Neutrino Masses and the SeeSaw mechanism in Noncommutative Geometry

Christoph A. Stephan
Institut für Mathematik
Universität Potsdam

Bayrischzell Workshop on Noncommutativity and Physics
17^{th} of May, 2008

Overview

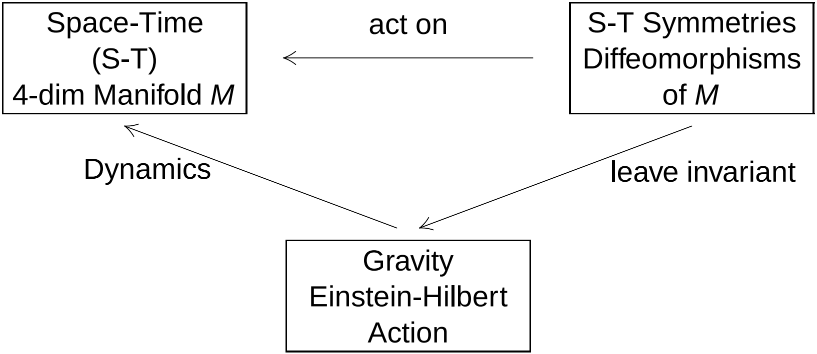
Overview

The aim of Noncommutative Geometry à la Connes:

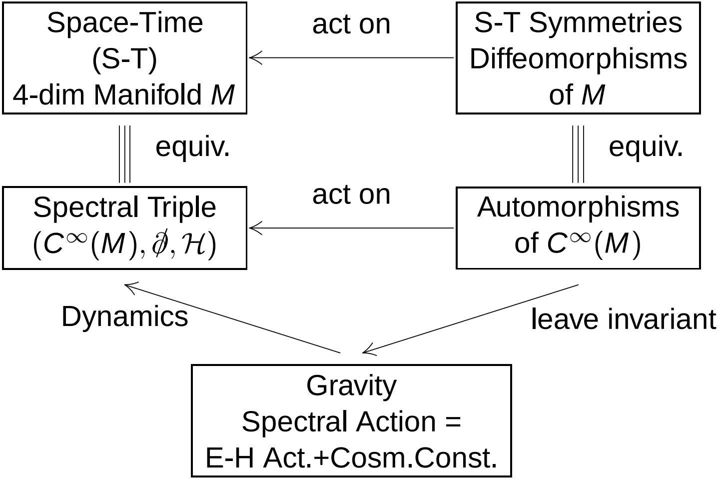
To unify general relativity (GR) and the standard model of particle physics (SM) on the same geometrical level.

This means to describe gravity and the electro-weak and strong forces as gravitational forces of a unified space-time.

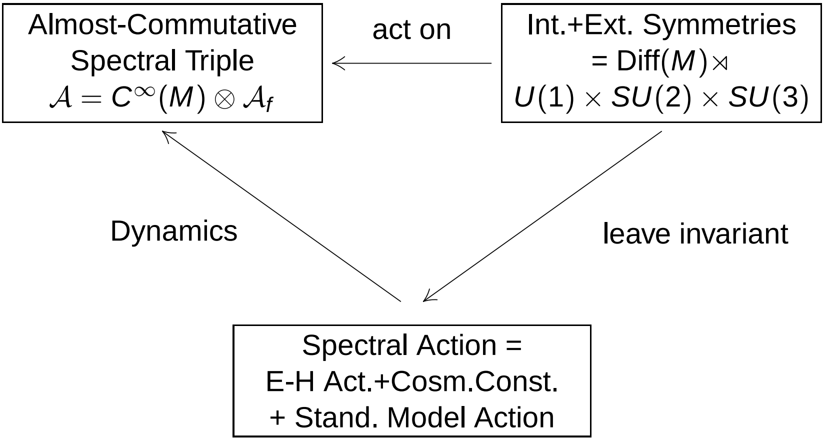
Gravity emerges as a pseudo-force associated to the space-time symmetries, i.e. the diffeomorphisms of the manifold M.



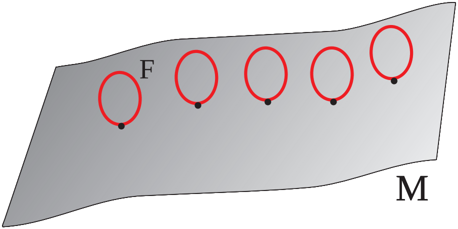
Euclidean space-time!



Almost-Commutative Spectral Action (A.Chamseddine, A.Connes 1996):

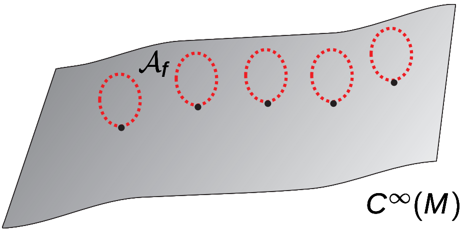


Analogy: Almost-comm. geometry $\leftrightarrow$ Kaluza-Klein space



Idea: $M \longrightarrow C^\infty(M)$, $F \longrightarrow$ some "finite space",
differential geometry $\longrightarrow$ spectral triple

Almost-commutative Geometry



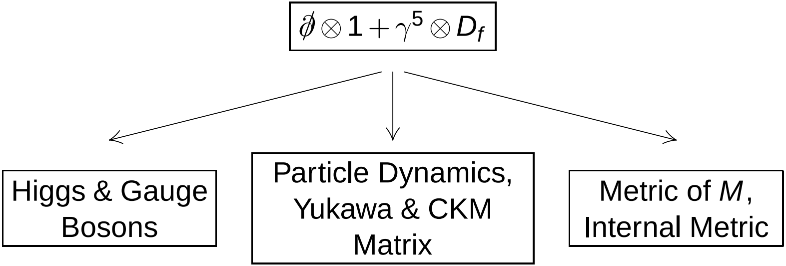
"finite space" $\rightarrow \mathcal{A}_f = M_1(\mathbb{K}) \oplus M_2(\mathbb{K}) \oplus \ldots$

Kaluza-Klein space $\rightarrow$ almost-com. geometry, $\mathcal{A} = C^\infty(M) \otimes \mathcal{A}_f$

The almost-commutative standard model automatically produces:

- The combined Einstein-Hilbert and standard model action
- A cosmological constant
- The Higgs boson with the correct quartic Higgs potential

The Dirac operator plays a multiple role:



Overview

An even, real spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$; the ingredients (A. Connes):

- A real, associative, unital pre-C^*-algebra $\mathcal{A}$
- A Hilbert space $\mathcal{H}$ on which the algebra $\mathcal{A}$ is faithfully represented via a representation ρ
- A self-adjoint operator $\mathcal{D}$ with compact resolvent, the Dirac operator
- An anti-unitary operator J on $\mathcal{H}$, the real structure (charge conjugation operator)
- A unitary operator γ on $\mathcal{H}$, the chirality

The conditions or axioms of noncommutative geometry (A. Connes 1996):

Condition 1: Classical Dimension n ($n = 0$ for the finite part)

Condition 2: Regularity

Condition 3: Finiteness

Condition 4: First Order of the Dirac Operator

Condition 5: Poincaré Duality

Condition 6: Orientability

Condition 7: Reality ($\rightarrow$ KO-dim = 0 or 6 for finite part)

The spectral action (A. Connes & A. Chamseddine 1996):

The spectral action is defined to be the number of eigenvalues of the Dirac operator up to a cut-off Λ.

$$S_{sp.} = \mathrm{Tr}(f(\frac{\mathcal{D}}{\Lambda})) + (\Psi, \mathcal{D}\Psi)$$

f: a positive test function

Heat-kernel expansion of the trace => bosonic action

Constraint: $g_2^2 = g_3^2 = \frac{\lambda}{8} = \frac{1}{4} Y_2$ at Λ

Robust predictions: $\Lambda \sim 10^{17}$GeV and $m_{Higgs} \sim 170$GeV

Overview

Pure SM with KO-dim. = 0 (Connes, Chamseddine 1996)

finite Algebra: $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$

- $N_{\nu_R} \neq 3$ (Poincaré duality)
- Neutrino masses are Dirac masses
- No SeeSaw mechanism
- Constraint: $3g_{top}^2 = 4\, g_2^2$ at $\Lambda \sim 10^{17}$GeV
  $=> m_{top} \sim 190$GeV
- Solution I: Need another Yukawa coupling $g \sim 1$
- Solution II: New particles

Pure SM with KO-dim. = 6 (Connes, Barrett 2006)

finite Algebra: $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$

- N_{ν_R} arbitrary
- Dirac and Majorana masses are allowed
- SeeSaw mechanism is *natural* with $M_{Maj.} \sim 10^{13}$GeV and $g_\nu \sim 1.6$ ($m_{top} \sim 170$GeV)
- Problem: Leptoquark masses (are put to zero by hand)
- Poincaré duality needs to be modified consider Leptons and Quarks separately
- Finite spectral triple violates Orientability axiom Solution (Connes 2006): enlarge finite Algebra to $\mathbb{C} \oplus \mathbb{H} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$

Pure SM with KO-dim. = 0 (Jureit, Schücker, C.S. 2005)

finite Algebra: $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}) \oplus \mathbb{C}$

- N_{ν_R} arbitrary
- Neutrino masses are Dirac masses
- No SeeSaw mechanism
- Constraint: $3g_{top}^2 = 4\, g_2^2$ at $\Lambda \sim 10^{17}$GeV
  $=> m_{top} \sim 190$GeV

Pure SM with KO-dim. = 6 (Jureit, C.S. 2006)

finite Algebra: $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}) \oplus \mathbb{C}$

- N_{ν_R} arbitrary
- Dirac and Majorana masses are allowed
- SeeSaw mechanism is *natural* => $m_{top} \sim$ 170GeV
- No Leptoquark masses!
- Poincaré duality needs not to be modified
- Finite spectral triple violates Orientability axiom

  (generic feature of right-handed neutrinos with Majorana mass)

SM + 2 neutral Fermions, KO-dim. = 6 (C.S., to appear)

finite Algebra: $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}) \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C}$

- N_{ν_R} arbitrary
- two new neutral particles X and Y (possibly in every generation)
- Dirac masses for all particles
- X and Y masses are vectorlike => $m_X \sim m_Y \sim \Lambda$
- vectorlike mass terms between X,Y and ν_R
  SeeSaw-like mechanism
- no problems with Axioms

New part in the SM-Langrangian:

$$\mathcal{L}_{new} = g_\nu \phi^0 \overline{\nu}_L \nu_R + m_X \overline{X}_L X_R + M_1 \overline{\nu}_R X_L + M_2 \overline{\nu}_R \overline{X}_R + m_Y \overline{Y}_R Y_L + h.c.$$

Mass eigenvalues for $M = M_1 = M_2 \sim m_X \sim \Lambda$, $m_\nu \sim 100\text{GeV}$

$$m_{1/2} \sim \pm m_\nu^2 \frac{m_X}{2M^2} \quad m_{3\ldots6} \sim \pm m_X \quad m_{7/8} \sim \pm 2 \frac{M^2}{m_X}$$

Successful SeeSaw mechanism with a detour!

Conclusions:

- Majorana masses and the SeeSaw mechanism problematic in Noncommutative Geometry à la Connes
- Physical constraint $Y_2 = 4g_2$ at Λ seems to suggest particles beyond the Standard Model
- SeeSaw mechanism requires either modification of Axioms or new particles

Open Questions & **Outlook:**

- How to distinguish the different models experimentally?
- Underlying theory? -> Quantisation?
- Lorentzian spectral triples (M. Paschke, A. Rennie, R. Verch to appear)