar\sigma^\mu_{\dot\alpha\alpha}\Lambda^\alpha,\\ \delta_\epsilon\Lambda_\alpha &= (\sigma^{\mu\nu})_\alpha{}^\beta\epsilon_\beta F_{\mu\nu} + i\epsilon_\alpha D,\\ \delta_\epsilon D &= -\epsilon^\alpha\sigma^\mu_{\alpha\dot\alpha}D_\mu\bar\Lambda^{\dot\alpha} + \bar\epsilon^{\dot\alpha}\bar\sigma^\mu_{\dot\alpha\alpha}D_\mu\Lambda^\alpha.\end{aligned}$$

and NC BRS transformations

$$\hat{s}A^\mu = -D_\mu\Omega,\ \hat{s}\Lambda_\alpha = -i[\Lambda_\alpha,\Omega]_*,\ \hat{s}D = -i[D,\Omega]_*, \hat{s}\Omega = i\Omega\star\Omega.$$

Dual of U(1) NC SYM under the SW map

Following Putz & Wulkeenhaar, IJMPA 18 (2003) 3325; Dayi, Ulker& Yapsikan, JHEP 10(2003)010 and Ulas Saka & Ulker PRD 75 085009 (2007), one uses the “standard" form of the SW map to obtain an ordinary dual of the U(1) NC SYM theory:

$$A_\mu = a_\mu - \frac{h}{2}\omega^{\nu\rho} a_\nu(\partial_\rho a_\mu + f_{\rho\mu}),$$
$$\Lambda_\alpha = \lambda_\alpha + h\omega^{\nu\rho} a_\rho \partial_\nu \lambda_\alpha,$$
$$\bar{\Lambda}_{\dot\alpha} = \bar{\lambda}_{\dot\alpha} + h\omega^{\nu\rho} a_\rho \partial_\nu \bar{\lambda}_{\dot\alpha},$$
$$D = d + h\omega^{\nu\rho} a_\rho \partial_\nu d.$$

The action, S_{dual}, of the ordinary dual reads:

$$S_{dual}(a_\mu, \lambda\, d; \omega) = S_{NCYM}[A_\mu[a_\sigma], \Lambda_\alpha[\lambda_a, a_\sigma], D[d, a_\sigma]]$$

BUT

the ordinary fields DO have non-ordinary –i.e.,deformed– “SUSY" transformations, although by construction they have ordinary U(1) gauge transformations!

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\begin{aligned}
A_\mu &= a_\mu - \tfrac{h}{2}\omega^{\nu\rho} a_\rho(\partial_\rho a_\mu + f_{\rho\mu}),\\
\Lambda_\alpha &= \lambda_\alpha + h\omega^{\nu\rho} a_\rho \partial_\nu \lambda_\alpha,\\
\bar{\Lambda}_{\dot\alpha} &= \bar{\lambda}_{\dot\alpha} + h\omega^{\nu\rho} a_\rho \partial_\nu \bar{\lambda}_{\dot\alpha},\\
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\end{aligned}
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Deformed “SUSY" transformations

The way a_μ transforms under δ_ϵ –the generator of the would-be SUSY transformations– is obtained by solving for $\delta_\epsilon a_\mu$ the following set of eqs.:

$$
\begin{aligned}
\delta_\epsilon A^\mu &= i\epsilon^\alpha \sigma^\mu_{\alpha\,\dot\alpha}\bar\Lambda^{\dot\alpha} + i\bar\epsilon^{\dot\alpha}\bar\sigma^\mu_{\dot\alpha\,\alpha}\Lambda^\alpha,\\
\delta_\epsilon A^\mu &= \delta_\epsilon a_\mu - \tfrac{h}{2}\omega^{\nu\rho}[\delta_\epsilon a_\nu(\partial_\rho a_\mu + f_{\rho\mu}) + a_\nu(2\partial_\rho\delta_\epsilon a_\mu - \partial_\mu\delta_\epsilon a_\rho)]\\
\Lambda_\alpha &= \lambda_\alpha + h\omega^{\nu\rho}a_\rho\partial_\nu\lambda_\alpha, \bar\Lambda_{\dot\alpha} = \bar\lambda_{\dot\alpha} + h\omega^{\nu\rho}a_\rho\partial_\nu\bar\lambda_{\dot\alpha}.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\delta_\epsilon a_\mu = {}& i\epsilon\sigma_\mu\bar\lambda + i\bar\epsilon\bar\sigma_\mu\lambda\\
& - \tfrac{h}{4}\omega^{\nu\rho}\,(i\epsilon\sigma_\nu\bar\lambda + i\bar\epsilon\bar\sigma_\nu\lambda)(f_{\rho\mu} + \partial_\rho a_\mu) - \tfrac{h}{2}\,\omega^{\nu\rho}\,(i\epsilon\sigma_\nu\partial_\mu\bar\lambda + i\bar\epsilon\bar\sigma_\nu\partial_\mu\lambda)a_\rho.
\end{aligned}
$$

Analogously,

$$
\begin{aligned}
\delta_\epsilon\lambda &= \sigma^{\mu\nu}\epsilon f_{\mu\nu} + i\epsilon D + h\omega^{\nu\rho}\,(i\epsilon\sigma_\nu\bar\lambda - i\bar\epsilon\bar\sigma_\nu\lambda)\partial_\rho\lambda + h\omega^{\rho\sigma}\sigma^{\mu\nu}\epsilon f_{\mu\rho}f_{\nu\sigma},\\
\delta_\epsilon d &= \bar\epsilon\bar\sigma^\mu\partial_\mu\lambda - \epsilon\sigma^\mu\partial_\mu\bar\lambda - h\omega^{\nu\rho}\,(i\epsilon\sigma_\nu\partial_\mu\bar\lambda + i\bar\epsilon\bar\sigma_\nu\partial_\mu\lambda)\partial_\rho d + h\omega^{\rho\sigma}(\epsilon\sigma^\mu f_{\rho\mu}\partial_\sigma\bar\lambda\\
&\qquad - \bar\epsilon\bar\sigma^\mu f_{\rho\mu}\partial_\sigma\lambda).
\end{aligned}
$$

THE δ_ϵ OF THE ORDINARY FIELDS GETS ω-DEPENDENT BITS!

Deformed Susy trans.: what may have gone awry?

We have shown that the "ordinary" fields undergo deformed "susy" transformations. What may have gone awry?

- Perhaps our insisting on having the Wess-Zumino gauge both in the NC theory and in the ordinary theory? In general, the SW map maps ordinary gauge orbits into noncommutative gauge orbits, but does not preserve the gauge choice, e.g., if we choose an ordinary gauge field in the temporal gauge its NC image under the SW map is not in the temporal gauge.
- So, perhaps, one should try to set up the formalism using superfields, thus dealing with all the components and no gauge choice and manifest SUSY in one single stroke.

NC and ordinary superfield BRS transformations

Let $V(x,\theta,\bar\theta)$ denote a U(1) NC vector superfield and let $v(x,\theta,\bar\theta)$ be a U(1) ordinary vector superfield. $\hat{s}$ and s implement the NC and ordinary BRS transformations, respectively:

$$\hat{s}\,V = -\tfrac{1}{2}L_V(\bar\Lambda+\Lambda)+\tfrac{1}{2}L_V\mathrm{coth}\tfrac{1}{2}L_V(\bar\Lambda-\Lambda), \qquad L_V=[V,\cdot\,]_\star,$$
$$\hat{s}\,\Lambda = \Lambda\star\Lambda,$$
$$\bar\Lambda = (\Lambda)^*,$$
$$s\,v = \bar\lambda-\lambda, \quad s\,\lambda = 0, \quad \bar\lambda=(\lambda)^*,$$

Λ and λ are, respectively, the NC and ordinary ghost chiral superfields. $\bar\Lambda$ and $\bar\lambda$ their antichiral partners.

Seiberg-Witten eqs. for superfields

We shall try to find a map –the Seiberg-Witten map for superfields–

$$v \longrightarrow V[v]$$

such that it maps ordinary U(1) gauge orbits of v in NC U(1) gauge orbits of V for infinitesimal gauge transformations. This map should be a solution to the following set of equations

$$\hat{s}\, V[v] = s\, V[v] \qquad \text{SW map eqs. for superfields,}$$
$$\hat{s}\, \Lambda[\lambda, v] = s\, \Lambda[\lambda, v] \qquad \text{Consistency conditions,}$$
$$\bar{D}_{\dot{\alpha}}\, \Lambda[\lambda, v] = 0, \quad \bar{D}_{\dot{\alpha}}\, \lambda = 0 \qquad \text{Chirality conditions.}$$

Before giving a family of solutions to the set of eqs. above and discuss its properties, we shall show that

- the standard SW is not the gauge vector component of a vector superfield which is a local function of v and
- at each order in $h\omega^{\mu\nu}$ there is no local solution to the previous set of eqs.

Local SW map field never a Superfield component. 1

Let us first consider NC superfields whose components are local functions of the ordinary components –with ordinary susy transformations– and the space-time derivatives of the latter. This is tantamount to saying that we shall first consider NC superfields that are local functions of the ordinary superfields and their susy derivatives.
$V(x,\theta,\bar\theta)$, at each order in $h\omega^{\mu\nu}$, is a polynomial in $v(x,\theta,\bar\theta)$ and its susy derivatives:

$$V = v - \Big[\tfrac{r_1}{16}\, h\omega^{\alpha\beta} D_\alpha \bar D^{\dot\delta} v \partial_{\beta\dot\delta} v + \tfrac{r_2}{16}\, h\omega^{\alpha\beta} D_\alpha v \partial_{\beta\dot\gamma} \bar D^{\dot\gamma} v - i\tfrac{r_3}{16}\, h\omega^{\alpha\beta} v D_\alpha w_\beta + \tfrac{r_4}{16}\, h\omega^{\alpha\beta} \bar D^{\dot\gamma} v \partial_{\alpha\dot\gamma} D_\beta v + -i\tfrac{r_5}{16}\, h\omega^{\alpha\beta}\, w_\alpha D_\beta v\Big] + \Big[c.c.\Big].$$

$$w_\alpha = -\frac{1}{4}\bar D^2 D_\alpha v$$

Local SW map field never a Superfield component. 2

Then the $O(h\omega^{\mu\nu})\ \theta\bar\theta-$ component of V is given by a linear combination, plus its dotted counterpart, of

$$\begin{aligned}
h\omega^{\alpha\beta}D_\alpha\bar D^{\dot\delta}v\partial_{\beta\dot\delta}v &\Longrightarrow -h\omega^{\alpha\beta}\sigma^{\rho\sigma}_{\alpha\beta}a_\rho\partial_\sigma a_\mu,\\
h\omega^{\alpha\beta}D_\alpha v\partial_{\beta\dot\gamma}\bar D^{\dot\gamma}v &\Longrightarrow \tfrac{h}{2}\omega^{\alpha\beta}\sigma^{\rho\sigma}_{\alpha\beta}(a_\rho f_{\sigma\mu}-\tfrac12 f_{\rho\sigma}a_\mu+a_\nu\partial_\rho a^\nu\delta^\mu_\sigma+\\
&\qquad a_\rho(\partial_\nu a^\nu)\delta^\mu_\sigma-a^\nu\partial_\nu a_\rho\delta^\mu_\sigma),\\
h\omega^{\alpha\beta}vD_\alpha w_\beta &\Longrightarrow i\tfrac{h}{2}\omega^{\alpha\beta}\sigma^{\rho\sigma}_{\alpha\beta}f_{\rho\sigma}a_\mu,\\
h\omega^{\alpha\beta}\bar D^{\dot\gamma}v\partial_{\alpha\dot\gamma}D_\beta v &\Longrightarrow -\tfrac{h}{2}\omega^{\alpha\beta}\sigma^{\rho\sigma}_{\alpha\beta}(a^\nu\partial_\nu a_\rho\delta^\mu_\sigma-\tfrac12 f_{\rho\sigma}a_\mu+a_\rho\partial_\mu a_\sigma+\\
&\qquad +a_\rho\partial_\sigma a_\mu+a_\nu\partial_\rho a^\nu\delta^\mu_\sigma+a_\rho\partial_\nu a^\nu\delta^\mu_\sigma),\\
h\omega^{\alpha\beta}w_\alpha D_\beta v &\Longrightarrow i\tfrac{h}{2}\omega^{\alpha\beta}\sigma^{\rho\sigma}_{\alpha\beta}(\tfrac12 f_{\rho\sigma}a_\mu-a_\rho f_{\sigma\mu}+a^\nu\partial_\nu a_\rho\delta^\mu_\sigma-a_\nu\partial_\rho a^\nu\delta^\mu_\sigma)
\end{aligned}$$

That sum is to be compared with the most general U(1) SW map

$$-\frac14 h\omega^{\rho\sigma}a_\rho(\partial_\sigma a_\mu+f_{\sigma\mu})+\text{ terms with 2 derivatives}$$

TERMS IN RED ARE NOT IN THE SW MAP AND EVENTUALLY LEAD TO

......

Local SW map field never a Superfield component. conclusion

- THE LOCAL SEIBERG-WITTEN MAP IS NOT THE $\theta\bar{\theta}$ COMPONENT OF A SUPERFIELD. SO, THE STANDARD SEIBERG-WITTEN MAP CANNOT BE OBTAINED FROM A LOCAL SOLUTION TO THE SEIBERG-WITTEN SUPERFIELD EQS.

  [This is in total disagreement with what was claimed by the authors of the paper JHEP05 (2000) 008.]

No local solution to the SW map superfield problem

We shall now show that, up to first order in $h\omega^{\mu\nu}$, there is no local solution to the first order Seiberg-Witten eqs + Consistency conditions + Chirality condition, i.e., to the set of eqs.

$$s\,V^{(1)}[v] = -\tfrac{i}{2}\,\omega^{\mu\nu}\partial_\mu v\partial_\nu(\bar\lambda+\lambda) + (\bar\Lambda^(1) - \Lambda^{(1)}), \qquad \text{SW map eqs}$$
$$s\,\Lambda[\lambda, v]^{(1)} = \tfrac{i}{2}\,\omega^{\mu\nu}\partial_\mu\lambda\partial_\nu\lambda, \qquad \text{Consistency conditions}$$
$$\bar D_{\dot\alpha}\,\Lambda^{(1)} = 0, \qquad \text{Chirality condition}$$

Notation: $V = v + hV^{(1)} + \ldots$ and $\Lambda = \lambda + h\Lambda^{(1)} + \ldots$.
One may show that the most general local solution to the Consistency conditions reads

$$\Lambda^{(1)} = -\tfrac{1}{64}\omega^{\alpha\beta}[(1+4\,c_1\,i)\bar D^{\dot\alpha}D_\alpha v\partial_{\beta\dot\alpha}\lambda - c_1\bar D^2(D_\alpha vD_\beta\lambda) + c_2(\bar D^{\dot\alpha}D_\alpha\partial_{\beta\dot\alpha}v)\lambda]$$
$$-\tfrac{1}{64}\omega^{\dot\alpha\dot\beta}[\bar D^{\dot\alpha}D_\alpha v\partial_{\beta\dot\alpha}\lambda + c_3\,(\partial^\alpha{}_{\dot\alpha}\bar D_{\dot\beta}D_\alpha v)\lambda],$$

Notation: $\omega_{\alpha\beta} = -2\sigma^{\mu\nu}_{\alpha\beta}\,\omega_{\mu\nu}, \quad \omega_{\dot\alpha\dot\beta} = 2\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}\,\omega_{\mu\nu}$

Unfortunately the $\omega^{\dot\alpha\dot\beta}$-bit of $\Lambda^{(1)}$ is not chiral for any value of c_3. So, ...

Introducing solutions with gauge dependent nonlocal terms

- A LOCAL $\Lambda^{(1)}$ IS NOT CHIRAL UNLESS $\omega^{\dot\alpha\dot\beta}=0$ —and $1+4\,c_1\,i=0,\;\; c_2=0$—, which is not possible since $\omega^{\mu\nu}$ is real.
  [This result was obtained first by Dzo Mikulovic JHEP01 (2004) 063 and it is also in total disagreement with what was claimed by the authors of the paper JHEP05 (2000) 008.]
- SINCE THE CHIRALITY CONDITION is not met, it is natural to look for solutions to the first order in $h\omega^{\mu\nu}$ SW eqs. that are polynomials in

$$v,\quad v_{+}=\frac{\bar D^2 D^2}{16\Box}v\quad \text{and/or}\quad v_{-}\frac{D^2\bar D^2}{16\Box}v$$

and theirs susy derivatives. Thus, non local contributions will be gauge artifacts!. Disappear in the SUSY Landau gauge $\bar D^2 D^2 v = D^2\bar D^2 v=0$.

Our superfield family of solutions

- By explicit computation, up to first order in $h\omega^{\mu\nu}$, we have found the most general solution to the superfield SW eqs. + Consistency conditions+ Chirality conditions that is quadratic in the superfields $v,\ v_+,\ v_-$. It run thus

$$\Lambda = \lambda + \tfrac{i}{32} h\omega^{\alpha\beta}\partial^{\dot\alpha}{}_{\alpha} v_+ \partial_{\dot\alpha\beta}\lambda + \tfrac{i}{32} h\omega^{\dot\alpha\dot\beta}\partial_{\dot\alpha}{}^{\alpha} v_+ \partial_{\alpha\dot\beta}\lambda + r\, h\omega^{\alpha\beta}\bar D^2(D_\alpha(v-v_+)D_\beta\lambda),$$

$$V = v + \tfrac{i}{32} h\omega^{\alpha\beta}[\partial^{\dot\alpha}{}_{\alpha}(v-v_-)\partial_{\dot\alpha\beta}(v-v_+)] - \tfrac{i}{32} h\omega^{\dot\alpha\dot\beta}[\partial_{\dot\alpha}{}^{\alpha}(v-v_+)\partial_{\dot\beta\alpha}(v-v_-)]$$
$$+ r h\omega^{\alpha\beta}\bar D^2(D_\alpha(v-v_+)D_\beta v) + \bar r h\omega^{\dot\alpha\dot\beta} D^2(\bar D_{\dot\alpha}(v-v_-)\bar D_{\dot\beta} v) + \mathcal{X}$$

$$\mathcal{X} = \Big[h\omega^{\alpha\beta}\{ c_1 D_\alpha \bar D^{\dot\gamma}\tilde v \partial_{\beta\dot\gamma}\tilde v + c_2 D_\alpha \tilde v D^{\dot\gamma}\partial_{\beta\dot\gamma}\tilde v + c_3(-i)\tilde v D_\alpha w_\beta + c_4 \bar D^{\dot\gamma}\tilde v \partial_{\alpha\dot\gamma} D_\beta \tilde v$$
$$+ c_5(-i) w_\alpha D_\beta \tilde v\}\Big] + \Big[\quad \text{c.c.} \quad \Big]$$

$$\tilde v = v - v_+ - v_-, \quad s\tilde v = 0 \Longrightarrow s\mathcal{X} = 0.$$

Comments to superfield nonlocal solution

COMMENTS:

- $\mathcal{X}$ corresponds to a field redefinition. It is local in the SUSY LANDAU GAUGE!.
- The first order contribution to V, in the SUSY LANDAU GAUGE, is the most general local contribution in v that one may write. Good for renormalisation?
- The superfield SW map found by Chekhov & Khizhnyakov, hep-th/0103048, is an element of our family of solutions.

Taking stock

Let's take stock. We have reached the following conclusions:

- In the WZ gauge, the local –standard– Seiberg-Witten map establishes a duality between noncommutative fields and "ordinary" fields. The noncommutative fields having the appropriate –i.e., NC– gauge gauge transformations and susy transformations, but the "ordinary" fields having ordinary gauge transformations and ω-deformed susy transformations –this is why we call them "ordinary".
- The nonlocal superfield Seiberg-Witten map establishes a duality between noncommutative superfields and ordinary superfields. By construction the noncommutative superfields transform appropriately under NC gauge transformations and SUSY transformations –which are actually supertranslations. Also by construction, the ordinary superfields undergo ordinary gauge transformations and ordinary –no deformation in ω– susy transformations. The nonlocal parts of the SW map in question are gauge artifacts: vanish in the supersymmetric Landau gauge.

A relationship?

- Since the field theory constructed from the "ordinary" fields and the field theory constructed from the ordinary fields are both dual to the same underlying NC field theory, there is natural question to ask:

ARE THEY RELATED AS ORDINARY THEORIES IN SOME WAY?

- We have answered this question in the affirmative. Actually both what we have called the "ordinary" theory and ordinary theory are the same ordinary field theory, but formulated in terms of different sets of field variables.

The relationship

Let a_μ denote the gauge field obtained from a NC field A_μ by using the nonlocal superfield SW map– and b_μ the "ordinary" field obtained from the very same NC field by employing the local Seiberg-Witten map. Let λ_α, d be the susy partners of a_μ. Then, we have

$$A_\mu = a_\mu + hA^{(1)}_\mu[a, \lambda_\alpha, \bar\lambda_{\dot\alpha}, d] + O(h^2) = b_\mu + hB^{(1)}_\mu[b_\mu] + O(h^2),$$
$$A^{(1)}_\mu = \tfrac{1}{2}\omega^{\rho\sigma}\Big(a_\rho\partial_\sigma a_\mu - \tfrac{1}{2}a_\rho\partial_\mu a_\sigma\Big) - 2\partial_\mu\mathcal{Z}[a_\mu] + \mathcal{A}_\mu[a_\mu, \lambda_\alpha, \bar\lambda_{\dot\alpha}, d]$$
$$B^{(1)}_\mu = \tfrac{1}{2}\omega^{\rho\sigma}\Big(b_\rho\partial_\sigma b_\mu - \tfrac{1}{2}b_\rho\partial_\mu b_\sigma\Big),$$

where $$s\mathcal{A}_\mu = 0.$$

Hence, we define the following field transformation which turns the local SW map into the nonlocal one:

$$b_\mu = a_\mu - 2h\partial_\mu\mathcal{Z}[a_\mu] + h\mathcal{A}_\mu[a_\mu, \lambda_\alpha, \bar\lambda_{\dot\alpha}, d] + O(h^2),$$

where $\Longrightarrow\Longrightarrow\Longrightarrow\Longrightarrow\Longrightarrow\Longrightarrow$

The relationship: formulae

$$\tilde{\mathcal{Z}} = -\frac{1}{128}\omega^{\alpha\beta}\left(a^{\dot\alpha}_{\alpha} - \frac{\partial^{\dot\alpha}_{\alpha}}{\Box}\partial a\right)a_{\dot\alpha\beta} - \frac{1}{128}\omega^{\dot\alpha\dot\beta}\left(a^{\alpha}_{\dot\alpha} - \frac{\partial^{\alpha}_{\dot\alpha}}{\Box}\partial a\right)a_{\dot\beta\alpha},$$

$$\begin{aligned}\mathcal{A}_{\dot\beta\gamma} = \frac{i}{64}\omega^{\alpha\beta}\Bigg[&\left(a^{\dot\alpha}_{\alpha} - \frac{\partial^{\dot\alpha}_{\alpha}}{\Box}\partial a\right)\partial_{\dot\beta\gamma}\left(a_{\dot\beta\beta} - \frac{\partial_{\dot\alpha\beta}}{\Box}\partial a\right)\\ &- 2\left(a^{\dot\alpha}_{\alpha} - \frac{\partial^{\dot\alpha}_{\alpha}}{\Box}\partial a\right)\partial_{\dot\alpha\beta}\left(a_{\dot\beta\gamma} - \frac{\partial_{\dot\beta\gamma}}{\Box}\partial a\right)\\ &+ \frac{1}{4}\frac{\partial^{\dot\alpha}_{\alpha}}{\Box}d\frac{\partial_{\dot\alpha\beta}\partial_{\dot\beta\gamma}}{\Box}d - \frac{i}{2}\frac{\partial^{\dot\alpha}_{\alpha}\partial_{\dot\beta\sigma}}{\Box}\lambda^{\sigma}\frac{\partial_{\dot\alpha\beta}\partial_{\gamma\dot\sigma}}{\Box}\bar\lambda^{\dot\sigma}\Bigg]\\ &+ (c.c.)|_{\beta\leftrightarrow\gamma},\end{aligned}$$

$$\mathcal{A}_{\dot\beta\gamma} = \mathcal{A}_{\mu}\bar\sigma^{\mu}_{\dot\beta\gamma}.$$

The relationship: Comments

- SINCE $\mathcal{A}_\mu$ IS BRS-CLOSED,THE MAP

$$b_\mu = a_\mu - 2h\partial_\mu \mathcal{Z}[a_\mu] + h\mathcal{A}_\mu[a_\mu, \lambda_\alpha, \bar{\lambda}_{\dot{\alpha}}, d] + O(h^2),$$

MAPS, FOR INFINITESIMAL GAUGE TRANSFORMATIONS, GAUGE ORBITS OF a_μ INTO GAUGE ORBITS OF b_μ :

$$a_\mu + \delta_\Omega a_\mu \longrightarrow b_\mu + \delta_{\hat{\Omega}} b_\mu,$$

$$\delta_\Omega a_\mu = -\partial_\mu \Omega,$$
$$\delta_{\hat{\Omega}} b_\mu = -\partial_\mu \hat{\Omega}, \qquad \hat{\Omega} = \Omega + 2h\delta_\Omega \tilde{\mathcal{Z}}[a_\mu, \Omega].$$

- WE HAVE OBTAINED ANALOGOUS RESULTS FOR THE "SUSY" PARTNERS OF b_μ .

Conclusions and outlook

- JUST ONE ORDINARY THEORY DUAL TO U(1) NC SYM.
- DIFFERENT SW MAPS FURNISH DIFFERENT REPRESENTATIONS OF THE SAME ORDINARY THEORY IN TERMS OF FIELD VARIABLES.
- GENERALIZE THE ANALYSIS TO ANY ORDER IN $h\omega^{\mu\nu}$ AND OTHER GAUGE GROUPS $\longrightarrow$ NC MSSM.
- GAIN A STRING THEORY UNDERSTANDING OF THE ISSUE: SUSY DBI ACTIONS, NC AND ORDINARY IN THE PRESENCE OF A CONSTANT $B_{\mu\nu}$:
  - NC DBI theory $\longleftarrow$ local SW map $\longrightarrow$ DBI theory in $B_{\mu\nu}$,
  - What about their susy generalizations?

FORMULAE 1

$\check{\lambda}_a$ and $\check{d}$ susy partners of b_μ, i.e., “ordinary" fields obtained from the NC WZ gauge susy multiplet A_μ, Λ_α, D by means of the standard SW map.

λ_a and d susy partners of a_μ, i.e., “ordinary" fields obtained from the NC WZ gauge susy multiplet A_μ, Λ_α, D by means of the nonlocal superfield SW map.

$$\Lambda_\alpha = \lambda_\alpha + h\Lambda_\alpha^{(1)} + O(h^2) = \check{\lambda}_\alpha + h\check{\Lambda}_\alpha^{(1)} + O(h^2)$$
$$\Lambda_\alpha^{(1)} = \frac{1}{2}\omega^{\rho\sigma} a_\rho \partial_\sigma \lambda_\alpha + \mathcal{L}_\alpha,$$
$$\check{\Lambda}_\alpha^{(1)} = \frac{1}{2}\omega^{\rho\sigma} b_\rho \partial_\sigma \check{\lambda}_\alpha,$$

Hence

$$\check{\lambda}_\alpha = \lambda_\alpha + h\mathcal{L}_\alpha[a_\mu, \lambda_\alpha, d] + O(h^2).$$

$\mathcal{L}_\alpha$ is BRS-closed, so $s\check{\lambda}_\alpha = s\lambda_\alpha = 0$, as it should be.

FORMULAE 2

$\check{\lambda}_a$ and $\check{d}$ susy partners of b_μ, i.e., “ordinary" fields obtained from the NC WZ gauge susy multiplet A_μ, Λ_α, D by means of the standard SW map.

λ_a and d susy partners of a_μ, i.e., “ordinary" fields obtained from the NC WZ gauge susy multiplet A_μ, Λ_α, D by means of the nonlocal superfield SW map.

$$D = d + hD^{(1)} + O(h^2) = \check{d} + h\check{D}^{(1)} + O(h^2)$$
$$D^{(1)} = \frac{1}{2}\omega^{\rho\sigma} a_\rho \partial_\sigma d + \mathcal{D},$$
$$\check{D}^{(1)} = \frac{1}{2}\omega^{\rho\sigma} b_\rho \partial_\sigma \check{d},$$

Hence

$$\check{d} = d + h\mathcal{D}[a_\mu, \lambda_\alpha, d] + O(h^2).$$

$\mathcal{D}_\alpha$ is BRS-closed, so $s\check{d} = sd = 0$, as it should be.