Emergent Gravity from Yang-Mills Matrix Models

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Bayrischzell, Mai 2008

Introduction

- Classical space-time inappropriate at Planck scale
  due to gravity $\leftrightarrow$ Quantum Mechanics
  $\Rightarrow$ "quantized" (noncommutative?) spaces:
- Physics on NC space:
  Noncommutative Quantum Field Theory
  strange feature: UV/IR mixing
- What about gravity on/for quantized spaces ??

  should be simple & naturally related to NC
  should improve quantization and/or "flatness" problem

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Main Message:

- $\exists$ simple models for dynamical NC space:

  Matrix Models

  - M. M. known to describe NC gauge theory
  - M. M. also contain gravity
    intrinsically NC mechanism

- metric emerges, not fundamental d.o.f.
  (Rivelles 2002, Yang 2006, ... NC gauge thy $\leftrightarrow$ gravity)
  reasonably close to GR at low energies (?)
  - gravitational waves
  - linearized metric: $R_{ab} \sim 0$

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Matrix Models and dynamical space(time)

Consider Matrix Model:

$$S_{YM} = -Tr[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'}, \qquad a = 0, 1, 2, 3$$

(toy candidate for fundamental theory)

dynamical objects: $X^a \in L(\mathcal{H})$ ... hermitian matrices

equation of motion: $[X^a, [X^{a'}, X^{b'}]]\eta_{aa'} = 0$

solutions:

- $[X^a, X^b] = 0$ ...classical objects; ignore here
- $[X^a, X^b] = i\bar{\theta}^{ab}\,\mathbb{1}$, "quantum plane"

  where $\bar{\theta}^{ab}$ ... antisymmetric tensor, nondegenerate
- many more, of type $[X^a, X^b] = i\theta^{ab}(x)$

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Noncommutative spaces and Poisson structure

$(\mathcal{M}, \theta^{\mu\nu}(x))$ ... $2n$-dimensional manifold with Poisson structure

Its quantization $\mathcal{M}_\theta$ is NC algebra such that

$$\begin{array}{ccc} \mathcal{C}(\mathcal{M}) & \to & \mathcal{A} \subset L(\mathcal{H}) \\ f(x) & \mapsto & \hat{f}(X) \qquad \text{(e.g. plane waves)} \\ i\{f,g\} & \mapsto & [\hat{f},\hat{g}] + O(\theta^2) \end{array}$$

Note

$$[X^\mu, f(X)] \sim i\theta^{\mu\nu}(x)\partial_\nu f(x)$$

simplest example: (Moyal-Weyl) quantum plane $\mathbb{R}^{2n}_\theta$

$$[X^\mu, X^\nu] = i\bar{\theta}^{\mu\nu}\, \mathbb{1}$$

cp. phase space in Quantum Mechanics, but $(X, P) \leftrightarrow (x^1, x^2)$

Effective geometry:

consider scalar field coupled to Matrix Model ("test particle")

$$
\begin{aligned}
S[\Phi] &= Tr\, \eta_{\mu\nu}[X^\mu, \Phi][X^\nu, \Phi] \\
&\sim \int d^4x\, \rho(x)\, \theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\, \eta_{\mu'\nu'}\partial_\mu\Phi\partial_\nu\Phi \\
&= \int d^4x\, \sqrt{\det G_{\mu\nu}}\; G^{\mu\nu}(x)\, \partial_\mu\Phi\partial_\nu\Phi
\end{aligned}
$$

where

$$\rho(x) = \mathrm{Pfaff}\,(\theta^{-1}_{\mu\nu})\ \ldots$$ symplectic volume

$$\boxed{G^{\mu\nu}(x) = \rho(x)\, \theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\, \eta_{\mu'\nu'}}, \qquad \det G \equiv 1$$

- Φ couples to effective metric $G^{\mu\nu}(x)$ determined by $\theta^{\mu\nu}(x)$
- $\theta^{\mu\nu}(x)$ ... vielbein ("gauge-fixed"!)

... quantized Poisson manifold with metric $(\mathcal{M}, \theta^{\mu\nu}(x), G_{\mu\nu}(x))$

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observe:

- preferred coordinates x^μ (defined by matrices), where background metric $\eta_{\mu\nu}$ is constant
- kinetic term always of form $[X^\mu, \psi] \sim i\theta^{\mu\nu}\partial_\nu\psi$

  $\rightarrow$ preferred frame $e^\mu = -i[X^\mu, .] = \theta^{\mu\nu}\partial_\nu$,
  universal coupling to $G^{\mu\nu}$
- natural (symplectic) volume $\mathrm{Tr} \sim \int d^4x\, (\det \theta^{-1}_{\mu\nu})^{1/2}$

  $\rightarrow$ stabilization of flat space,

$$\mathrm{Vol}(\mathcal{M}^4_\theta) = \int d^4x \sqrt{|G|} = (2\pi)^2 \frac{\mathcal{N}}{\Lambda^4_{NC}} .$$

flat case: Moyal-Weyl plane

e. o. m.

$$[X^a,[X^{a'},X^{b'}]]\eta_{aa'} = 0$$

solution

$$[\overline{X}^\mu,\overline{X}^\nu] = i\bar{\theta}^{\mu\nu}\,\mathbf{1}$$ ... "Moyal-Weyl quantum plane"

effective metric

$$G^{\mu\nu} = \rho\,\theta^{\mu\mu'}\theta^{\nu\nu'}\,\eta_{\mu'\nu'} =: \bar{\eta}^{\mu\nu}$$

... indeed flat, effective metric for all other fields

Deformations of Moyal-Weyl plane, I

consider configurations of form

$$X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu \qquad \text{("covariant coordinates")}$$

2 different points of view:

1. new (geometric) point of view

$$i\theta^{\mu\nu}(x) \sim [X^\mu, X^\nu], \qquad G^{\mu\nu} = \theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\,\rho\,\eta_{\mu'\nu'}$$

... nontrivial metric $\to$ gravity

2. old point of view: NC $U(1)$ gauge theory

$$\begin{aligned}[X^\mu, X^\nu] &= i\bar{\theta}^{\mu\mu'}\bar{\theta}^{\nu\nu'}(\partial_{\mu'}A_{\nu'} - \partial_{\nu'}A_{\mu'} + [A_{\mu'}, A_{\nu'}]) + i\bar{\theta}^{\mu\nu} \\ &= i\bar{\theta}^{\mu\mu'}\bar{\theta}^{\nu\nu'} F_{\mu'\nu'} + i\bar{\theta}^{\mu\nu} \qquad (= i\theta^{\mu\nu}(x))\end{aligned}$$

$$S_{YM} \sim \int d^4x\, F_{\mu\nu}F_{\mu'\nu'}\,\bar{\eta}^{\mu\mu'}\bar{\eta}^{\nu\nu'} \qquad (+\text{surface terms})$$

... $U(1)$ Yang-Mills on quantum plane

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deformations of Moyal-Weyl II: linearized gravity

small fluctuations: $X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu$

$$\begin{aligned}
\theta^{\mu\nu}(x) &= -i[X^\mu, X^\nu] = \bar{\theta}^{\mu\nu} + \bar{\theta}^{\mu\mu'}\bar{\theta}^{\nu\nu'} F_{\mu'\nu'} \\
G^{\mu\nu}(x) &= -(\bar{\theta}^{\mu\mu'} + \bar{\theta}^{\mu\eta}\bar{\theta}^{\mu'\kappa} F_{\eta\kappa})(\bar{\theta}^{\nu\nu'} + \bar{\theta}^{\nu\kappa}\bar{\theta}^{d\eta} F_{\kappa\eta})\rho(x)\eta_{\mu'\nu'} \\
&\approx \overline{\eta}^{\mu\nu} - h^{\mu\nu} \qquad (+O(F^2))
\end{aligned}$$

$F_{\mu\nu}(x)$ ... $\mathfrak{u}(1)$ field strength

therefore

$$\boxed{h_{\mu\nu} = \bar{\eta}_{\nu\nu'}\bar{\theta}^{\nu'\rho} F_{\rho\mu} + \bar{\eta}_{\mu\mu'}\bar{\theta}^{\mu'\eta} F_{\eta\nu} - \tfrac{1}{2}\bar{\eta}_{\mu\nu}(\bar{\theta}^{\rho\eta} F_{\rho\eta})}$$

... linearized metric fluctuation (cf. Rivelles [hep-th/0212262])

e.o.m for gravitational d.o.f.:

$$[X^\mu, [X^\nu, X^{\mu'}]]\eta_{\mu\mu'} = 0 \;\Leftrightarrow\; G^{\mu\rho}\partial_\rho\,\theta^{-1}_{\mu\eta}(x) = 0$$

implies linearized vacuum equations of motion

$$R_{\mu\nu}[G] = 0 \;+ O(\theta^2)$$

while $R_{\mu\nu\rho\eta} = O(\bar{\theta}) \neq 0$ ... nonvanishing curvature

$\Rightarrow$ on-shell d.o.f. of gravitational waves on Minkowski space

note

- $G^{\mu\nu} \sim \theta^{\mu\mu'}(x)\,\theta^{\nu\nu'}(x)\eta_{\mu'\nu'}$ ... restricted class of metrics
- same on-shell d.o.f. as general relativity (for vacuum)

i.e.: trace-$U(1)$ photons on $\mathbb{R}^4_\theta$ are actually gravitons e.o.m for gravitational d.o.f.:

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generalization to $su(n)$ gauge fields

basically

$$X^\mu \to X^\mu \otimes M_n(\mathbb{C}) \equiv Y^\mu$$

separate $\mathfrak{u}(1)$ and $\mathfrak{su}(n)$ components !

$$\begin{aligned} Y^\mu &= (\bar{X}^\mu + \bar{\theta}^{\mu\nu} A^0_\nu) \otimes \mathbb{1}_n + (\theta^{\mu\nu} A^\alpha_\nu \otimes \lambda_\alpha) \\ &=: X^\mu \otimes \mathbb{1}_n + \theta^{\mu\nu}(x) A^\alpha_\nu \otimes \lambda_\alpha \end{aligned}$$

will see:

$\mathfrak{u}(1)$ component X^μ ... dynamical geometry, gravity

$\mathfrak{su}(n)$ components A^α_μ ... $\mathfrak{su}(n)$ gauge field coupled to gravity

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effective action to leading order:

requires use of Seiberg-Witten map (technical)

$$S_{YM} = -\int d^4x\, \rho^{-1}\, G^{\mu\mu'} G^{\nu\nu'}\, tr\, F_{\mu\nu}\, F_{\mu'\nu'} + 2\int \eta(x)\, tr\, F \wedge F$$

where

$$\eta(x) = G^{\mu\nu}(x)\eta_{\mu\nu}$$

- indeed $\mathfrak{su}(n)$ YM coupled to metric $G^{\mu\nu}(x)$
- additional term $\int \eta(y)\, tr\, F \wedge F$, topological for $\theta^{\mu\nu} = const$

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Question: what about the Einstein-Hilbert action?

Answer:

- tree level: e.o.m. for gravity follow from $\mathfrak{u}(1)$ sector:

$$G^{\mu\rho}\partial_\rho\,\theta^{-1}_{\mu\nu}(y) = 0 \quad \text{implies} \quad \boxed{R_{\mu\nu}[G] \sim 0} \quad \text{(linearized)}$$

- one-loop: gauge or matter (scalar) fields couple to $G_{\mu\nu}$
$\Rightarrow$ (Sakharov) induced Einstein-Hilbert action:

$$S_{1-loop} \sim \int d^4x \sqrt{|G_{\mu\nu}|}\left(c_1\Lambda^4_{UV} + c_2\Lambda^2_{UV}R[G] + O(\log(\Lambda_{UV}))\right)$$

suggests

$$\frac{1}{G} \sim \Lambda^2_{UV}$$

note: $\det G_{\mu\nu} \equiv 1$, first term is huge but irrelevant

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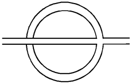
Relation with UV/IR mixing

UV/IR mixing of NC gauge theory (old point of view)

- Quantization of NC field theory $\rightarrow$ new IR - divergences
  nonplanar diagrams: UV-finite, except for $p \rightarrow 0$



$$\Gamma^{NC}[A] \sim g^2 \int d^4p\, (\theta^{ab} F_{ab})^2\, \Lambda_{eff}^4(p) + \ldots$$

$$\Lambda_{eff}^2(p) = \frac{1}{1/\Lambda^2 + \frac{1}{4} p^2/\Lambda_{NC}^4}$$

related to UV divergences; non-renormalizable ?

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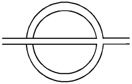

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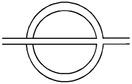

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  matter couples to G_{ab}

  ⇒ expect new divergences in IR due to induced gravity therefore:

- explanation for UV/IR mixing in terms of gravitational action

$$\Gamma_{eff}[A] \cong \int d^4x \left(\Lambda^4 + c\Lambda^2 R[G]\right)$$

detailed matching UV/IR mixing $\leftrightarrow$ gravity
(H. Grosse, H.S., M. Wohlgenannt, JHEP 0804:023,2008)

- finite UV cutoff $\frac{1}{G} \sim \Lambda^2 \iff \mathcal{N} = 4$ SUSY broken at Λ_{Planck}

  $\rightarrow$ IKKT model, suitable for quantization

- no cosm. const.: clear from NC gauge theory point of view

  Moyal-Weyl is solution of quantized NC $U(1)$ gauge theory therefore:

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Matter

Question: sufficient d. o. f. in $G^{\mu\nu}$ for gemetries with matter?

Consider Newtonian limit

$$ds^2 = -c^2 dt^2 \Big(1 + \frac{2U(x)}{c^2}\Big) + d\vec{x}^2 \Big(1 + O(\frac{1}{c^2})\Big)$$

where $\Delta_{(3)} U(x) = 4\pi G \rho(x)$ and ρ ...static mass density

can show: $\exists$ sufficient d.o.f. in $G^{\mu\nu}$ for arbitrary $\rho(x)$

but: gravitational field of e.g. point mass not correct
(metric too constrained ?)

need more d.o.f: $\to$ branes & extra dimensions

Extra dimensions, branes & compactification

recall action for scalar field (H.S., in preparation)

$$S[\Phi] = Tr\, [X^\mu, \Phi][X^\nu, \Phi]\, \eta_{\mu\nu}$$

interpret Φ as extra dimension → consider D –dim. M.M.

$$S_{YM} = -Tr[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'}, \qquad a, b = 0, ..., D-1$$

(in particluar: IKKT model in $D = 10$ ↔ NC SYM in $D = 4$)

split matrices $X^a = (X^\mu, \Phi^i), \qquad \mu = 0, ..., 2n-1$

so far: background $\Phi(x) = 0$

now: background with nontrivial $\Phi(x)$ → generic $\mathcal{M}_\theta^{2n} \subset \mathbb{R}^D$

$\mathcal{M}_\theta^{2n}$ carries Poisson structure $[X^\mu, X^\nu] = i\theta^{\mu\nu}(x)$

tangential VF $e^\mu = -i[X^\mu, .] = \theta^{\mu\nu}(x)\,\partial_\nu$

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so far: background $\Phi(x) = 0$

now: background with nontrivial $\Phi(x) \rightarrow$ generic $\mathcal{M}_\theta^{2n} \subset \mathbb{R}^D$

$\mathcal{M}_\theta^{2n}$ carries Poisson structure $[X^\mu, X^\nu] = i\theta^{\mu\nu}(x)$

tangential VF $e^\mu = -i[X^\mu, .] = \theta^{\mu\nu}(x)\,\partial_\nu$

effective geometry:
consider scalar field coupled to Matrix Model ("test particle")

$$
\begin{aligned}
S[\Phi] &= Tr\,[X^a,\phi][X^b,\phi]\,\eta_{ab} \\
&\sim \int d^{2n}x\,\sqrt{|G_{\mu\nu}|}\,G^{\mu\nu}(x)\,\partial_\mu\Phi\partial_\nu\Phi
\end{aligned}
$$

where

$$
\begin{aligned}
G^{\mu\nu}(x) &= \theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\,\tilde\rho(x)\,g_{\mu'\nu'}(x) && \text{effective metric} \\
g_{\mu\nu}(x) &= \eta_{\mu\nu} + \partial_\mu\Phi^i\partial_\nu\Phi^j\delta_{ij} && \text{induced metric on } \mathcal{M}^{2n}_\theta
\end{aligned}
$$

all fields couple to $G_{\mu\nu}$ $\sim$ open string metric
($g_{\mu\nu}$ $\sim$ closed string metric)

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Summary and outlook

- matrix-model $Tr[X^a, X^b][X^{a'}, X^{b'}]\,\eta_{aa'}\eta_{bb'}$

  describes dynamical NC spaces, & $SU(n)$ gauge theory
- simple, intrinsically NC mechanism to generate gravity

  NC spaces $\leftrightarrow$ gravity
- *not* same as G.R., but close to G.R. for small curvature
  - vacuum equation $R_{ab} \sim 0$ at least in linearized case
  - Newtonian limit o.k., but no Schwarzschild in minimal $D = 4$ (?)

  $\rightarrow$ extends to branes & compactification (IKKT model!)
- mechanism for stabilizing flat spaces
  suitable for quantizing gravity
- explanation for UV/IR mixing in NC gauge theory

Coupling to nonabelian gauge fields (heuristic)

set $Y^a = X^a + \theta^{ab}(x) A_b(x)$ obtain

$$
\begin{aligned}
[Y^a, Y^b] &= i\theta^{ab}(x) + i\theta^{ac}\theta^{bd}(\partial_c A_d - \partial_d A_c + [A_c, A_d] \; + O(\theta^{-1}\partial\theta)) \\
&= i\theta^{ab}(x) + i\theta^{ac}(x)\theta^{bd}(x) F_{cd} \; + O(\theta^{-1}\partial\theta))
\end{aligned}
$$

hence

$$
\begin{aligned}
S_{YM} &= -Tr[Y^a, Y^b][Y^{a'}, Y^{b'}]\eta_{aa'}\eta_{bb'} \\
&\approx Tr\left(G^{ab}(x)\eta_{ab} - G^{cc'}(x)\, G^{dd'}(x)\, (F_{cd}\, F_{c'd'} + O(\theta^{-1}\partial\theta))\right)
\end{aligned}
$$

using $Tr(\theta^{ab}(x) F^{ab}) \approx 0$

similar to $\mathfrak{su}(n)$ YM coupled to metric $G^{ab}(x)$

nonabelian gauge fields (correct)

Seiberg-Witten map:

$$Y^a = X^a + \theta^{ab} A_b - \frac{1}{2}(A_c[X^c, \theta^{ad} A_d] + A_c F^{ca}) \quad + O(\theta^3)$$

- expresses $\mathfrak{su}(n)$ d.o.f. in terms of commutative $\mathfrak{su}(n)$ gauge fields A_a
- relates NC g.t. $i[\Lambda, Y^a]$ in terms of standard $\mathfrak{su}(n)$ g.t. of A_a

Volume element:

$$\begin{aligned}(2\pi)^2 Tr\, f(x) &= \int d^4x\, \rho(x)\, f(x), \\ \rho(x) &= \sqrt{\det(\theta^{-1}_{ab})} = (\det(\eta_{ab}) \det(G_{ab}))^{1/4}\end{aligned}$$

(cp. Bohr-Sommerfeld quantization)

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