Comments on non-perturbative effects in noncommutative $\mathcal{N} = 1$ supersymmetric gauge theories

Carlos Tamarit Degenhardt

Ongoing work with Kenneth Intriligator

May 17th, 2008, Bayrischzell

Outline

Outline

Outline

Outline

Motivation

Why $\mathcal{N}=1$ noncommutative supersymmetric gauge theories?

- Free of dangerous IR quadratic divergences arising from non-planar diagrams.
- Supersymmetry allows to obtain some exact results.

Some previous results we will use

- "Anomalous" running of the wilsonian gauge coupling of non-traceless U(1) gauge couplings [Khoze, Travaglini]:
  $\frac{1}{g^2_{eff}} = \frac{1}{g^2_{eff,p}} + \frac{1}{g^2_{eff,np}}$,
  $\frac{1}{g^2_{eff,p}} = \frac{b_0}{(4\pi)^2} \log k^2$: Planar contribution, for a U(N) theory identical to the corresponding ordinary SU(N) running.
  $\frac{1}{g^2_{eff,np}} \sim \frac{-2b_0}{(4\pi)^2} \log k^2, k << \theta^{-1/2}$: Non-planar running (the scale $\theta^{-1/2}$ appears in the running!).
- There is strong motivation to believe that the effective actions are gauge invariant. Non-planar contributions are associated to operators involving Wilson loops. E. g., to lowest order in fields,

[Raamsdonk,Armoni,López,Levell,Travaglini]

$$\int \frac{d^4p}{(2\pi)^4} \mathcal{O}^{\mu\nu}(p) T(p) \mathcal{O}_{\mu\nu}(p),$$

$$\mathcal{O}_{\mu\nu}(p) = \mathrm{tr} \int d^4x \, L_\star \left(F_{\mu\nu}(x) e^{\int_0^1 d\sigma \bar{p}^\rho A_\rho(x+\bar{p}\sigma)} \right) \star e^{ipx}, \bar{p}^\mu = \theta^{\mu\nu} p_\nu,$$

- Planar $U(1)_A$ anomalies are given by $\star$ product generalization of ordinary ones [C. P. Martin].
- $U(N)$ theories, including $N = 1$, have instanton configurations [Nekrasov, Schwarz, Chu, Khoze, Travaglini].
- [Chu,Zamora]: Noncommutative superpotentials do not receive perturbative corrections.
- [Liao,Ruiz]: Goldstone's theorem is likely to hold, even in non-supersymmetric theories.

Some previous results we will debate about exact superpotentials at low energy

- [Chu, Khoze, Travaglini]: Brief comments about validity of $\star$ product generalisation of ADS type superpotentials. For $U(N)$ with $F < N$ flavours:

$$W(M) \sim \left(\frac{\Lambda^{b_0}}{detM}\right)^{\frac{1}{N-F}}, \quad M_{\bar{i}j} = \tilde{Q}_{\bar{i}} \star Q_j.$$

- [Ardalan, Sadooghi]: Incompatible superpotentials obtained. They use different $U(1)_A$ selection rules for $k >> \theta^{-1/2}$ and $k << \theta^{-1/2}$.

We will support the validity of ADS type superpotentials, clarifying the roles of the planar and non-planar runnings of the $U(1)$ gauge coupling.

We will also explicitly show how $U(1)$ instantons generate gaugino condensation for pure $\mathcal{N} = 1$ NC U(1). We will discuss the possibility of instantons generating a gaugino mass term.

Outline

Basic tools

- Classical Vacua (U(N) with F flavours $\tilde{Q}_{\bar{i}}, Q_i$). Same as in ordinary theory. Vacua parametrised by gauge invariant "meson fields" $M_{\bar{i}j} = \tilde{Q}_{\bar{i}} \star Q_j$. There are no baryon fields.
- Global anomalies:
  - Planar anomalies follow from invariance of functional integral measure under $\delta\Psi(x) = i\alpha(x) \star \gamma_5 \Psi(x)$.. The jacobian of the corresponding global transformations is given by the integral of the planar anomaly.
  - Planar anomalies are the $\star$ product generalisation of ordinary anomalies $\Rightarrow$ $U(1)_R$ is conserved, $U(1)_A$ is anomalous.
- Exact NSVZ β function. As pointed out by [Hollowood,Khoze,Travaglini], assumption of UV renormalisability + existence of instanton configurations $\Rightarrow$ Exact NSVZ β function, given by commutative $SU(N)$ expression (to be extrapolated to the $N = 1$ case).

- How does it fit with the "anomalous" running of the U(1) coupling?
  $\frac{1}{g^2_{eff}(k^2)}$ calculated by [Khoze,Travaglini] includes contributions from the running of 2 couplings:
  - τ, coupling to operator $\int d^4x \, d\theta^2 W_\alpha \star W^\alpha$, which is renormalised by planar diagrams and whose β function is the nc NSVZ one. This running generates the holomorphic scale Λ in the usual way.
  - $\hat{\tau}$ coupling to the supersymmetric generalisation of the operator mentioned earlier involving Wilson loops:

$$\int \frac{d^4p}{(2\pi)^4} \mathcal{O}^{\mu\nu}(p) T(p) \mathcal{O}_{\mu\nu}(p),$$

$$\mathcal{O}_{\mu\nu}(p) = \mathrm{tr} \int d^4x \, L_\star \left(F_{\mu\nu}(x) e^{\int_0^1 d\sigma \bar{p}^\rho A_\rho(x+\bar{p}\sigma)} \right) \star e^{ipx}, \bar{p}^\mu = \theta^{\mu\nu} p_\nu,$$

which is renormalised by nonplanar diagrams. The corresponding running is dependent on the noncommutative scale $\theta^{-1/2}$ and is unaffected by matter in the fundamental.

- Confusion can arise because both operators have same lowest order in derivatives-fields contributions. In contrast to ordinary case, we seem to have more than one marginal operator!.
- Consequences for $U(1)_A$ selection rules. Selection rules arise from incorporating the anomalous path integral $U(1)_A$ jacobian into transformations of the couplings. Since the jacobian is given by the integral of the planar anomaly, an anomalous $U(1)_A$ transformation is associated to a change $Q \rightarrow e^{i\alpha} Q, \tilde{Q} \rightarrow e^{i\alpha} \tilde{Q} \Rightarrow \theta_{YM} \rightarrow \theta_{YM} - 2F\alpha$ which yields the usual selection rule (recall $\lambda = \mu e^{2i\pi\tau/b_0}, \tau = \frac{4\pi i}{g^2} + \frac{\theta_{YM}}{2\pi}$) that

$$Q \rightarrow e^{i\alpha} Q, \tilde{Q} \rightarrow e^{i\alpha} \tilde{Q}, \Lambda^{b_0} \rightarrow e^{2i\alpha F} \Lambda^{b_0}$$

leaves the quantum theory invariant. Note that, as opposed to [Ardalan,Sadooghi].

  - b_0 is the one-loop coefficient of the NSVZ planar β function.
  - Selection rules involve integral, not local, forms of the anomaly.
  - Non-planar effects play no role in the selection rule.

Generating exact superpotentials

- Quantum symmetries:

  Local nc gauge symmetry: Superpotential dependent on $M_{ij} = \tilde{Q}_{\bar{i}} \star Q_j$.

  Global $SU(F) \times SU(F) \times U(1)_R$: Superpotential dependent on $\det M$.

  Periodicity in θ: Superpotential dependent on Λ^{b_0}.

- $F < N$. Quantum symmetry+Selection rule: Same reasoning as in ordinary SU(N) theories.

$$W(M) \sim \left(\frac{\Lambda^{b_0}}{detM}\right)^{\frac{1}{N-F}}, \quad M_{ij} = \tilde{Q}_{\bar{i}} \star Q_j.$$

- $F \geq N$. No superpotential: quantum moduli space. Can we have similar features as in ordinary SU(N) theories? NO:

$F \geq N$

As in the commutative SU(N) case, we will have zeros in the planar NSVZ β function for τ for appropriate combinations of N, F.

BUT

the nonplanar running of the coupling $\tilde{\tau}$ is insensitive to F, we will never have a zero in the corresponding β function.

$\Rightarrow$ We can not expect conformal fixed points and electric-magnetic duality.

Also note that baryon operators can not be constructed, and the commutative SU(N) anomaly matchings in different points of the moduli space can not be reproduced.

Outline

Gaugino condensation in pure U(1) from holomorphicity

Pure $\mathcal{N} = 1$ noncommutative U(1) has an anomaly under $U(1)_R$. In contrast to the commutative $U(1)$ case -but as in ordinary SU(N) theories- we get a selection rule:

$$\lambda \to e^{i\alpha}\lambda, \Lambda \to e^{2i\alpha/3}\Lambda$$

leaves the theory invariant. Thus, if no massless singlet are present, we generate

$$W_{eff} = a\Lambda^3$$

from which one can derive as in the commutative case

$$\langle\lambda\lambda\rangle = -32\pi^2 a\Lambda^3.$$

Generating gaugino condensation from instanton effects

- U(1) nc instanton [Nekrasov, Schwarz,Chu,Khoze,Travaglini]

We choose x_1, x_2 as noncommuting coordinates: time is commutative.

$$z_0 \equiv x_4 + ix_3, \;\; z_1 \equiv x_1 + ix_2, \;\; [z_1, \bar{z}_1] = 2\theta.$$

Anti-instanton from noncommutative ADHM construction
$(A_m) = \Phi^{\dagger\lambda}\partial_m\Phi_\lambda$,

$$\Phi = \begin{bmatrix} \sqrt{2\theta_+}\bar{z}_0 \\ i\sqrt{2\theta_+}z_1 \\ z_0\bar{z}_0 + \bar{z}_1 z_1 \end{bmatrix} \frac{1}{\sqrt{(z_0\bar{z}_0 + \bar{z}_1 z_1)(z_0\bar{z}_0 + z_1\bar{z}_1)}}.$$

Instanton field strength and Chern number

- Basis of anti-self-dual forms: $\alpha_3 \equiv \frac{i}{2}(dz_0 d\bar{z}_0 + dz_1 d\bar{z}_1)$, $\alpha_+ \equiv dz_0 dz_1$, $\alpha_- \equiv d\bar{z}_0 d\bar{z}_1$

$$F = \frac{1}{2} F_{mn} dx^m dx^n = 2i\Phi^\dagger f \begin{bmatrix} \alpha_3 & \alpha_- & 0 \\ \alpha_+ & -\alpha_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Phi, \quad f = \frac{1}{\bar{z}_0 z_0 + z_1 \bar{z}_1},$$

- Second Chern number can be explicitly evaluated [Kim,Lee,Yang], yielding -1 as expected:

$$I = \frac{-1}{8\pi^2} \int F \wedge F = -1.$$

- Anomaly equation implies the existence of two fermionic zero modes.

$$D^\mu j^5_\mu = t_2(R_f) \cdot \frac{1}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} \star F_{\rho\sigma}, \quad j^5_\mu = \psi_\beta \star \overline{\psi}_\alpha (\gamma_\mu \gamma_5)_{\alpha\beta}.$$

$$\Rightarrow \text{index}_{R_f}(\not{D}) \equiv n_L - n_R = t_2(R_f) I.$$

- Anti-instanton background: $n_L = 0$. Gauginos in U(1) "adjoint" $t_2(ad) = 2 \Rightarrow$ two gaugino zero modes, standard "supersymmetry zero modes," superpartners of translations

$$\lambda_{\alpha}^{(\beta)} \sim F_{\alpha\beta}^{inst}.$$

  There are no "superconformal zero modes" since there is not a dilatation modulus "ρ".

- Zero mode counting implies that instantons generate non-zero $S_{\alpha\beta}(x,y) = \langle \lambda_\alpha(x) \lambda_\beta(y) \rangle$. With Pauli-Villars regulator M_{PV}

$$S_{\alpha\beta} \sim M_{PV}^3 \exp(-8\pi^2 g^{-2}(M_{PV}) + i\theta_{YM}) \int d^4 z F_{\alpha\gamma}^{inst}(x-z) F_{\beta\delta}^{inst}(y-z) \epsilon^{\gamma\delta},$$

On symmetry grounds,

$$S_{\alpha\beta}(x,y) = S(x-y)\epsilon_{\alpha\beta},$$

$$S(x-y) \sim \Lambda^3 \int d^4 z F_{\alpha\gamma}^{inst}(x-y-z) F_{\beta\delta}^{inst}(-z) \epsilon^{\alpha\beta} \epsilon^{\gamma\delta}.$$

Gaugino condensate from instantons

Finally,

$$\langle\lambda_\alpha(x)\lambda^\alpha(x)\rangle \sim S(0) \sim \Lambda^3 I = -\Lambda^3.$$

- Gaugino mass term? The instanton-generated gaugino condensation can be reproduced by a non-local effective 2-fermion interaction, such that:

$$S(x-y) = \int d^4z \int d^4z' \mathfrak{S}(x-z) m(z-z') \mathfrak{S}(z'-y).$$

$\mathfrak{S}$: Free fermion propagator. Doing a derivative expansion, in principle this could include a mass term (as happens for example in ordinary QCD with one flavour).

$$m_\lambda = \int d^4x (\partial_x)^2 S(x) \sim \Lambda^3 \int d^4x d^4z (\partial_x)^2 F^{inst}_{\alpha\beta}(x-z)(F^{inst})^{\alpha\beta}(z).$$

Gaugino mass term?

If this were true, in the absence of massless fermions, the Goldstone theorem would imply that supersymmetry can not be broken, and therefore we would have a massive photon: nc $\mathcal{N} = 1\, U(1)$ would be confining. However, it seems that the corresponding integral is zero, so that instantons might only yield higher derivative terms.