Bayrischzell Workshop 2008

Noncommutativity and Physics:
Quantum Geometry and Gravity

Renormalizability as a condition to determine noncommutative deformation parameter for θ-expanded NCGFT and related Phenomenology

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Introduction

Example of noncommutativity (NC): Heisenberg algebra

$$[\widehat{x}^\mu, p^\nu] = i\hbar\delta^{\mu\nu}, \quad [p^\mu, p^\nu] = 0$$

Constructing models on noncommutative space-time

Motivations: String Theory
Quantum Gravity
Lorentz invariance breaking
Heuristic

* The star product: $[x^\mu, x^\nu]_\star = x^\mu \star x^\nu - x^\nu \star x^\mu = ih\theta^{\mu\nu}$

$$(f \star g)(x) = e^{-\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}} f(x)g(y)|_{y\to x}$$

* Noncommutative space is flat Minkowski space:

$$x^\mu \to \widehat{x}^\mu \Rightarrow [\widehat{x}^\mu, \widehat{x}^\nu] = ih\theta^{\mu\nu},$$

θ - constant, antisymmetric and real 4×4 matrix
$h = 1/\Lambda^2_{\mathrm{NC}}$ - NC deformation parameter

* Symmetry extended to enveloping algebra
* Seiberg-Witten map (SW)

There are 2 essential points in which NCGFT differ from standard gauge theories:

* The breakdown of Lorentz invariance with respect to a fixed $\neq 0$ background field $\theta^{\mu\nu}$ (which fixes preferred directions)
* The appearance of new interactions and the modification of standard ones. For exapmle, triple–neutral–gauge boson, 2 fermion–2 gauge bosons, photon-neutrino, etc.

Both properties have a common origin and appear in a number of phenomena

AT VERY HIGH ENERGIES AND/OR VERY SHORT DISTANCES.

CONSTRUCTION
VIA SEIBERG-WITTEN MAP

[N. Seiberg and E. Witten; String theory and non-commutative geometry, JHEP **9909**, 032 (1999)]

[J. Madore, S. Schraml, P. Schupp and J. Wess; Gauge theory on noncommutative spaces, Eur. Phys. J. C**16** (2000) 161]

[B. Jurčo, S. Schraml, P. Schupp and J. Wess; Enveloping algebra valued gauge transformations for non-Abelian gauge groups on non-commutative spaces, Eur. Phys. J. C **17**, 521 (2000)

[X. Calmet, B. Jurčo, P. Schupp, J. Wess and M. Wohlgenannt; The standard model on non-commutative space-time, EPJ C**23** (2002) 363]

[W. Behr, N. G. Deshpande, G. Duplančić, P. Schupp, J.T. and J. Wess; The Z $\rightarrow$ gamma gamma, g g decays in the non-commutative standard model, Eur. Phys. J. C **29**, 441 (2003)]

[G. Duplančić, P. Schupp and J. Trampetić; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C**32** (2003) 141]

[B. Melić, K. Passek-Kumerički, J.T., P. Schupp and M. Wohlgenannt; The Standard Model on Non-Commutative Space-Time: Electroweak Currents and Higgs Sector, EPJ C**24** (2005) 483 ibid. 499]

[F. Brandt, C.P. Martin and F. Ruiz Ruiz; Anomaly freedom in Seiberg-Witten noncommutative gauge theories JHEP **07** (2003) 068]

[M. Buric, D. Latas and V. Radovanovic, Renormalizability of noncommutative SU(N) gauge theory; JHEP **0602** (2006) 046]

[M. Buric, V. Radovanovic and J.T., The one-loop renormalization of the gauge sector in the noncommutative standard model; JHEP**03** (2007) 030]

[D. Latas, V. Radovanovic and J.T., Non-commutative SU(N) gauge theories and asymptotic freedom; Phys.Rev. D**76**, 085006 (2007).]

[M. Buric, D. Latas, V. Radovanovic and J.T., Absence of the 4ψ divergence in noncommutative chiral models; Phys.Rev. D**77**, 045031 (2008).]

* Models based on the Seiberg-Witten mapping
* Expansion in power series in θ $\rightarrow$ new vertices
* Any gauge groups
* Arbitrary matter representation
* No charge quantization problem
* No UV/IR mixing due to θ expansion
* Unitarity is OK for: $\theta^{ij} \neq$, $\theta^{0i} = 0$;

carefull cannonical quantization produces always unitary theory: (*Bahns, Fredenhagen, Doplicher, Piaticelli: Time in S matrix treated in form of slices*)

* By covariant generalization of $\theta^{0i} = 0$ to:

$$\theta_{\mu\nu}\theta^{\mu\nu} \;=\; -\theta^2 = \frac{2}{\Lambda_{\mathrm{NC}}^4}\left(\vec{B}_\theta^2 - \vec{E}_\theta^2\right) > 0$$

known as *perturbative unitarity condition* one avoids potential difficulties with unitarity in noncommutative gauge field theories

* Covariant NCSM Yukawa couplings OK
* Models 1 & 2: mNCSM & nmNCSM constructed as an effective, anomaly free, with 1-loop renormalizable gauge sector, GFT at first order in noncommutative parameter θ
* Model 3: SU(N) GFT constructed as an renormalizable theory via renormalization of $\theta \rightarrow$ RGE for noncommutative deformation parameter h.
* In noncommutative chiral model for fermions there is NO typical 4ψ divergence, as for Dirac fermions:

$$\mathcal{D}|_{\mathrm{div}} = \frac{1}{(4\pi)^2\epsilon}\frac{9}{32}h\theta^{\mu\nu}\varepsilon_{\mu\nu\rho\sigma}(\bar{\psi}\gamma^\rho\gamma_5\psi)(\bar{\psi}\gamma^\sigma\psi).$$

NC gauge transformation

Consider infinitesimal NC local gauge transformation $\widehat{\delta}$ of a fundamental matter field that carries a representation ρ_Ψ

$$\widehat{\delta}\widehat{\Psi} = i\rho_\Psi(\widehat{\Lambda}) \star \widehat{\Psi}$$

In Abelian case ρ_Ψ fixed by the hypercharge.

Covariant coordinates in NC theory introduced in analogy to covariant derivatives in ordinary theory

$$\widehat{x}^\mu = x^\mu + \theta^{\mu\nu}\widehat{A}_\nu$$

Locality

A $\star -$ product of ordinary functions f, g, determined by a Poisson tensor $\theta^{\mu\nu}(x)$, is local function of f, g with finite number of derivatives at each order in θ:

$$f \star g = f \cdot g + \frac{i}{2}\theta^{\mu\nu}(x)\partial_\mu f \cdot \partial_\nu g + \mathcal{O}(\theta^2)$$

Gauge equivalence, and consistency conditions

Ordinary gauge transformations $\delta A_\mu = \partial_\mu \Lambda + i[\Lambda, A_\mu]$ and $\delta\Psi = i\Lambda \cdot \Psi$ induce non-commutative gauge transformations of the fields $\widehat{A}$, $\widehat{\Psi}$ with gauge parameter $\widehat{\Lambda}$

$$\delta\widehat{A}_\mu = \widehat{\delta}\widehat{A}_\mu \qquad \delta\widehat{\Psi} = \widehat{\delta}\widehat{\Psi}$$

Consistency require that any pair of non-commutative gauge parameters $\widehat{\Lambda}$, $\widehat{\Lambda'}$ satisfy

$$[\widehat{\Lambda}, \widehat{\Lambda'}] + i\delta_\Lambda \widehat{\Lambda'} - i\delta_{\Lambda'}\widehat{\Lambda} = \widehat{[\Lambda, \Lambda']}.$$

Enveloping algebra-valued gauge transformation

The commutator

$$[\widehat{\Lambda}, \widehat{\Lambda}'] = \frac{1}{2}\{\Lambda_a(x) \stackrel{\star}{,} \Lambda'_b(x)\}[T^a, T^b] + \frac{1}{2}[\Lambda_a(x), \Lambda'_b(x)]\{T^a, T^b\}$$

of two Lie algebra-valued NC gauge parameters $\widehat{\Lambda} = \Lambda_a(x)T^a$ and $\widehat{\Lambda}' = \Lambda'_a(x)T^a$ does not close in Lie. For NC SU(N) & Lie algebra traceless condition incompatible with commutator. Extension to enveloping algebra-valued NC gauge parameters and fields.

$$\widehat{\Lambda} = \Lambda^0_a(x)T^a + \Lambda^1_{ab}(x) : T^aT^b : + \Lambda^2_{abc}(x) : T^aT^bT^c : + \dots$$

Clousing condition for gauge transformation algebra are homogenos differential equations which are solved by iteration, order by order in θ. This solution is known as Seiberg–Witten map. However solution is NOT UNIQUE due to the homogenus differential equation which gauge transformation satisfies:

$$\begin{aligned}
\widehat{\Lambda} &= \Lambda + \frac{1}{4}\theta^{\mu\nu}\{V_\nu, \partial_\mu\Lambda\} + \dots \\
\widehat{V}_\mu &= V_\mu + \frac{1}{4}\theta^{\alpha\beta}\{\partial_\alpha V_\mu + F_{\alpha\mu}, V_\beta\} + \dots \\
\widehat{F}_{\mu\nu} &= \partial_\mu \widehat{V}_\nu - \partial_\nu \widehat{V}_\mu - i[\widehat{V}_\mu \stackrel{\star}{,} \widehat{V}_\nu] \\
&= F_{\mu\nu} + \frac{h}{4}\theta^{\rho\sigma}\Big(2\{F_{\rho\mu}, F_{\sigma\nu}\} - \{V_\rho, (\partial_\sigma + D_\sigma)F_{\mu\nu}\}\Big), \\
\widehat{\psi} &= \psi - \frac{1}{2}\theta^{\alpha\beta}\left(V_\alpha \partial_\beta - \frac{i}{4}[V_\alpha, V_\beta]\right)\psi + \dots
\end{aligned}$$

THE FIRST AND SECOND ORDER SW MAPS

[M. Wohlgenannt and J.T. Phys.Rev. D**76**, 127703 (2007).]

Hermicity requirement: $c = \frac{1}{2}$, and choice $d = -\frac{1}{8}$, above ref.

$$\Lambda^\theta[V] = \frac{1}{2}\theta^{\mu\nu}\{V_\nu, \partial_\mu\Lambda\}_c = \frac{1}{2}\theta^{\mu\nu}\Big(cV_\nu \cdot \partial_\mu\Lambda + (1-c)\partial_\mu\Lambda \cdot V_\nu\Big)$$

$$\Lambda^{\theta^2}[V] = \frac{1}{32}\theta^{\mu\nu}\theta^{\kappa\lambda}\Big(\{V_\mu, \{\partial_\nu V_\kappa, \partial_\lambda\Lambda\}\} + \{V_\mu, \{V_\kappa, \partial_\nu\partial_\lambda\Lambda\}\}$$
$$+\{\{V_\mu, \partial_\nu V_\kappa\}, \partial_\lambda\Lambda\}\} - \{\{F_{\mu\kappa}, V_\nu\}, \partial_\lambda\Lambda\} - 2i[\partial_\mu V_\kappa, \partial_\nu\partial_\lambda\Lambda]\Big)$$

$$V_\mu^\theta[V] = \frac{1}{2}\theta^{\alpha\beta}\Big(\{V_\beta, \partial_\alpha V_\mu\}_c + \{V_\beta, F_{\alpha\mu}\}_c\Big)$$

$$V_\mu^{\theta^2}[V] = \frac{1}{64}\theta^{\alpha\beta}\theta^{\gamma\delta} \times$$
$$\Big(8\{V_\alpha, \{F_{\mu\gamma}, F_{\beta\delta}\}\} + 8\{V_\alpha, \{\partial_\beta F_{\mu\gamma}, V_\delta\}\} + 2i\{V_\alpha, \{\partial_\mu V_\beta, V_\gamma V_\delta\}\}$$
$$-2\{V_\alpha, \{\partial_\beta\partial_\mu V_\gamma, V_\delta\}\} - \{V_\mu, \{F_{\alpha\gamma}, F_{\beta\delta}\}\} - 4\{V_\mu, \{\partial_\alpha V_\gamma, \partial_\delta V_\beta\}\}$$
$$+2\{V_\mu, \{V_\alpha V_\beta, V_\gamma V_\delta\}\} + 2\{\partial_\alpha V_\gamma, \{V_\beta, \partial_\mu V_\delta\}\} + 4\{\partial_\alpha V_\gamma, \{V_\mu, \partial_\delta V_\beta\}\}$$
$$+8\{\partial_\alpha V_\mu, \{\partial_\gamma V_\beta, V_\delta\}\} + 2\{\partial_\mu V_\alpha, \{F_{\beta\gamma}, V_\delta\}\} - 2\{V_\alpha V_\beta, \{V_\mu, V_\gamma V_\delta\}\}$$
$$-4\{V_\alpha V_\gamma, \{V_\mu, V_\beta V_\delta\}\} + 8\{V_\alpha V_\mu V_\gamma, V_\beta V_\delta\} + 8i[\partial_\alpha\partial_\gamma V_\mu, \partial_\beta V_\delta]$$
$$-2i[\partial_\mu F_{\alpha\gamma}, F_{\beta\delta}] - 4i[\partial_\alpha\partial_\mu V_\gamma, \partial_\delta V_\beta] - 4V_\alpha\partial_\beta V_\gamma\partial_\mu V_\delta$$
$$+4F_{\alpha\gamma}V_\mu F_{\beta\delta} - 4\partial_\mu V_\alpha\partial_\gamma V_\beta V_\delta + 2iV_\alpha V_\gamma(\partial_\beta V_\delta)V_\mu$$
$$-4iV_\alpha V_\gamma(\partial_\mu V_\beta)V_\delta - 2iV_\alpha(\partial_\beta V_\gamma)V_\delta V_\mu - 2iV_\alpha(\partial_\gamma V_\beta)V_\delta V_\mu$$
$$+4iV_\alpha(\partial_\mu V_\gamma)V_\beta V_\delta - 2iV_\mu V_\alpha V_\gamma(\partial_\beta V_\delta) + 2iV_\mu V_\alpha(\partial_\beta V_\gamma)V_\delta$$
$$+2iV_\mu V_\alpha(\partial_\gamma V_\beta)V_\delta - 2iV_\mu(\partial_\alpha V_\gamma)V_\delta V_\beta + 2i(\partial_\alpha V_\gamma)V_\delta V_\beta V_\mu\Big)$$

$$\psi^\theta[\psi, V] = \frac{1}{2}\theta^{\alpha\beta}\Big(V_\alpha\,\partial_\beta + (1-c)\partial_\beta V_\alpha + 2dF_{\alpha\beta}\Big)\psi$$
$$= -\frac{1}{2}\theta^{\alpha\beta}\Big(V_\alpha\,\partial_\beta - \frac{i}{4}[V_\alpha, V_\beta]\Big)\psi$$

$$\psi^{\theta^2}[\psi, V] = \frac{1}{32}\theta^{\mu\nu}\theta^{\kappa\lambda} \times$$
$$\Big(-4i\partial_\kappa V_\mu\partial_\nu\partial_\lambda + 4V_\kappa V_\mu\partial_\nu\partial_\lambda - 4\partial_\kappa V_\mu V_\nu\partial_\lambda + 4F_{\kappa\mu}V_\nu\partial_\lambda$$
$$-4V_\nu\partial_\kappa V_\mu\partial_\lambda + 8V_\nu F_{\kappa\mu}\partial_\lambda - 8iV_\mu V_\kappa V_\nu\partial_\lambda + 4iV_\mu V_\nu V_\kappa\partial_\lambda$$
$$-2\partial_\kappa V_\mu\partial_\lambda V_\nu + 2i\partial_\kappa V_\mu V_\lambda V_\nu - 2iV_\nu V_\lambda\partial_\kappa V_\mu$$
$$-i[[\partial_\kappa V_\mu, V_\nu], V_\lambda] - 4iV_\nu F_{\kappa\mu}V_\lambda + V_\kappa V_\lambda V_\mu V_\nu - 2V_\kappa V_\mu V_\nu V_\lambda\Big)\psi$$

The non-commutative Higgs field $\widehat{\Phi}$ is given by the hybrid SW map

$$\widehat{\Phi} \equiv \widehat{\Phi}[\Phi, V, V'] = \Phi + \Phi^{\theta}[V, V'] + \Phi^{\theta^2}[V, V'] + \mathcal{O}(\theta^3)$$

$\widehat{\Phi}$ is a functional of two gauge fields V and V' and transforms covariantly under gauge transformations:

$$\delta\widehat{\Phi}[\Phi, V, V'] = i\widehat{\Lambda} * \widehat{\Phi} - i\widehat{\Phi} * \widehat{\Lambda}'$$

$\widehat{\Lambda}$ and $\widehat{\Lambda}'$ are the corresponding gauge parameters. Hermitian conjugation yields $\widehat{\Phi}[\Phi, V, V']^\dagger = \widehat{\Phi}[\Phi^\dagger, V', V]$.

$$\widehat{D}_\mu\widehat{\Phi} = \partial_\mu\widehat{\Phi} - i\,\widehat{V}_\mu * \widehat{\Phi} + i\,\widehat{\Phi} * \widehat{V}'_\mu\,.$$

The precise representations of the gauge fields V and V' in the Yukawa couplings are inherited from the fermions on the left ($\bar{\psi}$) and on the right side (ψ) of the Higgs field, respectively.

The hybrid Seiberg-Witten map for the Higgs boson up to second order is not unique. One solution up to first order is given by

$$\Phi^{\theta}[\Phi, V, V'] = \frac{1}{2}\theta^{\alpha\beta} \times$$

$$\left[V_\beta\left(\partial_\alpha\Phi - \frac{i}{2}(aV_\alpha\Phi - \Phi V'_\alpha)\right) + \left(\partial_\alpha\Phi - \frac{i}{2}(V_\alpha\Phi - b\Phi V'_\alpha)\right)V'_\beta\right.$$

$$\left.+\frac{1}{4}\Big((1-a)(\partial_\alpha V_\beta)\Phi + (1-b)\Phi(\partial_\alpha V'_\beta)\Big)\right]$$

Conventionaly we choose $a = b = 1$, (above ref.)

$$\Phi^{\theta}[\Phi, V, V'] = \frac{1}{2}\theta^{\alpha\beta} \times$$

$$\left[V_\beta \left(\partial_\alpha \Phi - \frac{i}{2}(V_\alpha \Phi - \Phi V'_\alpha)\right) + \left(\partial_\alpha \Phi - \frac{i}{2}(V_\alpha \Phi - \Phi V'_\alpha)\right) V'_\beta\right]$$

and for the second order we have

$$\Phi^{\theta^2}[\Phi, V, V'] = -\frac{i}{32}\theta^{\alpha\beta}\theta^{\gamma\delta} \times$$

$$\Big(V_\alpha \Big[V_\beta \big(V_\gamma \big(4\partial_\delta \Phi - 3iV_\delta \Phi + 4i\Phi V'_\delta \big) + \big(-4\partial_\gamma \Phi - 2i\Phi V'_\gamma \big) V'_\delta \big)$$

$$+ V_\gamma \Big[4i\partial_\beta \partial_\delta \Phi + V_\beta \big(-4\partial_\delta \Phi + 2i \big(V_\delta \Phi - 2\Phi V'_\delta \big) \big) + V_\delta \big(4\partial_\beta \Phi + 4i\Phi V'_\beta \big)$$

$$+ 3\partial_\beta V_\delta \Phi - 4\partial_\beta \Phi V'_\delta - 4\partial_\delta \Phi V'_\beta + \Phi \big(4 \big(-2\partial_\beta V'_\delta + \partial_\delta V'_\beta + i \big(V'_\beta V'_\delta - 2V'_\delta V'_\beta \big) \big) \big) \Big]$$

$$+ \partial_\beta V_\gamma \big(8i\partial_\delta \Phi + 5V_\delta \Phi - 8\Phi V'_\delta \big) + \partial_\gamma V_\beta \big(-4i\partial_\delta \Phi - 3V_\delta \Phi \big)$$

$$+ \partial_\gamma \Phi \big(4 \big(-i\partial_\beta V'_\delta + i\partial_\delta V'_\beta + V'_\beta V'_\delta + V'_\delta V'_\beta \big) \big) + \big(-8i\partial_\beta \partial_\gamma \Phi + 4\partial_\beta \Phi V'_\gamma \big) V'_\delta$$

$$+ \Phi \big(V'_\gamma \big(4\partial_\beta V'_\delta - 4\partial_\delta V'_\beta - 4iV'_\beta V'_\delta + 4iV'_\delta V'_\beta \big) + \big(8\partial_\gamma V'_\beta + 4iV'_\beta V'_\gamma \big) V'_\delta \big) \Big]$$

$$+ \partial_\alpha V_\gamma \Big[4\partial_\beta \partial_\delta \Phi + V_\beta \big(-4i\partial_\delta \Phi + 4\Phi V'_\delta \big) + V_\delta \big(-V_\beta \Phi - 4\Phi V'_\beta \big) + 4i\partial_\beta \Phi V'_\delta$$

$$-2i\partial_\delta V_\beta \Phi - 4i\partial_\delta \Phi V'_\beta + \Phi \big(4i\partial_\delta V'_\beta - 4V'_\beta V'_\delta + 8V'_\delta V'_\beta \big) \Big]$$

$$+ \partial_\alpha \Phi \Big[V'_\gamma \big(-4i\partial_\delta V'_\beta + 4V'_\beta V'_\delta - 4V'_\delta V'_\beta \big) + \big(-4i\partial_\beta V'_\gamma + 8i\partial_\gamma V'_\beta - 4V'_\beta V'_\gamma \big) V'_\delta \Big]$$

$$+ \partial_\alpha \partial_\gamma \Phi \big(-4\partial_\delta V'_\beta + 4iV'_\delta V'_\beta \big) + \Phi \Big[V'_\alpha \big[V'_\gamma \big(\partial_\beta V'_\delta + 2iV'_\beta V'_\delta \big)$$

$$+ \big(3\partial_\beta V'_\gamma - 5\partial_\gamma V'_\beta - 3iV'_\beta V'_\tau \big) V'_\nu \big] + \partial_\alpha V'_\gamma \big(-2i\partial_\delta V'_\beta - 3V'_\delta V'_\beta \big) \Big] \Big)$$

Note that above Eqs., representing SW mapps up to second order in θ for fermion and Higgs fields respectively, in the case of $V' = 0$ and for two fields only, are identical.

GAUGE SECTOR

FRAMEWORK PROPOSAL

1: Commutative GFT, that are renormalizable are extended to the NC space with deformed gauge transformations. These deformations are not unique. For instance deformed action S_g depends on the choice of representation. This derives from the fact that $\widehat{F}^{\mu\nu}$ is enveloping algebra, not Lee algebra valued.

$$
\begin{aligned}
S_{\mathrm{NC}} &= S_g + S_\psi = S_g^0 + S_g^\theta + S_\psi^0 + S_\psi^\theta \\
S_g &= -\frac{1}{2}\,\mathrm{Tr}\int d^4x \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} \\
S_\psi &= +\mathrm{i}\int \mathrm{d}^4x\,\widehat{\bar{\varphi}} \star \bar{\sigma}^\mu(\partial_\mu + \mathrm{i}\widehat{A}_\mu) \star \widehat{\varphi}\,.
\end{aligned}
$$

The trace $\mathbf{Tr}$ is over all representations.
$\widehat{\varphi}$'s are the noncommutative Weyl spinors.
2: Seiberg-Witten map up to 1st order in θ.
Points 1: and 2: leads to:

$$
\begin{aligned}
S_g^0 &= -\frac{1}{2}\mathrm{Tr}\int d^4x\,F_{\mu\nu}F^{\mu\nu} \\
S_g^\theta &= h\,\theta^{\rho\sigma}\mathrm{Tr}\int d^4x\,\left[\left(\frac{1}{4}F_{\rho\sigma}F_{\mu\nu} - F_{\rho\mu}F_{\sigma\nu}\right)F^{\mu\nu}\right] \\
S_\psi^0 &= \mathrm{i}\int d^4x\,\bar{\varphi}\sigma^\mu(\partial_\mu + \mathrm{i}A_\mu)\varphi \\
S_\psi^\theta &= -\frac{h}{8}\theta^{\mu\nu}\Delta^{\alpha\beta\gamma}_{\mu\nu\rho}\int d^4x\,F_{\alpha\beta}\,\bar{\varphi}\,\bar{\sigma}^\rho(\partial_\gamma + \mathrm{i}A_\gamma)\varphi \\
\Delta^{\alpha\beta\gamma}_{\mu\nu\rho} &= \varepsilon^{\alpha\beta\gamma\lambda}\varepsilon_{\lambda\mu\nu\rho}\,.
\end{aligned}
$$

3: Clearly we do not know the meaning of 'minimal coupling concept' for some NCGFT in the NC space. However, renormalization is the principle that help us to find such acceptable couplings. We learned that the renormalizability condition of some specific NCGFT requires introduction of the higher order NC gauge interaction by expanding general NC action in terms of NC field strengths. This lead us to the extension of 'minimal' action S_g to higher order

$$
\begin{aligned}
S_g &= \mathrm{Tr}\int \mathrm{d}^4x \Big[-\frac{1}{2}\widehat{F}_{\mu\nu}(x)\star\widehat{F}^{\mu\nu}(x) \\
&+ \mathrm{i}(a-1)\, x^\mu \star x^\nu \star \widehat{F}_{\mu\nu}(x)\star\widehat{F}_{\rho\sigma}(x)\star\widehat{F}^{\rho\sigma}(x)\Big],
\end{aligned}
$$

with a being free parameter determining renormalizable deformation.

4: SW map for NC field strength up to the first order in $h\theta^{\mu\nu}$ gives

$$
\begin{aligned}
S &= \mathrm{Tr}\int \mathrm{d}^4x \,\Big[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} \\
&\qquad + h\theta^{\mu\nu}\left(\frac{a}{4}F_{\mu\nu}F_{\rho\sigma} - F_{\mu\rho}F_{\nu\sigma}\right)F^{\rho\sigma}\Big]
\end{aligned}
$$

5: Choice of Majorana spinors for the U(1) case gives

$$
\begin{aligned}
S_\psi &= \frac{\mathrm{i}}{2}\int d^4x\Big[\,\bar\psi\gamma^\mu(\partial_\mu - \mathrm{i}\gamma_5 A_\mu)\psi \\
&- h\frac{1}{16}\theta^{\mu\nu}\Delta^{\alpha\beta\gamma}_{\mu\nu\rho}F_{\alpha\beta}\,\bar\psi\,\gamma^\rho(\partial_\gamma - \mathrm{i}\gamma_5 A_\gamma)\psi\Big]
\end{aligned}
$$

More complicated expression for the SU(2) case.

[J. T. Renormalizability and phenomenology of θ-expanded noncommutative gauge field theory Fortschr. Phys. **56**, No. 4-5, 521-531 (2008).]

Proposed framework 1,...,5 gives starting action for gauge and fermion sectors!

REQUIREMENT OF RENORMALIZABILITY
fixes the freedom parameter a $\Rightarrow$
PRINCIPLE OF RENORMALIZATION
DETERMINES
NC RENORMALIZABLE DEFORMATION

Trace of three generators in the above action lead to dependence of the gauge group representation!
The choice of the trace corresponds to the choice of the representation of the gauge group

Choosing however vector field in the adjoint representation, i.e. using a sum of three traces over the SM gauge group we have:
$\Rightarrow$ Model 1: mNCSM
Choosing a trace over all massive particle multiplets with different quantum numbers in the model that have covariant derivative acting on them we found:
$\Rightarrow$ Model 2: nmNCSM
Choosing however vector field in the adjoint representation SU(N) we have:
$\Rightarrow$ Model 3: NC SU(N) GFT
Noncommutative chiral model for fermions:
$\Rightarrow$ fermions coupled to the U(1) gauge boson
$\Rightarrow$ fermions in the fundamental representation of SU(2)

Gauge sector Model 1: mNCSM

Short rewiev of mNCSM gauge sector

The mNCSM gauge action is given by

$$S_{\text{gauge}}^{\text{mNCSM}} = -\frac{1}{2}\int d^4x \left(\frac{1}{g'^2}\text{Tr}_\mathbf{1} + \frac{1}{g^2}\text{Tr}_\mathbf{2} + \frac{1}{g_s^2}\text{Tr}_\mathbf{3}\right) \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu} .$$

In the definition of $\text{Tr}_\mathbf{1}$:

$$Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The fundamental representations for $SU(2)$ and $SU(3)$ generators in $\text{Tr}_\mathbf{2}$ and $\text{Tr}_\mathbf{3}$, respectively. In terms of physical fields, the action then reads

$$\begin{aligned} S_{\text{gauge}}^{\text{mNCSM}} &= -\frac{1}{2}\int d^4x \left(\frac{1}{2}\mathcal{A}_{\mu\nu}\mathcal{A}^{\mu\nu} + \text{Tr}\,\mathcal{B}_{\mu\nu}\mathcal{B}^{\mu\nu} + \text{Tr}\,G_{\mu\nu}G^{\mu\nu}\right) \\ &\quad + \frac{1}{4} g_s\, d^{abc}\, \theta^{\rho\sigma} \int d^4x \left(\frac{a}{4} G^a_{\rho\sigma} G^b_{\mu\nu} - G^a_{\rho\mu} G^b_{\sigma\nu}\right) G^{\mu\nu,c} , \end{aligned}$$

where $\mathcal{A}_{\mu\nu}$, $\mathcal{B}_{\mu\nu}(= B^a_{\mu\nu}T^a_L)$ and $G_{\mu\nu}(= G^a_{\mu\nu}T^a_S)$ denote the $U(1)$, $SU(2)_L$ and $SU(3)_c$ field strengths, respectively:

$$\begin{aligned} \mathcal{A}_{\mu\nu} &= \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu , \\ \mathcal{B}^a_{\mu\nu} &= \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g\, \epsilon^{abc} B^b_\mu B^c_\nu , \\ G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s\, f^{abc} G^b_\mu G^c_\nu . \end{aligned}$$

For adjoint representation $\Rightarrow$

* NO NEW NEUTRAL EW TGB INTERACTIONS

Gauge sector Model 2: nmNCSM

The action $S^{\text{nmNCSM}}_{\text{gauge}}$ up to linear order in θ:

$$
\begin{aligned}
S^{\text{nmNCSM}}_{\text{gauge}} &= S_{cl} = S^0_{\text{SM}} + S^{\theta} = -\frac{1}{2}\int d^4x\, \mathbf{Tr}\frac{1}{\mathbf{G}^2} F_{\mu\nu}F^{\mu\nu} \\
&+\theta^{\rho\sigma}\int d^4x\, \mathbf{Tr}\frac{1}{\mathbf{G}^2}\left[\left(\frac{a}{4}F_{\rho\sigma}F_{\mu\nu} - F_{\rho\mu}F_{\sigma\nu}\right)F^{\mu\nu}\right]
\end{aligned}
$$

where $\mathbf{Tr}\frac{1}{\mathbf{G}^2}$ is trace over all massive particle multiplets with different quantum numbers in the model that have covariant derivative acting on them; 5 multiplets for each generation of fermions and 1 Higgs multiplet (Table). Here $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu]$ is SM field strenght, i.e. V_μ is the SM gauge potential:

$$
V^{\mu} = g'\mathcal{A}^{\mu}(x)Y + g\sum_{a=1}^{3} B^{\mu}_a(x)T^a_L + g_s\sum_{b=1}^{8} G^{\mu}_b(x)T^b_S
$$

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_Q$	T_3
$e^{(i)}_R$	$\mathbf{1}$	$\mathbf{1}$	-1	-1	0
$L^{(i)}_L = \begin{pmatrix} \nu^{(i)}_L \\ e^{(i)}_L \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$u^{(i)}_R$	$\mathbf{3}$	$\mathbf{1}$	$2/3$	$2/3$	0
$d^{(i)}_R$	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	$-1/3$	0
$Q^{(i)}_L = \begin{pmatrix} u^{(i)}_L \\ d^{(i)}_L \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$1/6$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$
W^+, W^-, Z	$\mathbf{1}$	$\mathbf{3}$	0	$(\pm 1, 0)$	$(\pm 1, 0)$
A	$\mathbf{1}$	$\mathbf{1}$	0	0	0
G^b	$\mathbf{8}$	$\mathbf{1}$	0	0	0

The SM fields. Here $i \in \{1,2,3\}$ denotes the generation index. The electric charge is given by the Gell-Mann-Nishijima relation $Q = (T_3 + Y)$. The physical electroweak fields A, W^+, W^- and Z are expressed through the unphysical $U(1)_Y$ and $SU(2)$ fields $\mathcal{A}$ and B_a ($a \in \{1,2,3\}$). The gluons G^b ($b \in \{1,2,\ldots,8\}$) are in the octet representation of $SU(3)_C$.

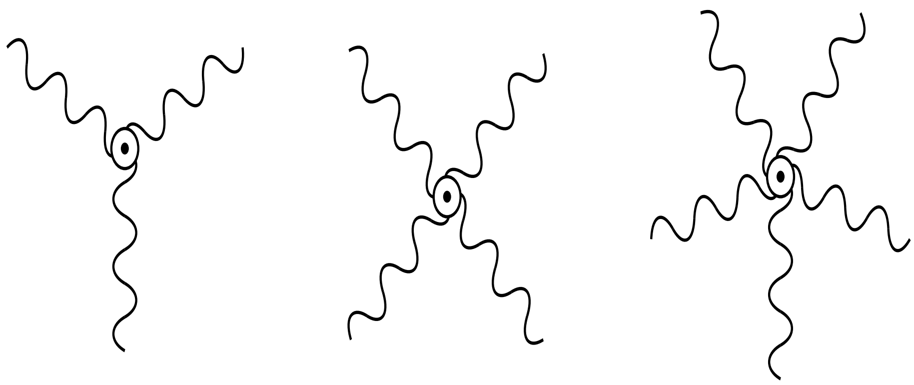
For SM gauge group we denote field strenght as:

$$F^{\mu\nu} = g' f^{\mu\nu}\mathcal{R}(Y) + g\sum_{a=1}^{3} B_a^{\mu\nu}\mathcal{R}(T_L^a) + g_s\sum_{b=1}^{8} G_b^{\mu\nu}\mathcal{R}(T_S^b)$$

Lagrangian linear correction in θ has trace of products of 3 field strenghts. Written genericaly that is:

$$\begin{aligned}
F^3 \sim\ & g'^3\, f^3\, Tr\mathcal{R}(Y)^3\, TrI\, TrI \neq 0 \\
+\ & g^3\, B^3\, Tr\mathcal{R}(T^i)^3\, TrI \qquad \sim d^{ijk} \text{ for SU(2)} \\
+\ & {g_s}^3\, G^3\, TrI\, Tr\mathcal{R}(T^a)^3\, TrI \qquad \sim d^{abc} \text{ for SU(3)} \\
+\ & {g_s}^3\, f^2 B\, TrT^i\, TrI = 0 \\
+\ & g' g^2\, f B^2\, Tr(T^i)^2\, TrI \neq 0 \\
+\ & g'^2 g_s\, f G^2\, TrI\, Tr(T^a)^2 = 0 \\
+\ & g' g_s^2\, f G^2\, TrI\, Tr(T^a)^2\, TrI \neq 0 \\
+\ & g {g_s}^2\, B G^2\, TrT^i\, Tr(T^a)^2 = 0 \\
+\ & g^2 g_s\, B^2 G\, Tr(T^i)^2\, TrT^a = 0
\end{aligned}$$

Nonzero are only 3 terms containinig 3, 4 and 5 fields linear in θ



The NC couplings $\rightarrow$ additional vertices.
The lines are gauge fields $\mathcal{A}_\mu$, B^i_μ and G^a_μ

Matching the SM action at zeroth order in θ, three consistency conditions are imposed

$$\begin{aligned}\frac{1}{g'^2} &= \frac{2}{g_1^2}+\frac{1}{g_2^2}+\frac{8}{3g_3^2}+\frac{2}{3g_4^2}+\frac{1}{3g_5^2}+\frac{1}{g_6^2},\\ \frac{1}{g^2} &= \frac{1}{g_2^2}+\frac{3}{g_5^2}+\frac{1}{g_6^2},\\ \frac{1}{g_s^2} &= \frac{1}{g_3^2}+\frac{1}{g_4^2}+\frac{2}{g_5^2}.\end{aligned}$$

giving final expression for TGB action

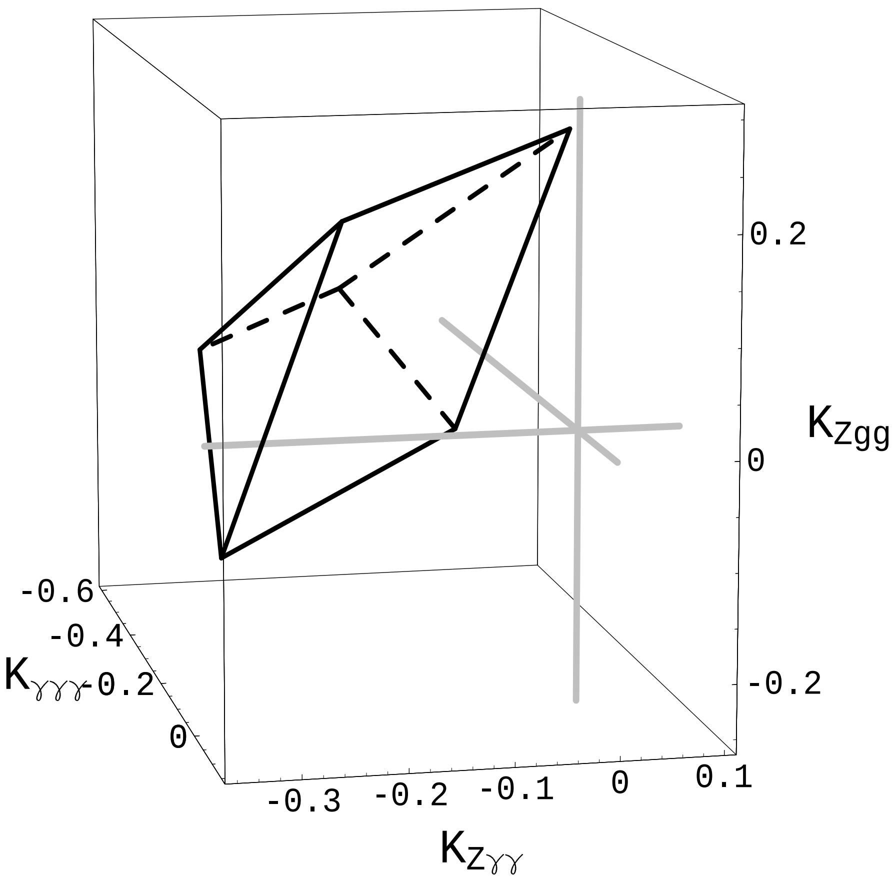
$$\begin{aligned}S_{gauge} &= S_{cl} = S^0_{\text{SM}} + S^\theta = -\frac{1}{4}\int d^4x f_{\mu\nu}f^{\mu\nu} - \frac{1}{2}\int d^4x\, \text{Tr}\,(B_{\mu\nu}B^{\mu\nu})\\ &- \frac{1}{2}\int d^4x\, \text{Tr}\,(G_{\mu\nu}G^{\mu\nu})\\ &+ g'^2\kappa_1\theta^{\rho\tau}\int d^4x\, \left(\frac{a}{4}f_{\rho\tau}f_{\mu\nu} - f_{\mu\rho}f_{\nu\tau}\right)f^{\mu\nu}\\ &+ g'g^2\kappa_2\,\theta^{\rho\tau}\int d^4x\sum_{a=1}^{3}\left[\left(\frac{a}{4}f_{\rho\tau}B^a_{\mu\nu} - f_{\mu\rho}B^a_{\nu\tau}\right)B^{\mu\nu,a} + c.p.\right]\\ &+ g'g_s^2\kappa_3\,\theta^{\rho\tau}\int d^4x\sum_{b=1}^{8}\left[\left(\frac{a}{4}f_{\rho\tau}G^b_{\mu\nu} - f_{\mu\rho}G^b_{\nu\tau}\right)G^{\mu\nu,b} + c.p.\right]\end{aligned}$$

Above three consistenty conditions together with the requirement that $1/g_i^2 > 0$ define a 3D pentahedron in the six-dimensional moduli space spanned by $1/g_1^2, ..., 1/g_6^2$

$$\frac{2K_{\gamma\gamma\gamma}}{gg'} = -\frac{1}{g_1^2} - \frac{1}{g_2^2} + \frac{8}{9g_3^2} - \frac{1}{9g_4^2} + \frac{7}{9g_5^2} + \frac{1}{g_6^2},$$

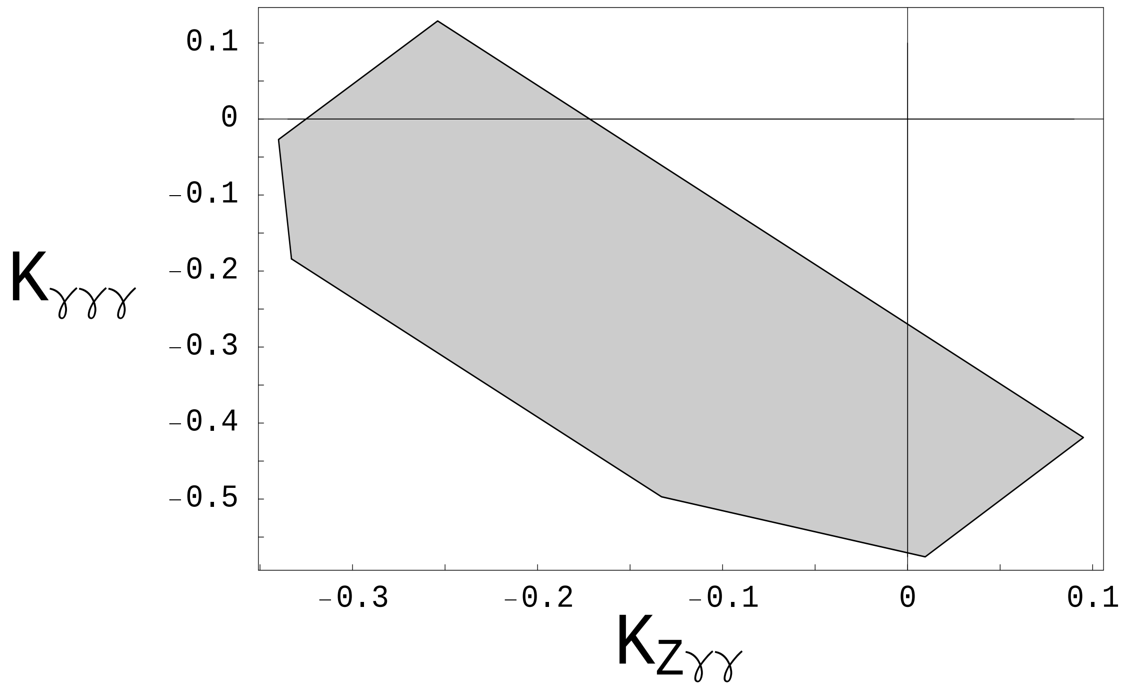
$$\frac{2K_{Z\gamma\gamma}}{g'^2} = -\frac{1}{g_1^2} - \left(1 - (\frac{g}{g'})^2\right)\frac{1}{2g_2^2} + \frac{8}{9g_3^2} - \frac{1}{9g_4^2} + \left(5 - 9(\frac{g}{g'})^2\right)\frac{1}{18g_5^2} + \left(1 - (\frac{g}{g'})^2\right)\frac{1}{2g_6^2},$$

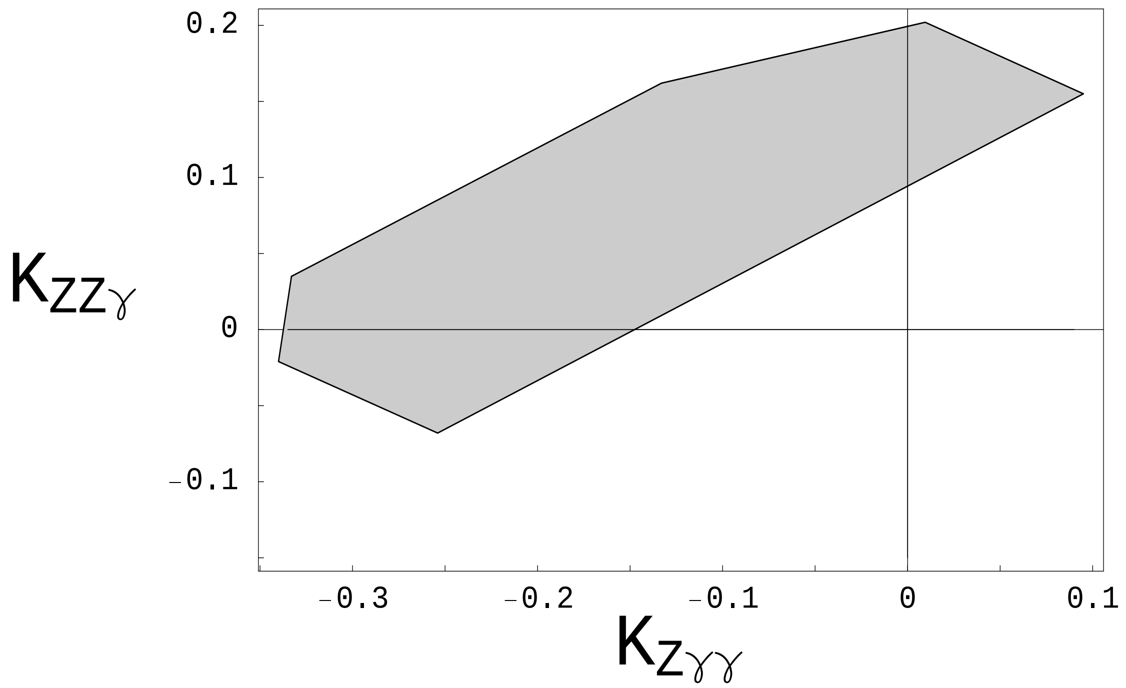
$$\frac{2K_{Zgg}}{g_s^2} = \left(1 + (\frac{g'}{g})^2\right)\left(\frac{1}{3g_3^2} - \frac{1}{6g_4^2} + \frac{1}{6g_5^2}\right).$$



$K_{\gamma\gamma\gamma}$	$K_{Z\gamma\gamma}$	K_{Zgg}	$K_{ZZ\gamma}$	K_{ZZZ}	$K_{\gamma gg}$
−0.184	−0.333	0.054	0.035	−0.213	−0.098
−0.027	−0.340	−0.108	−0.021	−0.337	0.197
0.129	−0.254	0.217	−0.068	−0.362	−0.396
−0.576	0.010	−0.108	0.202	0.437	0.197
−0.497	−0.133	0.054	0.162	0.228	−0.098
−0.419	0.095	0.217	0.155	0.410	−0.396

[G. Duplančić, P. Schupp and J. Trampetić; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C**32** (2003) 141]





The interactions $\mathcal{L}^\theta$ in terms of physical fields (A, Z, W, G)

$$\mathcal{L}^\theta_{\gamma\gamma\gamma} = \frac{e}{4}\sin 2\theta_W \, \mathsf{K}_{\gamma\gamma\gamma}\theta^{\rho\tau} A^{\mu\nu}\left(a A_{\mu\nu}A_{\rho\tau} - 4 A_{\mu\rho}A_{\nu\tau}\right)$$

$$\mathsf{K}_{\gamma\gamma\gamma} = \frac{1}{2}\, gg'(\kappa_1 + 3\kappa_2)$$

$$\mathcal{L}^\theta_{Z\gamma\gamma} = \frac{e}{4}\sin 2\theta_W \, \mathsf{K}_{Z\gamma\gamma}\,\theta^{\rho\tau}\left[2Z^{\mu\nu}\left(2A_{\mu\rho}A_{\nu\tau} - a A_{\mu\nu}A_{\rho\tau}\right) + 8 Z_{\mu\rho}A^{\mu\nu}A_{\nu\tau} - a Z_{\rho\tau}A_{\mu\nu}A^{\mu\nu}\right]$$

$$\mathsf{K}_{Z\gamma\gamma} = \frac{1}{2}\left[{g'}^2\kappa_1 + \left({g'}^2 - 2g^2\right)\kappa_2\right]$$

$$\mathcal{L}^\theta_{WW\gamma} = \frac{e}{4}\sin 2\theta_W \, \mathsf{K}_{WW\gamma}\,\theta^{\rho\tau}\Big\{A^{\mu\nu}\Big[2\left({W^+}_{\mu\rho}{W^-}_{\nu\tau} + {W^-}_{\mu\rho}{W^+}_{\nu\tau}\right) - a\left({W^+}_{\mu\nu}{W^-}_{\rho\tau} + {W^-}_{\mu\nu}{W^+}_{\rho\tau}\right)\Big] + 4A_{\mu\rho}\left(W^{+\mu\nu}{W^-}_{\nu\tau} + W^{-\mu\nu}{W^+}_{\nu\tau}\right) - a A_{\rho\tau}{W^+}_{\mu\nu}W^{-\mu\nu}\Big]\Big\}$$

$$\mathsf{K}_{WW\gamma} = -\frac{g}{g'}\left[{g'}^2 + g^2\right]\kappa_2$$

$$\mathcal{L}^\theta_{WWZ} = \mathcal{L}_{WW\gamma}(A \leftrightarrow Z)$$

$$\mathsf{K}_{WWZ} = -\frac{g'}{g}\,\mathsf{K}_{WW\gamma}$$

$$\mathcal{L}^\theta_{ZZ\gamma} = \mathcal{L}_{Z\gamma\gamma}(A \leftrightarrow Z)$$

$$\mathsf{K}_{ZZ\gamma} = \frac{-1}{2gg'}\left[{g'}^4\kappa_1 + g^2\left(g^2 - 2{g'}^2\right)\kappa_2\right]$$

$$\mathcal{L}^\theta_{ZZZ} = \mathcal{L}_{\gamma\gamma\gamma}(A \to Z)$$

$$\mathsf{K}_{ZZZ} = \frac{-1}{2g^2}\left[{g'}^4\kappa_1 + 3g^4\kappa_2\right]$$

$$\mathcal{L}^\theta_{Zgg} = \mathcal{L}_{Z\gamma\gamma}(A \to G^b)$$

$$\mathsf{K}_{Zgg} = \frac{g_s^2}{2}\left[1 + \left(\frac{g'}{g}\right)^2\right]\kappa_3$$

$$\mathcal{L}^\theta_{\gamma gg} = \mathcal{L}_{Zgg}(Z \to A)$$

$$\mathsf{K}_{\gamma gg} = \frac{-g_s^2}{2}\left[\frac{g}{g'} + \frac{g'}{g}\right]\kappa_3; \quad A_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \ldots$$

Renormalization Model 2: nmNCSM

⋆ Advantage of the background field method is the guarantee of covariance, because by doing the path integral the local symmetry of the quantum field Φ_V is fixed, while the gauge symmetry of the background field ϕ_V is manifestly preserved.

⋆ Quantization is performed by the functional integration over the quantum vector field Φ_V in the saddle-point approximation around classical (background) configuration. Our case $\phi_V = constant$.

⋆ The main contribution to the functional integral is given by the Gaussian integral.

⋆ Split the vector potential into the classical background plus the quantum-fluctuation parts, that is: We replace, $\phi_V \to \phi_V + \Phi_V$, and than compute the terms quadratic in the quantum fields.

⋆ Interactions are of the polynomial type.

⋆ Proper quantization requires the presence of the gauge fixing term $S_{\mathsf{gf}}[\phi]$. Adding to the SM part in the usual way, FFP ghost appears in the effective action. Result of functional integration

$$\Gamma[\phi] = S_{\mathsf{cl}}[\phi] + S_{\mathsf{gf}}[\phi] + \Gamma^{(1)}[\phi],$$
$$S_{\mathsf{gf}}[\phi] = -\frac{1}{2}\int \mathsf{d}^4x (D_\mu \phi_V^\mu)^2 \,,$$

produce the standard result of the commutative part of our action:

$$\Gamma^{(1)}[\phi] = \frac{\mathsf{i}}{2} \log \det S^{(2)}[\phi] = \frac{\mathsf{i}}{2} \mathsf{Tr} \log S^{(2)}[\phi].$$

The $S^{(2)}[\phi]$ is the 2^{nd}-functional derivative of the classical action,

$$S^{(2)}[\phi] = \frac{\delta^2 S_{cl}}{\delta\phi_{V_1}\delta\phi_{V_2}}.$$

After making the splitting

$$\mathcal{A}_\mu \to \mathcal{A}_\mu + \mathbf{A}_\mu, \quad B^i_\mu \to B^i_\mu + \mathbf{B}^i_\mu, \quad G^a_\mu \to G^a_\mu + \mathbf{G}^a_\mu,$$

we obtain for the quadratic part of the action :

$$\frac{1}{2}\left(\mathbf{A}_\alpha\, \mathbf{B}^i_\alpha\, \mathbf{G}^a_\alpha\right)\begin{pmatrix} g^{\alpha\beta}\Box + M^{\alpha\beta} & * & * \\ * & g^{\alpha\beta}\delta^{ij}\Box + V^{\alpha\beta;ij} & 0 \\ * & 0 & g^{\alpha\beta}\delta^{ab}\Box + W^{\alpha\beta ab}\end{pmatrix}\begin{pmatrix}\mathbf{A}_\beta \\ \mathbf{B}^j_\beta \\ \mathbf{G}^b_\beta\end{pmatrix}.$$

$\Box$ - propagator of any field

$*$ - terms which will not contribute θ^1: they give only higher-order corrections.

$$M^{\alpha\beta} = \overleftarrow{\partial_\mu} M^{\mu\alpha,\nu\beta}(x)\overrightarrow{\partial_\nu}$$

$$\begin{aligned} M^{\mu\rho,\nu\sigma} &= \frac{1}{2}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho})\theta^{\alpha\beta}f_{\alpha\beta} \\ &+ g^{\mu\nu}(\theta^{\alpha\rho}f^{\sigma}{}_{\alpha} + \theta^{\alpha\sigma}f^{\rho}{}_{\alpha}) + g^{\rho\sigma}(\theta^{\alpha\mu}f^{\nu}{}_{\alpha} + \theta^{\alpha\nu}f^{\mu}{}_{\alpha}) \\ &- g^{\mu\sigma}(\theta^{\alpha\rho}f^{\nu}{}_{\alpha} + \theta^{\alpha\nu}f^{\rho}{}_{\alpha}) - g^{\nu\rho}(\theta^{\alpha\sigma}f^{\mu}{}_{\alpha} + \theta^{\alpha\mu}f^{\sigma}{}_{\alpha}) \\ &+ \theta^{\mu\rho}f^{\nu\sigma} + \theta^{\nu\sigma}f^{\mu\rho} - \theta^{\rho\sigma}f^{\mu\nu} - \theta^{\mu\nu}f^{\rho\sigma} - \theta^{\nu\rho}f^{\mu\sigma} - \theta^{\mu\sigma}f^{\nu\rho} \end{aligned}$$

The structure of $V^{\alpha\beta;ij}$ is as follows:

$$V^{\alpha\beta;ij} = (N_1 + N_2 + T_1 + T_2 + T_3)^{\alpha\beta;ij}.$$

The operators N_1 and N_2 come from the commutative 3-vertex and 4-vertex interactions:

$$\begin{aligned} (N_1)^{ij}_{\alpha\beta} &= -2ig_{\alpha\beta}(B_\mu)^{ij}\partial^\mu - i(\partial^\mu B_\mu)^{ij}g_{\alpha\beta}, \\ (N_2)^{ij}_{\alpha\beta} &= -(B_\mu B^\mu)^{ij}g_{\alpha\beta} - 2i(B_{\alpha\beta})^{ij}, \end{aligned}$$

the notation $(X_\mu)^{ij} = -if^{ijk}X^k_\mu$. The operators T_1, T_2 and T_3 describe the θ^1, noncommutative vertices.

$$
\begin{aligned}
(T_1)^{ij}_{\alpha\beta} &= g'g^2\kappa_2\delta^{ij}\Big[a(\overleftarrow{\partial_\mu}\theta^{\rho\sigma}f_{\rho\sigma}g_{\alpha\beta}\overrightarrow{\partial_\mu} - \overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\rho\sigma}\overrightarrow{\partial_\alpha}) \\
&- 2(\overleftarrow{\partial_\beta}\theta_{\rho\alpha}f^{\mu\rho}\overrightarrow{\partial_\mu} - \overleftarrow{\partial^\nu}\theta^\rho{}_\alpha f_{\beta\rho}\overrightarrow{\partial_\nu} - \overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}\overrightarrow{\partial^\mu} + \overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\beta\rho}\overrightarrow{\partial_\alpha} \\
&+ \overleftarrow{\partial_\mu}\theta_{\rho\beta}f^{\mu\rho}\overrightarrow{\partial_\alpha} - \overleftarrow{\partial^\nu}\theta^\rho{}_\beta f_{\alpha\rho}\overrightarrow{\partial_\nu} - \overleftarrow{\partial^\mu}\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}\overrightarrow{\partial_\sigma} \\
&+ \overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\alpha\rho}\overrightarrow{\partial_\sigma}) + 2a(\overleftarrow{\partial_\rho}\theta^\rho{}_\alpha f_{\mu\beta}\overrightarrow{\partial^\mu} + \overleftarrow{\partial^\mu}\theta^\rho{}_\beta f_{\mu\alpha}\overrightarrow{\partial_\rho}) \\
&- 2(\overleftarrow{\partial_\mu}\theta_{\alpha\beta}f^{\mu\nu}\overrightarrow{\partial_\nu} - \overleftarrow{\partial^\mu}\theta_{\alpha\sigma}f_{\mu\beta}\overrightarrow{\partial^\sigma} - \overleftarrow{\partial^\sigma}\theta_{\beta\sigma}f_{\mu\alpha}\overrightarrow{\partial^\mu} + \overleftarrow{\partial_\rho}\theta^{\rho\sigma}f_{\alpha\beta}\overrightarrow{\partial_\sigma})\Big], \\
(T_2)^{ij}_{\alpha\beta} &= g'g^2i\kappa_2\Big[a(-\overleftarrow{\partial_\mu}\theta^{\rho\sigma}g_{\alpha\beta}f_{\rho\sigma}(B^\mu)^{ij} - \theta^{\rho\sigma}f_{\rho\sigma}g_{\alpha\beta}(B^\mu)^{ji}\overrightarrow{\partial_\mu} \\
&+ \overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\rho\sigma}(B_\alpha)^{ij} + \theta^{\rho\sigma}f_{\rho\sigma}(B_\beta)^{ji}\overrightarrow{\partial_\alpha} + \theta_{\rho\sigma}f^{\rho\sigma}(B_{\alpha\beta})^{ij}) \\
&- 2(-\overleftarrow{\partial_\beta}\theta_{\rho\alpha}f^{\mu\rho}(B_\mu)^{ij} - \theta_{\rho\beta}f^{\mu\rho}(B_\mu)^{ji}\overrightarrow{\partial_\alpha} + \overleftarrow{\partial_\nu}\theta_{\rho\alpha}f_\beta{}^\rho(B^\nu)^{ij} \\
&+ \theta_{\rho\beta}f_\alpha{}^\rho(B^\nu)^{ji}\overrightarrow{\partial_\nu} + \overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B^\mu)^{ij} + \theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B^\mu)^{ji}\overrightarrow{\partial_\sigma} \\
&- \overleftarrow{\partial_\sigma}\theta^{\rho\sigma}f_{\beta\rho}(B_\alpha)^{ij} - \theta^{\rho\sigma}f_{\alpha\rho}(B_\beta)^{ji}\overrightarrow{\partial_\sigma} - \overleftarrow{\partial_\mu}\theta_{\rho\beta}f^{\mu\rho}(B_\alpha)^{ij} - \theta_{\rho\alpha}f^{\mu\rho}(B_\beta)^{ji}\overrightarrow{\partial_\mu} \\
&+ \overleftarrow{\partial_\mu}\theta^{\rho\sigma}g_{\alpha\beta}f^\mu{}_\rho(B_\sigma)^{ij} + \theta^{\rho\sigma}f_{\mu\rho}g_{\alpha\beta}(B_\sigma)^{ji}\overrightarrow{\partial^\mu} + \overleftarrow{\partial_\mu}\theta^\rho{}_\beta f_{\alpha\rho}(B^\mu)^{ij} \\
&+ \theta_{\rho\alpha}f_\beta{}^\rho(B^\mu)^{ji}\overrightarrow{\partial_\mu} - \overleftarrow{\partial_\beta}\theta^{\rho\sigma}f_{\alpha\rho}(B_\sigma)^{ij} - \theta^{\rho\sigma}f_{\beta\rho}(B_\sigma)^{ji}\overrightarrow{\partial_\alpha} + \theta^{\rho\sigma}f_{\alpha\rho}(B_{\beta\sigma})^{ij} \\
&+ \theta_{\rho\beta}f^{\mu\rho}(B_{\mu\alpha})^{ij} + \theta^{\rho\sigma}f_{\beta\rho}(B_{\alpha\sigma})^{ji} \\
&+ \theta_{\rho\alpha}f^{\mu\rho}(B^\mu{}_\beta)^{ji}) - 2a(\overleftarrow{\partial^\rho}\theta_{\rho\alpha}f_{\mu\beta}(B^\mu)^{ij} + \theta_{\rho\beta}f_{\mu\alpha}(B^\mu)^{ji}\overrightarrow{\partial^\rho} \\
&+ \overleftarrow{\partial^\mu}\theta_{\rho\beta}f_{\mu\alpha}(B^\rho)^{ij} + \theta_{\rho\alpha}f_{\mu\beta}(B^\rho)^{ji}\overrightarrow{\partial^\mu} - \frac{1}{2}\theta_{\rho\sigma}f_{\alpha\beta}(B^{\rho\sigma})^{ij} \\
&- \frac{1}{2}\theta_{\alpha\beta}f_{\rho\sigma}(B^{\rho\sigma})^{ij}) - 2(-\overleftarrow{\partial^\mu}\theta_{\alpha\beta}f_{\mu\nu}(B^\nu)^{ij} - \theta_{\beta\alpha}f_{\mu\nu}(B^\nu)^{ji}\overrightarrow{\partial^\mu} \\
&+ \overleftarrow{\partial^\mu}\theta_{\alpha\sigma}f_{\mu\beta}(B^\sigma)^{ij} + \theta_{\beta\sigma}f_{\mu\alpha}(B^\sigma)^{ji}\overrightarrow{\partial^\mu} + \overleftarrow{\partial^\rho}\theta_{\rho\beta}f_{\alpha\nu}(B^\nu)^{ij} + \theta_{\rho\alpha}f_{\beta\nu}(B^\nu)^{ji}\overrightarrow{\partial^\rho} \\
&- \overleftarrow{\partial_\rho}\theta_{\beta\sigma}f_{\alpha\beta}(B_\sigma)^{ij} - \theta^{\rho\sigma}f_{\beta\alpha}(B_\sigma)^{ji}\overrightarrow{\partial_\rho} + \theta_{\beta\sigma}f_{\alpha\nu}(B^{\nu\sigma})^{ij} + \theta_{\alpha\sigma}f_{\beta\nu}(B^\nu{}_\sigma)^{ji})\Big], \\
(T_3)^{ij}_{\alpha\beta} &= g'g^2\kappa_2\Big[a(\theta^{\rho\sigma}f_{\rho\sigma}(B_\mu B^\mu)^{ij}g_{\alpha\beta} - \theta^{\rho\sigma}f_{\rho\sigma}(B_\beta B_\alpha)^{ij}) \\
&- 2(\theta_{\rho\alpha}f^{\mu\rho}(B_\beta B_\mu)^{ij} - \theta^\rho{}_\alpha f_{\beta\rho}(B_\nu B^\nu)^{ij} - \theta^{\rho\sigma}f_{\mu\rho}(B_\sigma B^\mu)^{ij}g_{\alpha\beta} \\
&+ \theta^{\rho\sigma}f_{\beta\rho}(B_\sigma B_\alpha)^{ij} + (\alpha \leftrightarrow \beta \quad i \leftrightarrow j)) \\
&+ 2a(\theta_{\rho\alpha}f_{\mu\beta}(B^\rho B^\mu)^{ij} + 2\theta_{\rho\beta}f_{\mu\alpha}(B^\rho B^\mu)^{ji}) \\
&- 2((\theta_{\alpha\beta}f^{\mu\nu}(B_\mu B_\nu)^{ij} \\
&- \theta_{\alpha\sigma}f_{\mu\beta}(B^\mu B^\sigma)^{ij} - \theta_{\beta\sigma}f_{\mu\alpha}(B^\mu B^\sigma)^{ji} + \theta^{\rho\sigma}f_{\alpha\beta}(B_\rho B_\sigma)^{ij})\Big].
\end{aligned}
$$

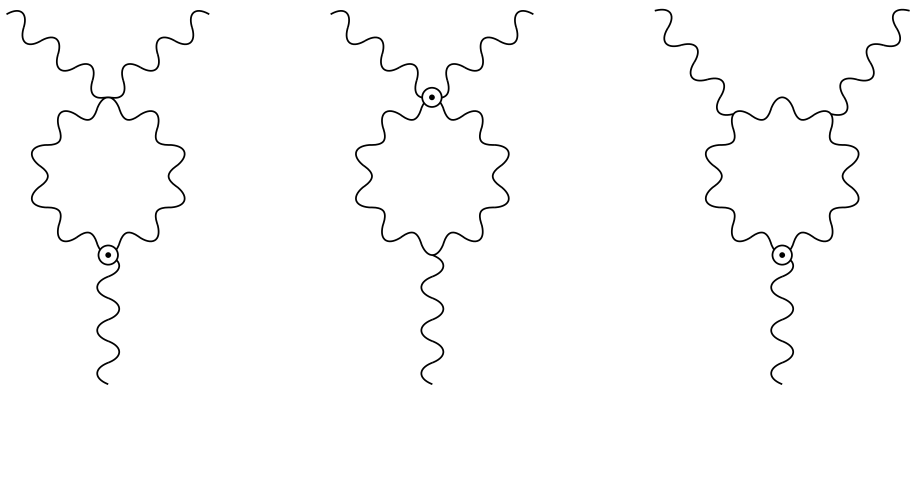
The matrix $W^{\alpha\beta,ab}$ analogous to $V^{\alpha\beta,ij}$ up to the change $B^i_\mu \leftrightarrow G^a_\mu$.

The one-loop effective action is

$$\Gamma^{(1)}_{\theta,2} = \frac{i}{2}\text{Tr}\log\left(\mathcal{I} + \square^{-1}(N_1 + N_2 + T_1 + T_2 + T_3)\right)$$
$$= \frac{i}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\text{Tr}\left(\square^{-1}N_1 + \square^{-1}N_2 + \square^{-1}T_1 + \square^{-1}T_2 + \square^{-1}T_3\right)^n.$$

the divergences in θ-linear order are all of the form $\theta f B^2$. Need to extract and compute only terms that contain three external fields.

$$\Gamma^{(1)}_{\theta,2} = \frac{i}{2}\text{Tr}\left[(\square^{-1}N_1)^2\square^{-1}T_1 - \square^{-1}N_1\square^{-1}T_2 - \square^{-1}N_2\square^{-1}T_1\right].$$



1-loop divergent corrections to the θ-3-vertex also contains the contributions to the θ-4-vertex and θ-5-vertex.

Computed divergences due to the U(1)$_Y$ − SU(2)$_L$ part of the noncommutative action, S_2^θ using background field method; divergent part calculated in momentum representation by dimensional regularization.

$$\begin{aligned}
\mathrm{Tr}\,(\Box^{-1}N_1\Box^{-1}T_2) &= \frac{4i}{3(4\pi)^2\epsilon}g'g^2\kappa_2 \\
&\times \Big[(6-2a)(\theta^{\rho\sigma}f_{\alpha\rho}+\theta_{\rho\alpha}f^{\sigma\rho})(B^{\alpha i}\partial_\mu\partial_\sigma B^{\mu i}-B^{\alpha i}\Box B^i_\sigma) \\
&+ (3a-4)\theta^{\rho\sigma}f_{\rho\sigma}(B^{\nu i}\partial_\mu\partial_\nu B^{\mu i}-B^i_\mu\Box B^{\mu i})\Big],
\end{aligned}$$

$$\begin{aligned}
\mathrm{Tr}\,(\Box^{-1}N_2\Box^{-1}T_1) &= \frac{4i}{3(4\pi)^2\epsilon}g'g^2\kappa_2 \\
&\times \Big[(2a-6)(\theta^{\rho\sigma}f_{\alpha\rho}+\theta_{\rho\alpha}f^{\sigma\rho})(B^{\nu i}\partial_\sigma\partial^\alpha B^i_\nu+\partial_\sigma B^{\mu i}\partial^\alpha B^i_\mu) \\
&+ \theta^{\rho\sigma}f_{\rho\sigma}(18-11a)(\partial_\nu B^{\nu i}\partial_\mu B^{\mu i}+B^i_\mu\Box B^{\mu i})\Big],
\end{aligned}$$

$$\begin{aligned}
\mathrm{Tr}\,(\Box^{-1}N_1^2\Box^{-1}T_1) &= \frac{4i}{3(4\pi)^2\epsilon}g'g^2\kappa_2\Big[\theta^{\rho\sigma}f_{\rho\sigma}\Big((22-14a)B^i_\mu\Box B^{\mu i} \\
&+ (15-10a)\partial_\nu B^{\mu i}\partial^\nu B^i_\mu \\
&+ (3a-4)B^{\mu i}\partial_\mu\partial_\nu B^{\nu i}+(3-a)\partial_\mu B^{\nu i}\partial_\nu B^{\mu i}\Big) \\
&+ (\theta^{\rho\sigma}f_{\alpha\rho}+\theta_{\rho\alpha}f^{\sigma\rho})\Big((2a-6)(B^i_\sigma\Box B^{\alpha i}-B^i_\sigma\partial^\alpha\partial_\mu B^{\mu i} \\
&+ B^{\mu i}\partial_\sigma\partial^\alpha B^i_\mu-\partial_\sigma B^{\mu i}\partial_\mu B^{\alpha i})+(a-3)\partial_\mu B^{\alpha i}\partial^\mu B^i_\sigma \\
&+ (3a-9)\partial_\sigma B^{\mu i}\partial^\alpha B^i_\mu\Big)\Big].
\end{aligned}$$

The result for U(1)$_Y$ − SU(3)$_C$ is analogous and follows immediately. Finally

$$\begin{aligned}
\Gamma^{(1)}_{\rm div} &= \frac{11}{3(4\pi)^2\epsilon}\int d^4x B^i_{\mu\nu}B^{\mu\nu i}+\frac{11}{2(4\pi)^2\epsilon}\int d^4x G^a_{\mu\nu}G^{\mu\nu a} \\
&+ \frac{4}{3(4\pi)^2\epsilon}g'g^2\kappa_2(3-a)\theta^{\mu\nu}\int d^4x\Big(\frac{1}{4}f_{\mu\nu}B^i_{\rho\sigma}B^{\rho\sigma i}-f_{\mu\rho}B^i_{\nu\sigma}B^{\rho\sigma i}\Big) \\
&+ \frac{6}{3(4\pi)^2\epsilon}g'g_S^2\kappa_3(3-a)\theta^{\mu\nu}\int d^4x\Big(\frac{1}{4}f_{\mu\nu}G^a_{\rho\sigma}G^{\rho\sigma a}-f_{\mu\rho}G^a_{\nu\sigma}G^{\rho\sigma a}\Big).
\end{aligned}$$

Renormalization via Counterterms & $a = 3$

$$
\begin{aligned}
\mathcal{L} + \mathcal{L}_{ct} &= -\frac{1}{4} f_{0\mu\nu} f_0{}^{\mu\nu} - \frac{1}{4} B_{0}{}^{i}_{\mu\nu} B_0{}^{\mu\nu i} - \frac{1}{4} G_{0}{}^{a}_{\mu\nu} G_0{}^{\mu\nu a} \\
&+ g'^3 \kappa_1 \theta^{\mu\nu} \left(\frac{3}{4} f_{0\mu\nu} f_{0\rho\sigma} f_0{}^{\rho\sigma} - f_{0\mu\rho} f_{0\nu\sigma} f_0{}^{\rho\sigma} \right) \\
&+ g_0' g_0^2 \kappa_2 \theta^{\mu\nu} \left(\frac{3}{4} f_{0\mu\nu} B_{0}{}^{i}_{\rho\sigma} B_0^{\rho\sigma i} - f_{0\mu\rho} B_{0}{}^{i}_{\nu\sigma} B_0^{\rho\sigma i} + c.p. \right) \\
&+ g_0' (g_S)_0^2 \kappa_3 \theta^{\mu\nu} \left(\frac{3}{4} f_{0\mu\nu} G_{0}{}^{a}_{\rho\sigma} G_0^{\rho\sigma a} - f_{0\mu\rho} G_{0}{}^{a}_{\nu\sigma} G_0^{\rho\sigma a} + c.p. \right),
\end{aligned}
$$

Bare quantities are:

$$
\begin{aligned}
\mathcal{A}_0^\mu &= \mathcal{A}^\mu, \qquad g_0' = g', \\
B_0{}^{\mu i} &= B^{\mu i} \sqrt{1 + \frac{44 g^2}{3(4\pi)^2 \epsilon}}, \quad g_0 = \frac{g\, \mu^{\epsilon/2}}{\sqrt{1 + \frac{44 g^2}{3(4\pi)^2 \epsilon}}}, \\
G_0{}^{\mu a} &= G^{\mu a} \sqrt{1 + \frac{22 g_S^2}{(4\pi)^2 \epsilon}}, \quad (g_S)_0 = \frac{g_S\, \mu^{\epsilon/2}}{\sqrt{1 + \frac{22 g_S^2}{(4\pi)^2 \epsilon}}}.
\end{aligned}
$$

κ_1, κ_2 and κ_3 unchanged under renormalization

$$
\kappa_1 = (\kappa_1)_0, \quad \kappa_2 = (\kappa_2)_0, \quad \kappa_3 = (\kappa_3)_0,
$$

replacement:

$$
\begin{aligned}
\frac{1}{g_1^2} &= \left(\frac{1}{g_1^2}\right)_0 + \frac{33}{18(4\pi)^2\epsilon}, \quad \frac{1}{g_2^2} = \left(\frac{1}{g_2^2}\right)_0 + \frac{-11}{18(4\pi)^2\epsilon}, \quad \frac{1}{g_3^2} = \left(\frac{1}{g_3^2}\right)_0 + \frac{-11}{18(4\pi)^2\epsilon}, \\
\frac{1}{g_4^2} &= \left(\frac{1}{g_4^2}\right)_0 + \frac{-143}{18(4\pi)^2\epsilon}, \quad \frac{1}{g_5^2} = \left(\frac{1}{g_5^2}\right)_0 + \frac{-121}{18(4\pi)^2\epsilon}, \quad \frac{1}{g_6^2} = \left(\frac{1}{g_6^2}\right)_0 + \frac{110}{18(4\pi)^2\epsilon}.
\end{aligned}
$$

NC parameter θ need not be renormalized

because $\mathcal{L}^\theta$ is free from divergences.

Gauge sector Model 3: NC SU(N) GFT

[D. Latas, V. Radovanovic and J.T., Non-commutative SU(N) gauge theories and asymptotic freedom; Phys.Rev. D**76**, 085006 (2007).]

$$
\begin{aligned}
S_{cl} &= S_{\mathrm{NCYM}} = \int \mathrm{d}^4x \left(-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}\right. \\
&\left. +\frac{1}{4}h\theta^{\mu\nu}d^{abc}\left(\frac{a}{4}F^a_{\mu\nu}F^b_{\rho\sigma} - F^a_{\mu\rho}F^b_{\nu\sigma}\right)F^{c\rho\sigma}\right),
\end{aligned}
$$

Here earlier introduced noncommutativity deformation parameter h becomes very important.

Renormalization:

Second functional derivative $S^2[\phi]$ of S_{cl}

$$
S^2 = \Box + N_1 + N_2 + T_2 + T_3 + T_4 \ ,
$$

N_1, N_2 - commutative vertices
T_2, T_3, T_4 non-commutative vertices

The 1-loop effective action computed by using BFM

$$
\begin{aligned}
\Gamma^{(1)}_{\theta,2} &= \frac{\mathrm{i}}{2}\mathrm{Tr}\,\log\left(\mathcal{I} + \Box^{-1}(N_1 + N_2 + T_2 + T_3 + T_4)\right) \\
&= \frac{\mathrm{i}}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\mathrm{Tr}\left(\Box^{-1}N_1 + \Box^{-1}N_2 + \Box^{-1}T_2 + \Box^{-1}T_3 + \Box^{-1}T_4\right)^n .
\end{aligned}
$$

Vertices are

$$
\begin{aligned}
(N_1)^{ab\alpha\beta} &= -2\mathrm{i}(A^\mu)^{ab}g^{\alpha\beta}\partial_\mu, \\
(N_2)^{ab\alpha\beta} &= -2f^{abc}F^{c\alpha\beta} - (A^\mu A_\mu)^{ab}g^{\alpha\beta},
\end{aligned}
$$

Noncommutative vertices are

$$
\begin{aligned}
(T_2)^{ab\alpha\beta} &= \frac{h}{8} d^{abc}\Big\{\Big[\Big(a\theta^{\rho\sigma}F^c_{\rho\sigma}g^{\alpha\nu}g^{\beta\mu} - 2(a-1)\theta^{\alpha\mu}F^{c\beta\nu} + 4\theta^{\alpha}{}_{\rho}F^{c\beta\rho}g^{\mu\nu} \\
&\qquad + 4\theta^{\mu}{}_{\rho}F^{c\nu\rho}g^{\alpha\beta}\Big) - (\beta \leftrightarrow \nu)\Big] + [\alpha \leftrightarrow \beta]\Big\}\,\partial_\mu\partial_\nu\,, \\
(T_3)^{ab\alpha\beta} &= \frac{ih}{4}\Big\{d^{acd}\Big[-2a\theta^{\alpha\mu}(A_\nu)^{bc}F^{d\beta\nu} - 2a\theta^{\beta\nu}(A_\nu)^{bc}F^{d\alpha\mu} \\
&\qquad - a\theta^{\rho\sigma}(A^\mu)^{bc}F^d_{\rho\sigma}g^{\alpha\beta} + a\theta^{\rho\sigma}(A^\alpha)^{bc}F^d_{\rho\sigma}g^{\beta\mu} \\
&\qquad - 2\theta^{\alpha}{}_{\rho}(A_\nu)^{bc}F^{d\nu\rho}g^{\beta\mu} + 2\theta^{\alpha}{}_{\nu}(A^\mu)^{bc}F^{d\beta\nu} \\
&\qquad + 2\theta^{\mu\rho}(A^\nu)^{bc}F^d_{\nu\rho} - 2\theta^{\mu}{}_{\rho}(A^\alpha)^{bc}F^{d\beta\rho} - 2\theta^{\beta}{}_{\rho}(A^\alpha)^{bc}F^{d\mu\rho} \\
&\qquad + 2\theta^{\beta}{}_{\rho}(A^\mu)^{bc}F^{d\alpha\rho} + 2\theta^{\nu}{}_{\rho}(A_\nu)^{bc}F^{d\mu\rho}g^{\alpha\beta} \\
&\qquad + 2\theta^{\alpha\beta}(A_\nu)^{bc}F^{d\mu\nu} + 2\theta^{\alpha\nu}(A_\nu)^{bc}F^{d\beta\mu} + 2\theta^{\beta\mu}(A_\nu)^{bc}F^{d\alpha\nu} \\
&\qquad - 2\theta^{\nu}{}_{\rho}(A_\nu)^{bc}F^{d\alpha\rho}g^{\beta\mu} + 2\theta^{\mu\nu}(A_\nu)^{bc}F^{d\alpha\beta}\Big] \\
&\qquad - [a \leftrightarrow b, \alpha \leftrightarrow \beta]\Big\}\,\partial_\mu\,, \\
(T_4)^{ab\alpha\beta} &= \frac{h}{8} d^{cde}\Big[\Big(-4a\theta^{\alpha\rho}(A_\rho)^{ac}(A_\mu)^{bd}F^{e\beta\mu} - a\theta^{\rho\sigma}(A^\mu)^{ac}(A_\mu)^{bd}F^e_{\rho\sigma}g^{\alpha\beta} \\
&\qquad + a\theta^{\rho\sigma}(A^\beta)^{ac}(A^\alpha)^{bd}F^e_{\rho\sigma} - 4\theta^{\alpha\rho}(A^\beta)^{ad}(A^\mu)^{bc}F^e_{\mu\rho} \\
&\qquad + 4\theta^{\alpha}{}_{\rho}(A^\mu)^{ad}(A_\mu)^{bc}F^{e\beta\rho} + 4\theta^{\nu}{}_{\rho}(A_\nu)^{ad}(A_\mu)^{bc}F^{e\mu\rho}g^{\alpha\beta} \\
&\qquad + 2\theta^{\alpha\beta}(A_\mu)^{ad}(A_\nu)^{bc}F^{e\mu\nu} + 4\theta^{\alpha\rho}(A_\mu)^{ad}(A_\rho)^{bc}F^{e\beta\mu} \\
&\qquad + 2\theta^{\rho\sigma}(A_\rho)^{ad}(A_\sigma)^{bc}F^{e\alpha\beta}\Big) + (a \leftrightarrow b\,, \alpha \leftrightarrow \beta) \\
&\qquad + f^{abc}(2a\theta^{\rho\sigma}F^d_{\rho\sigma}F^{e\alpha\beta} + a\theta^{\alpha\beta}F^d_{\rho\sigma}F^{e\rho\sigma} \\
&\qquad + 4\theta_{\rho\sigma}F^{d\alpha\rho}F^{e\beta\sigma} + 8\theta^{\alpha\rho}F^{d\beta\mu}F^e_{\mu\rho})\Big]\,.
\end{aligned}
$$

The divergent parts are calculated in the momentum representation via dimensional regularization

$$\begin{aligned}
D_1{}^{\text{div}} &= \frac{i}{2}\text{Tr}\left(\left(\Box^{-1}N_1\right)^2\left(\Box^{-1}T_4\right)\right)^{\text{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int d^4x\left[\frac{a-3}{4}(\theta^{\alpha\rho}F^a_{\alpha\nu}+\theta_{\alpha\nu}F^{a\alpha\rho})(V^\nu V^\mu V_\mu V_\rho)^{bc}\right. \\
&\qquad \left.+\frac{3a-4}{4}\theta^{\rho\sigma}F^a_{\rho\sigma}(V^\mu V^\nu V_\nu V_\mu)^{bc}\right], \\
D_2{}^{\text{div}} &= -\frac{i}{2}\text{Tr}\left(\left(\Box^{-1}N_1\right)^3\left(\Box^{-1}T_3\right)\right)^{\text{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int d^4x\left[\frac{7-3a}{6}\theta^{\rho\sigma}F^a_{\rho\sigma}(V_\mu V^\mu V_\nu V^\nu+V_\mu V_\nu V^\mu V^\nu+V_\mu V_\nu V^\nu V^\mu)^{bc}\right. \\
&\qquad +\frac{3-2a}{6}(\theta^{\alpha\rho}F^a_{\alpha\sigma}+\theta_{\alpha\sigma}F^{a\alpha\rho})(V_\rho V^\sigma V^\mu V_\mu+V_\rho V_\mu V^\sigma V^\mu \\
&\qquad \left.+V_\rho V^\mu V_\mu V^\sigma)^{bc}\right], \\
D_3{}^{\text{div}} &= \frac{i}{2}\text{Tr}\left(\left(\Box^{-1}N_1\right)^4\left(\Box^{-1}T_2\right)\right)^{\text{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int d^4x\left[\frac{7a-11}{12}\theta^{\rho\sigma}F^a_{\rho\sigma}(V_\mu V^\mu V_\nu V^\nu+V_\mu V_\nu V^\mu V^\nu+V_\mu V_\nu V^\nu V^\mu)^{bc}\right. \\
&\qquad +\frac{a-3}{12}(\theta^{\alpha\rho}F^a_{\alpha\sigma}+\theta_{\alpha\sigma}F^{a\alpha\rho})(2V^\mu V_\mu V_\rho V^\sigma+2V^\mu V_\rho V_\mu V^\sigma \\
&\qquad \left.+V^\mu V_\rho V^\sigma V_\mu+V_\rho V^\mu V_\mu V^\sigma)^{bc}\right],
\end{aligned}$$

$$
\begin{aligned}
\mathrm{D_4}^{\mathrm{div}} &= -\frac{\mathrm{i}}{2}\mathrm{Tr}\left(\left(\Box^{-1}N_2\right)\left(\Box^{-1}T_4\right)\right)^{\mathrm{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc}\int \mathrm{d}^4x\left[\frac{4-3a}{4}\theta^{\rho\sigma}F^a_{\rho\sigma}(V_\mu V_\nu V^\nu V^\mu)^{bc}\right. \\
&\quad + \frac{2-a}{2}(\theta^{\alpha\rho}F^a_{\alpha\mu}+\theta_{\alpha\mu}F^{a\alpha\rho})(V^\mu V^\nu V_\nu V_\rho)^{bc} \\
&\quad + i(2(a+1)\theta_{\alpha\nu}F^a_{\beta\mu}(V^\mu F^{\alpha\beta}V^\nu)^{bc}+\theta^{\alpha\beta}F^a_{\mu\nu}(V^\mu F_{\alpha\beta}V^\nu)^{bc}) \\
&\quad +2i\theta^{\alpha\beta}F^a_{\beta\mu}\Big(V_\nu F^{\mu\nu}V_\alpha - V^\mu F^{\alpha\beta}V^\nu\Big)^{bc} \\
&\quad -i\theta^{\alpha\beta}F^a_{\mu\nu}(V_\alpha F^{\mu\nu}V_\beta)^{bc} - 2N\theta^{\beta\mu}F^a_{\mu\nu}F^{b\alpha\nu}V^c_{\alpha\beta} \\
&\quad \left.+N\theta^{\mu\nu}F^a_{\alpha\mu}F^b_{\beta\nu}F^{c\alpha\beta} - a\frac{3}{4}N\theta^{\mu\nu}F^a_{\mu\nu}F^b_{\alpha\beta}F^{c\alpha\beta}\right], \\
\mathrm{D_5}^{\mathrm{div}} &= \frac{\mathrm{i}}{2}\mathrm{Tr}\left(\left(\Box^{-1}N_2\right)^2\left(\Box^{-1}T_2\right)\right)^{\mathrm{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int \mathrm{d}^4x\left[\frac{a-3}{2}(\theta_{\alpha\rho}F^{a\alpha\mu}+\theta^{\alpha\mu}F^a_{\alpha\rho})(F_{\mu\nu}F^{\nu\rho})^{bc}\right. \\
&\quad \left.+\theta^{\rho\sigma}F^a_{\rho\sigma}\left(\frac{3a-4}{4}(F_{\mu\nu}F^{\mu\nu})^{bc}+\frac{3a-7}{4}(V^\mu V_\mu V^\nu V_\nu)^{bc}\right)\right], \\
\mathrm{D_6}^{\mathrm{div}} &= \frac{\mathrm{i}}{2}\mathrm{Tr}\left(\left[\left(\Box^{-1}N_1\right)\left(\Box^{-1}N_2\right)+\left(\Box^{-1}N_2\right)\left(\Box^{-1}N_1\right)\right]\left(\Box^{-1}T_3\right)\right)^{\mathrm{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int \mathrm{d}^4x\left[\frac{a-3}{2}\theta^{\alpha\sigma}F^a_{\rho\sigma}(V_\alpha V_\beta V^\beta V^\rho)^{bc} - \frac{a}{4}\theta^{\rho\sigma}F^a_{\rho\sigma}(F_{\alpha\beta}F^{\alpha\beta})^{bc}\right. \\
&\quad - \theta_{\alpha\sigma}F^a_{\beta\rho}(a+1)\Big(2\mathrm{i}(V^\alpha F^{\rho\sigma}V^\beta)^{bc} - (F^{\rho\sigma}F^{\alpha\beta})^{bc}\Big) \\
&\quad + \frac{a}{2}\theta^{\beta\sigma}F^a_{\rho\sigma}(V_\alpha V^\alpha V_\beta V^\rho + V_\alpha V^\alpha V^\rho V_\beta)^{bc} \\
&\quad \left.+\theta^{\rho\sigma}F^a_{\rho\sigma}(\frac{3a-4}{4}V_\alpha V_\beta V^\beta V^\alpha + \frac{5a-4}{4}V_\alpha V^\alpha V_\beta V^\beta - 2V_\alpha V_\beta V^\alpha V^\beta)^{bc}\right] + \ldots,
\end{aligned}
$$

$$\begin{aligned}
\mathsf{D}_7^{\mathrm{div}} &= -\frac{\mathrm{i}}{2}\mathrm{Tr}\left(\sum\left(\Box^{-1}N_1\right)^2\left(\Box^{-1}N_2\right)\left(\Box^{-1}T_2\right)\right)^{\mathrm{div}} \\
&= \frac{h}{(4\pi)^2\epsilon}d^{abc} \\
&\times \int \mathrm{d}^4x\left[\frac{18-11a}{12}\theta^{\alpha\beta}F^a_{\alpha\beta}(2V^\mu V_\mu V^\nu V_\nu + V^\mu V^\nu V_\nu V_\mu)^{bc}\right. \\
&+ \left.\frac{3-a}{6}(\theta^{\alpha\mu}F^a_{\beta\mu}+\theta_{\beta\mu}F^{a\alpha\mu})(V_\alpha V^\beta V^\nu V_\nu + V^\nu V_\nu V^\beta V_\alpha + V_\alpha V^\nu V_\nu V^\beta)^{bc}\right].
\end{aligned}$$

$$\sum_{i=1}^{7}\mathsf{D}_i^{\mathrm{div}} = \frac{N}{(4\pi)^2\epsilon}h\theta^{\mu\nu}d^{abc}\int \mathrm{d}^4x\left(-\frac{25a-3}{48}F^a_{\mu\nu}F^b_{\rho\sigma}+\frac{a+21}{12}F^a_{\mu\rho}F^b_{\nu\sigma}\right)F^{c\rho\sigma},$$

Renormalization of the theory:

To cancel divergences, counter terms should be added to the starting action, which produces the bare Lagrangian

$$\begin{aligned}
\mathcal{L}_0 &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{11Ng^2}{6(4\pi)^2\epsilon}F^a_{\mu\nu}F^{a\mu\nu} \\
&+ \frac{1}{4}g\mu^{\epsilon/2}h\theta^{\mu\nu}d^{abc}\left(\frac{a}{4}F^a_{\mu\nu}F^b_{\rho\sigma} - F^a_{\mu\rho}F^b_{\nu\sigma}\right)F^{c\rho\sigma} \\
&- \frac{Ng^3\mu^{\epsilon/2}}{(4\pi)^2\epsilon}h\theta^{\mu\nu}d^{abc}\left(\frac{3-25a}{48}F^a_{\mu\nu}F^b_{\rho\sigma}+\frac{21+a}{12}F^a_{\mu\rho}F^b_{\nu\sigma}\right)F^{c\rho\sigma} \\
&= -\frac{1}{4}F_0{}^a_{\mu\nu}F_0{}^{a\mu\nu} + \frac{1}{4}g\mu^{\epsilon/2}h\theta^{\mu\nu}d^{abc} \\
&\times \left[\frac{a}{4}\left(1-\frac{3-25a}{3a}\frac{Ng^2}{(4\pi)^2\epsilon}\right)F^a_{\mu\nu}F^b_{\rho\sigma}\right. \\
&\quad \left. - \left(1+\frac{21+a}{3}\frac{Ng^2}{(4\pi)^2\epsilon}\right)F^a_{\mu\rho}F^b_{\nu\sigma}\right]F^{c\rho\sigma}.
\end{aligned}$$

To obtain the same structure as in starting Lagrangian we have to impose the condition

$$\left(\frac{25a-3}{48}\right):\left(-\frac{a+21}{12}\right)=\frac{a}{4}:(-1).$$

Solutions, $a=1$ and $a=3$.

The case $a=1$ correspondes to Model 1 : mNCSM; the deformation parameter h need not to be renormalized. Renormalization obtained through the renormalizations of gauge fields the coupling constant.

The case $a=3$ is different since the NC deformation parameter h has to be renormalized.

The bare gauge field, the coupling constant and the NC deformation parameter are :

$$\begin{aligned} V_0^\mu &= V^\mu\sqrt{1+\frac{22Ng^2}{3(4\pi)^2\epsilon}}, \\ g_0 &= \frac{g\mu^{\epsilon/2}}{\sqrt{1+\frac{22Ng^2}{3(4\pi)^2\epsilon}}}, \\ h_0 &= \frac{h}{1-\frac{2Ng^2}{3(4\pi)^2\epsilon}}, \end{aligned}$$

Ultraviolet asymptotic behaviour of NC SU(N) GFT via RGE

Gauge coupling constant g in our theory depends on energy i.e., the renormalization point μ, *satisfying the same beta function* as in QCD

$$\beta_g = \mu \frac{\partial}{\partial \mu} g(\mu) = -\frac{11 N g^3(\mu)}{3(4\pi)^2},$$

our theory is UV *stable, i.e.* $asymptotically\ free$:

$$\alpha_s(\mu) = \frac{g^2(\mu)}{4\pi} = \frac{6\pi}{11N} \frac{1}{\ln \frac{\mu}{\Lambda}}.$$

Λ is an integration constant not predicted by the theory; free parameter to be determined from experiment: hadronic production in e^+e^- annihilation at the Z resonance has given $\alpha_s(m_Z) = 0.12$ corresponding to $\Lambda = \Lambda_{\mathrm{QCD}} \simeq 250$ MeV.

$$\beta_h = \mu \frac{\partial}{\partial \mu} h(\mu) = -\frac{11 N g^2(\mu)}{24\pi^2} h(\mu).$$

Both β functions are *negative* $\rightarrow$ decrese with increasing energy μ. Solution to β_h:

$$h(\mu) = \frac{h_0}{\ln \frac{\mu}{\Lambda}}, \quad \Rightarrow$$

$running\ deformation\ parameter\ h$. h_0 is an additional integration constant, physical interpretation later.

By increase of μ the h decreases, $\Rightarrow$ modification of Heisenberg uncertainty relations at high energy

$$
\begin{aligned}
[x, p] &= i\hbar(1 + \beta p^2), \\
\Delta x &= \frac{\hbar}{2}\left(\frac{1}{\Delta p} + \beta \Delta p\right).
\end{aligned}
$$

Large momenta $\rightarrow$ distance Δx grows linearly: So large energies do not necessarily correspond to small distances. Running h does not imply that noncommutativity vanishes at small distances. Related to UV/IR correspondence. By assuming

$$
h(\mu) = \frac{1}{\Lambda_{\mathrm{NC}}^2(\mu)}
$$

Λ_{NC} becomes a function of energy μ too giving:

$$
\begin{aligned}
\mu \frac{d}{d\mu} \Lambda_{\mathrm{NC}}(\mu) &= \frac{11N}{3(4\pi^2)} g^2(\mu)\, \Lambda_{\mathrm{NC}}(\mu)\,, \\
\Lambda_{\mathrm{NC}}(\mu) &= \Lambda_\theta \sqrt{\ln \frac{\mu}{\Lambda}}.
\end{aligned}
$$

Λ_{NC} becomes the running scale of non-commutativity.
* Physical interpretation of h_0 and/or Λ_θ is not quite clear; they have to be proportional to the scale of noncommutativity Λ_{NC}.
* Assume that in a first approximation $h_0 = 1/\Lambda_\theta^2 = 1/\Lambda_{\mathrm{NC}}^2$.
* Considering typical QCD energies, $\mu = m_Z$, factor $\sqrt{\ln(m_Z/\Lambda_{\mathrm{QCD}})} \simeq 2.4$

Fermion sector Absence of 4ψ divergences for noncommutative chiral fermions

The 1-loop effective action is computed by using BFM

$$\begin{aligned}\Gamma^{(1)}_{\theta,2} &= \frac{\mathrm{i}}{2}\mathrm{STr}\log\left(\mathcal{I}+\Box^{-1}(N_1+N_2+T_1+T_2+T_3)\right)\\ &= \frac{\mathrm{i}}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\mathrm{STr}\left(\Box^{-1}N_1+\Box^{-1}T_1+\Box^{-1}T_2\right)^n .\end{aligned}$$

Divergent contributions comes from

$$\mathcal{D}_1=\mathrm{STr}\left((\Box^{-1}N_1)^3(\Box^{-1}T_1)\right)\ ,\ \ \mathcal{D}_2=\mathrm{STr}\left((\Box^{-1}N_1)^2(\Box^{-1}T_2)\right)\ .$$

Our computations shows that term $\mathcal{D}_1$in both, the U(1) and the SU(2), cases.

NC chiral electrodynamics, U(1), with Majorana spinors

$$\mathcal{D}_2^{\mathrm{U}(1)}|_{\mathrm{div}}=\frac{1}{(4\pi)^2\epsilon}\frac{3\mathrm{i}}{8}h\theta^{\mu\nu}\varepsilon_{\mu\nu\rho\sigma}(\bar\psi\gamma^\rho\gamma_5\psi)(\bar\psi\gamma^\sigma\gamma_5\psi)\equiv 0\,.$$

Chiral fermions in the fundamental representation of SU(2). Choosing Majorana spinors we apparently break the SU(2) symmetry. So we have to work in the framework of the components for the vector potential. Divergent part of $\mathcal{D}_2$ is

$$\begin{aligned}\mathcal{D}_2^{\mathrm{SU}(2)}|_{\mathrm{div}} &= \frac{-1}{(4\pi)^2\epsilon}\frac{9\mathrm{i}}{64}h\theta^{\mu\nu}\varepsilon_{\mu\nu\rho\sigma}(\bar\psi_1\gamma^\rho\gamma_5\psi_1+\bar\psi_2\gamma^\rho\gamma_5\psi_2)\\ &\quad\times(\bar\psi_1\gamma^\sigma\gamma_5\psi_1+\bar\psi_2\gamma^\sigma\gamma_5\psi_2),\end{aligned}$$

and it vanishes identically, too.

FORBIDDEN DECAYS

GAUGE SECTOR: $Z \to \gamma\gamma$ decay

[W. Behr, N. G. Deshpande, G. Duplančić, P. Schupp, J.T. and J. Wess; The Z $\to \gamma\gamma$, g g decays in the non-commutative standard model, Eur. Phys. J. C **29**, 441 (2003)]

[G. Duplančić, P. Schupp and J. Trampetić; Comment on triple gauge boson interactions in the non-commutative electroweak sector, Eur. Phys. J. C**32** (2003) 141]

[M. Buric, V. Radovanovic and J.T., The one-loop renormalization of the gauge sector in the noncommutative standard model; JHEP**03** (2007) 030]

[M. Buric, D. Latas, V. Radovanovic and J.T., Nonzero Z $\to \gamma\gamma$ decay in the renormalizable NCSM; Phys. Rev. **D 75**, 097701 (2007).]

From $\mathcal{L}_{Z\gamma\gamma} \Rightarrow$ the gauge-invariant amplitude $\mathcal{A}_{Z\to\gamma\gamma}$

$$
\begin{aligned}
\mathcal{A}^{\theta}(Z \to \gamma\gamma) &= -2e \sin 2\theta_W \, \mathrm{K}_{Z\gamma\gamma} \Theta_3^{\mu\nu\rho}(a; k_1, -k_2, -k_3) \\
&\times \epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(k_3);
\end{aligned}
$$

$k_1 + k_2 + k_3 = 0$;

$$
\begin{aligned}
\Theta_3^{\mu\nu\rho}(a; k_1, k_2, k_3) &= -(k_1\theta k_2) \\
& \times[(k_1 - k_2)^\rho g^{\mu\nu} + (k_2 - k_3)^\mu g^{\nu\rho} + (k_3 - k_1)^\nu g^{\rho\mu}] \\
& - \theta^{\mu\nu}\,[k_1^\rho\,(k_2k_3) - k_2^\rho\,(k_1k_3)] \\
& - \theta^{\nu\rho}\,[k_2^\mu\,(k_3k_1) - k_3^\mu\,(k_2k_1)] \\
& - \theta^{\rho\mu}\,[k_3^\nu\,(k_1k_2) - k_1^\nu\,(k_3k_2)] \\
& + (\theta k_2)^\mu\,\left[g^{\nu\rho}\,k_3^2 - k_3^\nu k_3^\rho\right] + (\theta k_3)^\mu\,\left[g^{\nu\rho}\,k_2^2 - k_2^\nu k_2^\rho\right] \\
& + (\theta k_3)^\nu\,\left[g^{\mu\rho}\,k_1^2 - k_1^\mu k_1^\rho\right] + (\theta k_1)^\nu\,\left[g^{\mu\rho}\,k_3^2 - k_3^\mu k_3^\rho\right] \\
& + (\theta k_1)^\rho\,\left[g^{\mu\nu}\,k_2^2 - k_2^\mu k_2^\nu\right] + (\theta k_2)^\rho\,\left[g^{\mu\nu}\,k_1^2 - k_1^\mu k_1^\nu\right] \\
& + \theta^{\mu\alpha}(ak_1 + k_2 + k_3)_\alpha\,\left[g^{\nu\rho}\,(k_3k_2) - k_3^\nu k_2^\rho\right] \\
& + \theta^{\nu\alpha}(k_1 + ak_2 + k_3)_\alpha\,\left[g^{\mu\rho}\,(k_3k_1) - k_3^\mu k_1^\rho\right] \\
& + \theta^{\rho\alpha}(k_1 + k_2 + ak_3)_\alpha\,\left[g^{\mu\nu}\,(k_2k_1) - k_2^\mu k_1^\nu\right] .
\end{aligned}
$$

Summations and averaging over that initial and final spins we compute branching ratio, i.e. from:

$$\sum_{\text{spins}} |\mathcal{A}_{Z\to\gamma\gamma}|^2 = -\theta^2 + (5a^2 - 22a + 25)\frac{(p\theta^2 p)}{M_Z^2} - (a^2 + 2a - 3)\frac{(k\theta^2 k + k'\theta^2 k')}{M_Z^2} - 4(a-3)(3a-5)\frac{(k\theta k')^2}{M_Z^4}.$$

we have for general a

Z-boson at rest

$$\Gamma_{Z\to\gamma\gamma} = \frac{\alpha}{12}\frac{M_Z^5}{\Lambda_{\rm NC}^4}\sin^2 2\theta_W \mathrm{K}^2_{Z\gamma\gamma} \times \frac{1}{6}\left[(13a^2 - 50a + 51)\vec{E}_\theta^2 + (a^2 + 2a + 3)\vec{B}_\theta^2\right],$$

$$\vec{E}_\theta = (\theta^{01}, \theta^{02}, \theta^{03}), \;\; \vec{B}_\theta = (\theta^{23}, \theta^{13}, \theta^{12})$$

Covariant generalization of the unitarity condition $\theta^{0i} = 0$ is

$$\theta_{\mu\nu}\theta^{\mu\nu} = -(\theta^2)^\mu_\mu = -\theta_{\mu\nu}\theta^{\nu\mu} = -\theta^2 = \frac{2}{\Lambda_{\rm NC}^4}\left(\vec{B}_\theta^2 - \vec{E}_\theta^2\right)$$

$$\equiv \frac{2}{\Lambda_{\rm NC}^4}\left(\sum_{i,j=1\,;\,i<j}^{3}(\theta^{ij})^2 - \sum_{i=1}^{3}(\theta^{0i})^2\right) > 0$$

known as *perturbative unitarity condition.*

For $a = 1$

Z-boson at rest

$$\Gamma_{Z\to\gamma\gamma} = \frac{\alpha}{12}\frac{M_Z^5}{\Lambda_{\rm NC}^4}\sin^2 2\theta_W \mathsf{K}_{Z\gamma\gamma}^2\left[\frac{7}{3}\vec{E}_\theta^2 + \vec{B}_\theta^2\right]$$

Z-boson at rest and polarized along the 3-axis

$$\Gamma_{Z^3\to\gamma\gamma} = \frac{\alpha}{4}\frac{M_Z^5}{\Lambda_{\rm NC}^4}\sin^2 2\theta_W\, \mathsf{K}_{Z\gamma\gamma}^2 \times\left[\frac{2}{5}\left((\theta^{01})^2 + (\theta^{02})^2\right) + \frac{23}{15}(\theta^{03})^2 + (\theta^{12})^2\right]$$

The same Lorentz structure of $\mathcal{L}_{Z\gamma\gamma}$ and $\mathcal{L}_{Zgg}$:

$$\frac{\Gamma_{Z\to gg}}{\Gamma_{Z\to\gamma\gamma}} = \frac{\Gamma_{Z^3\to gg}}{\Gamma_{Z^3\to\gamma\gamma}} = 8\frac{\mathsf{K}_{Zgg}^2}{\mathsf{K}_{Z\gamma\gamma}^2}.$$

The factor of eight in the above ratios is due to color.

For $a = 3$

Z-boson at rest

$$\Gamma_{Z\to\gamma\gamma} = \frac{\alpha}{4}\frac{M_Z^5}{\Lambda_{\rm NC}^4}\sin^2 2\theta_W \mathsf{K}_{Z\gamma\gamma}^2\left(\vec{B}_\theta^2 + \vec{E}_\theta^2\right)$$

$$\Gamma_{Z^3\to\gamma\gamma} = \frac{\alpha}{60}\frac{M_Z^5}{\Lambda_{\rm NC}^4}\sin^2 2\theta_W \mathsf{K}_{Z\gamma\gamma}^2\Big[\vec{B}_\theta^2 + \vec{E}_\theta^2 + 42\left((\theta^{12})^2 + (\theta^{03})^2\right)\Big]$$

Experiments

Decay mode: $Z \to \gamma\gamma$, old mesurements:

$$BR = \frac{\Gamma(Z \to \gamma\gamma)}{\Gamma_{tot}(Z)} \begin{cases} < 5.2 \times 10^{-5} & \text{L3} & 1995 \\ < 5.5 \times 10^{-5} & \text{DELPHI} & 1994 \\ < 1.4 \times 10^{-4} & \text{OPAL} & 1991 \end{cases}$$

$e^+e^- \to \gamma\gamma$ near Z resonance is an ideal process to test QED. The present statistic enables comparison of data with the QED up to $\mathcal{O}(\alpha^3)$.

Deviation of the experimentaly measured cross sections from the QED prediction $\longrightarrow$ evidence for $Z \to \gamma\gamma$ (SM forbidden) and $Z \to \pi^0\gamma\,/\,\eta\gamma$.

The forbidden decay $Z \to \gamma\gamma$ and the real decays $Z \to \pi^0\gamma\,/\,\eta\gamma$ would have the same experimental signature as the SM forbidden process

$$e^+e^- \to Z^* \to \gamma\gamma$$

Rare decays at high energies, the two photons from π^0 or η decays are very close seen in EM calorimeter as a single high energy photon:

$$e^+e^- \to Z^* \to (\pi^0, \eta)\gamma \to (\gamma\gamma)\gamma$$

Theoretical estimates $Br(Z \to \pi^0\gamma\,/\,\eta\gamma) \sim 10^{-10}$.
(Arnellos at al. Nucl.Phys.B 196 (1982) 378)

$Z \to \gamma\gamma$ LHC experimental possibilities:

CMS Physics Technical Design Report:

10^7 events of $Z \to e^+e^-$ for 10 fb^{-1} in 2 years of LHC

Assuming $BR(Z \to \gamma\gamma) \sim 10^{-8}$ and using $BR(Z \to e^+e^-) = 0.03 \Rightarrow \sim 3$ events of $Z \to \gamma\gamma$ with 10 fb^{-1}

Background sources (CMS Note 2006/112, Fig.3):
1. Study for $Higgs \to \gamma\gamma$ showes that, when e^- from $Z \to e^+e^-$ radiates very high energy Bremsstrahlung photon into pixel detector, for similar energies of e^- and γ, there is a huge probability of misidentification of e^- with γ !
2. Irreducible di-photon background may kill signal.

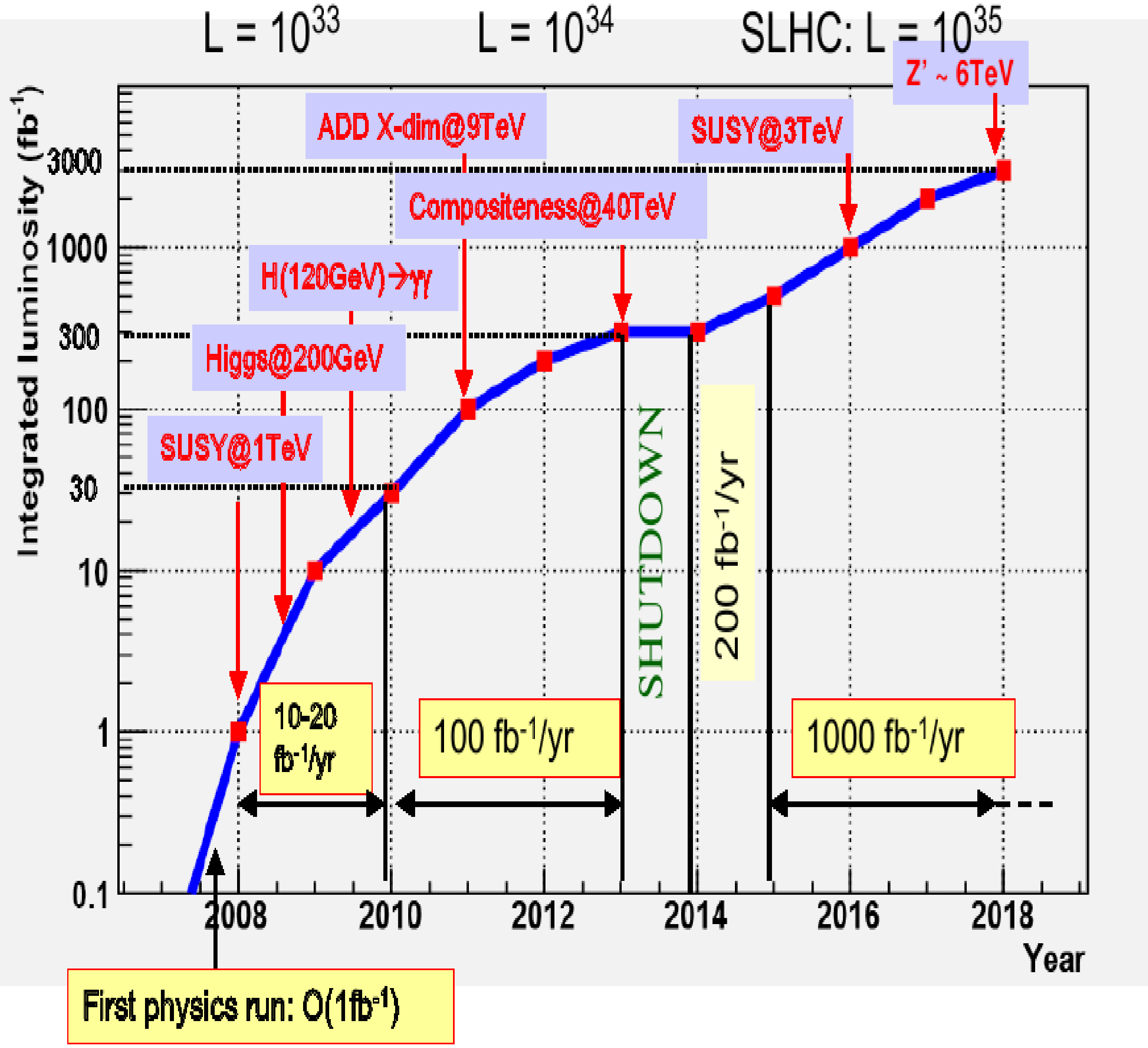
After 10 years of LHC running Int. L $\sim$ 1000 fb^{-1} and assuming $BR(Z \to \gamma\gamma) \sim 10^{-8}$

$\Rightarrow \sim$ 300 events of $Z \to \gamma\gamma$ decays, OR

$\Rightarrow \sim$ 3 events with $BR(Z \to \gamma\gamma) \sim 10^{-10}$

$\Rightarrow$ NC scale $\Lambda_{NC} \gtrsim 3.0$ TeV

Probable/possible LHC luminosity profile - need for L-upgrade in a longer term



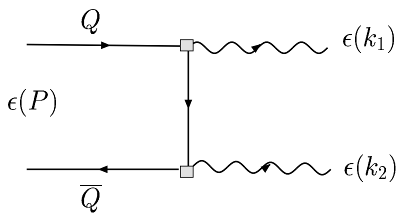
for the 2008 run likely to get from $100pb^{-1}$ to $1fb^{-1}$

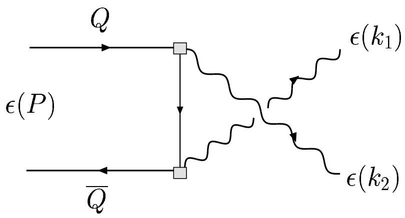
HADRON SECTOR

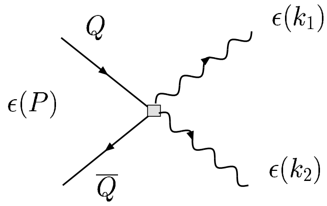
* NEUTRAL CURRENT DECAYS:

$\overline{Q}Q_{1^{--}}(J/\psi,\,\Upsilon) \to \gamma\gamma$

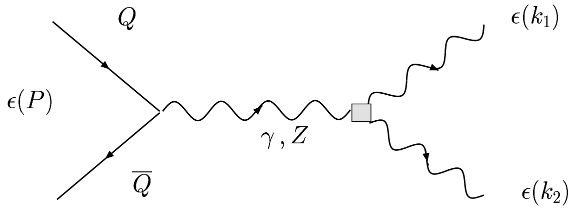
[B. Melic, K. Passek-Kumericki and J.T.; Quarkonia decays into two photons induced by the space-time noncommutativity, PRD **72** (2005) 054004]



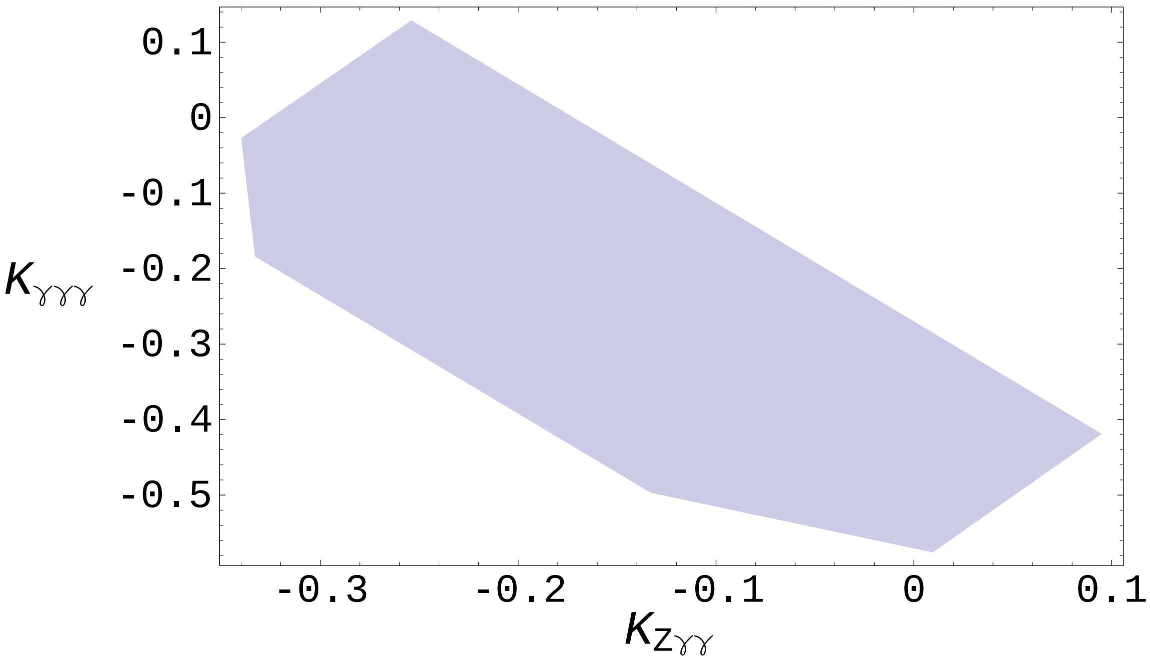




$$\mathcal{M}_{\text{mNCSM}} = i\,\pi\,4\sqrt{3\,M}\,\alpha\,e_Q^2|\Psi_{\overline{Q}Q}(0)|\,\epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(P)$$
$$\times\Big\{-(k_1-k_2)^\rho\left[\theta^{\mu\nu}-2g^{\mu\nu}\frac{(k_1\theta k_2)}{M^2}\right]$$
$$+2g^{\mu\rho}\left[(k_1\theta)^\nu-2k_1^\nu\frac{(k_1\theta k_2)}{M^2}\right]+2g^{\nu\rho}\left[(k_2\theta)^\mu+2k_2^\mu\frac{(k_1\theta k_2)}{M^2}\right]\Big\}$$



$$\mathcal{M}_{\text{nmNCSM}} = -\,i\,\pi\frac{16\sqrt{3\,M}}{M^2}\alpha|\Psi_{\overline{Q}Q}(0)|\,\epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(P)$$
$$\times\Theta_3((\mu,k_1),(\nu,k_2),(\rho,P))\left[e_Q\sin 2\theta_W K_{\gamma\gamma\gamma}+\left(\frac{M}{M_Z}\right)^2 c_V^Q K_{Z\gamma\gamma}\right]$$



The allowed region for the $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ coupling constants.

Hadronization: collinear quarks; annihilation and the WFO:

$$\langle 0|q_i^{\alpha}\overline{q}_j^{\beta}|\overline{Q}Q_{1^{--}}(P)\rangle = -\frac{|\Psi_{\overline{Q}Q}(0)|}{\sqrt{12M}}\,[(\not{P} + M)\not{\epsilon}]^{\alpha\beta}\,\delta_{ij},$$

$$|\Psi_{\overline{Q}Q}(0)|^2 = \frac{\Gamma(\overline{Q}Q_{1^{--}} \to \ell^+\ell^-)M^2}{16\pi\alpha^2 e_Q^2}$$

$$c_V^c = \frac{1}{2}\left(1 - \frac{8}{3}\sin^2\theta_W\right),\; c_V^b = -\frac{1}{2}\left(1 - \frac{4}{3}\sin^2\theta_W\right)$$

$$\Gamma^{\text{nmNCSM}}(\overline{Q}Q_{1^{--}} \to \gamma\gamma) = \frac{4\alpha^2\pi}{3}|\Psi_{\Upsilon}(0)|^2\frac{M^2}{\Lambda_{\text{NC}}^4}\left[7\vec{E}_\theta^2 + 3\vec{B}_\theta^2\right]$$

$$\times\left[\frac{e_Q^2}{2} - e_Q\sin 2\theta_W K_{\gamma\gamma\gamma} - (\frac{M}{M_Z})^2 c_V^Q K_{Z\gamma\gamma}\right]^2$$

Maximal noncommutativity for $\vec{E}_\theta^2 \simeq \vec{B}_\theta^2 \simeq 1$:

$$\frac{\Gamma^{\rm nmNCSM}(\overline{Q}Q_{1^{--}} \to \gamma\gamma)}{\Gamma(\overline{Q}Q_{1^{--}} \to \ell^+\ell^-)} = \frac{5}{24}\, e_Q^2 \left(\frac{M}{\Lambda_{\rm NC}}\right)^4$$
$$\times \left[1 - \frac{2}{e_Q} \sin 2\theta_W K_{\gamma\gamma\gamma} - \frac{2}{e_Q^2}\left(\frac{M}{M_Z}\right)^2 c_V^Q K_{Z\gamma\gamma}\right]^2$$

The range of the scale of non-commutativity:

$1 \geq \Lambda_{\rm NC}/\text{TeV} \geq 0.25$

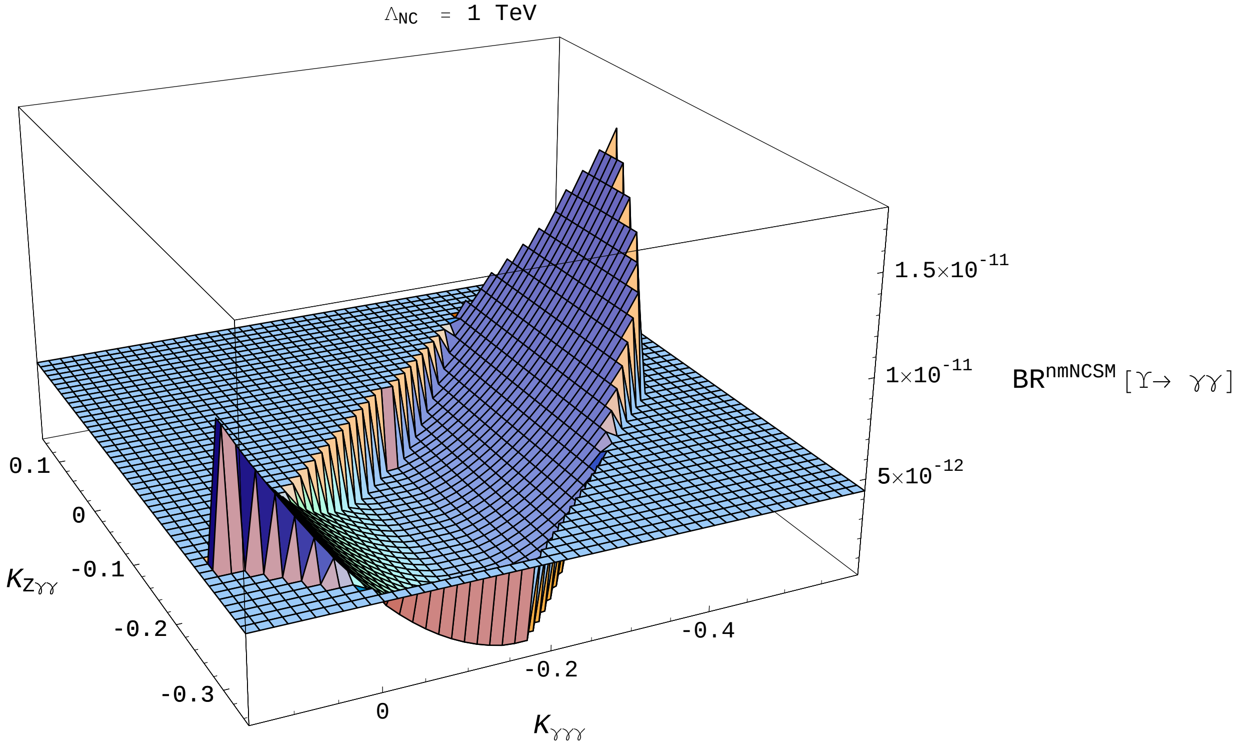
then gives

$$2 \times 10^{-10} \lesssim \frac{\Gamma^{\rm mNCSM}(\Upsilon(1S) \to \gamma\gamma)}{\Gamma(\Upsilon(1S) \to \ell^+\ell^-)} \lesssim 5 \times 10^{-8}$$

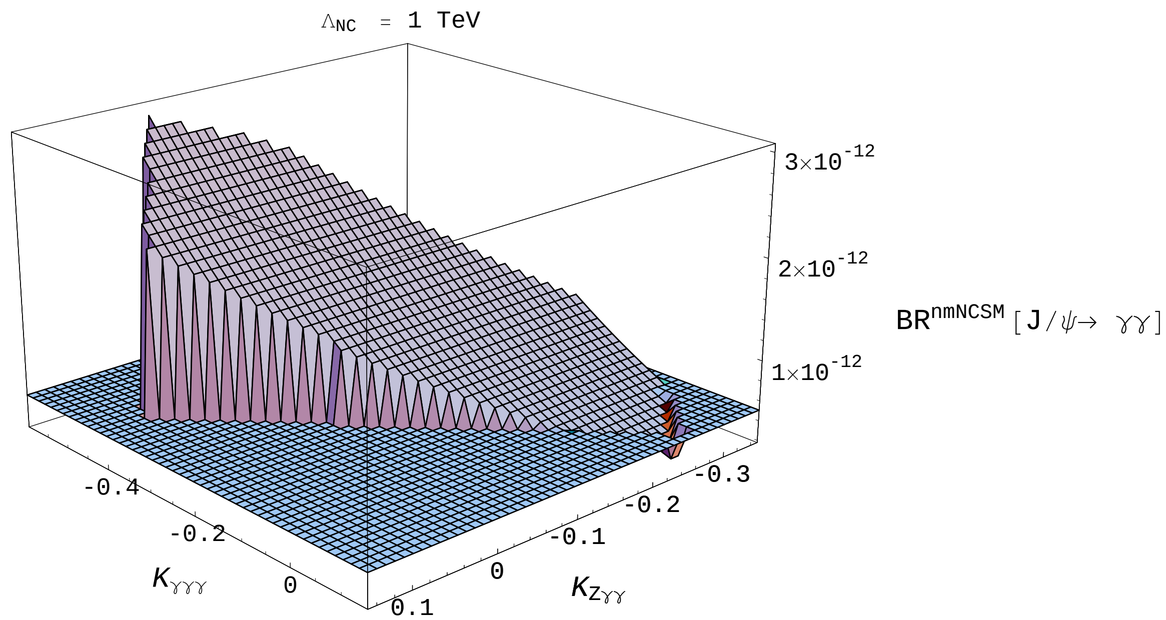
$$9 \times 10^{-12} \lesssim \frac{\Gamma^{\rm mNCSM}(J/\psi \to \gamma\gamma)}{\Gamma(J/\psi \to \ell^+\ell^-)} \lesssim 2 \times 10^{-9}$$

$$7 \times 10^{-10} \lesssim \frac{\Gamma^{\rm nmNCSM}(\Upsilon(1S) \to \gamma\gamma)}{\Gamma(\Upsilon(1S) \to \ell^+\ell^-)} \lesssim 2 \times 10^{-7}$$

$$5 \times 10^{-11} \lesssim \frac{\Gamma^{\rm nmNCSM}(J/\psi \to \gamma\gamma)}{\Gamma(J/\psi \to \ell^+\ell^-)} \lesssim 1 \times 10^{-8}$$



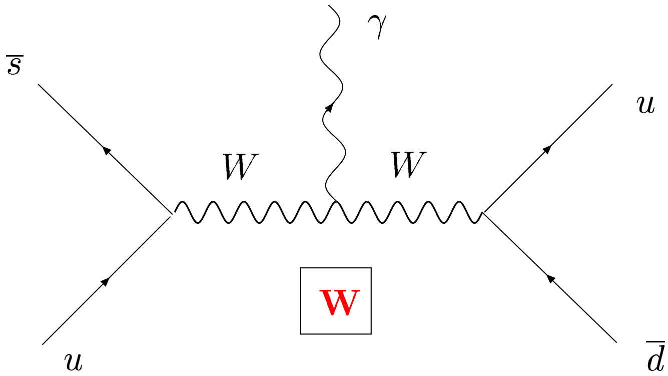
Branching ratio $BR^{\text{nmNCSM}}(\Upsilon \to \gamma\gamma)$ as a function of $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ coupling constants, at the scale of non-commutativity $\Lambda_{\text{NC}} = 1$ TeV. The horizontal plane at the value of 4.7×10^{-12} indicates the $BR^{\text{mNCSM}}(\Upsilon \to \gamma\gamma)$, which one obtains by setting $K_{\gamma\gamma\gamma} = K_{Z\gamma\gamma} = 0$.



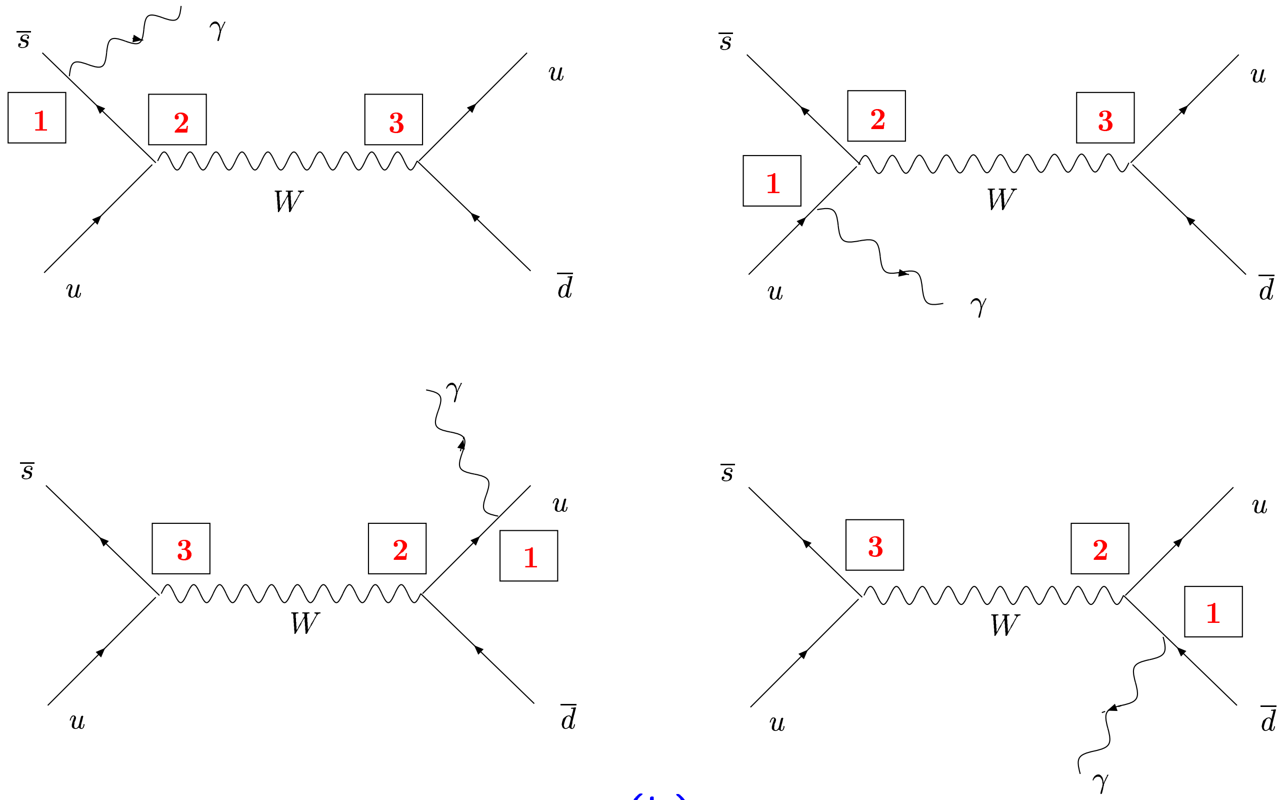
Branching ratio $BR^{\text{nmNCSM}}(J/\psi \to \gamma\gamma)$ as a function of $K_{\gamma\gamma\gamma}$ and $K_{Z\gamma\gamma}$ coupling constants, at the scale of non-commutativity $\Lambda_{\text{NC}} = 1$ TeV. The horizontal plane at the value of 5.1×10^{-13} indicates the $BR^{\text{mNCSM}}(J/\psi \to \gamma\gamma)$, which one obtains by setting $K_{\gamma\gamma\gamma} = K_{Z\gamma\gamma} = 0$.

* CHARGED CURRENT DECAYS: $K \to \pi\gamma$,...

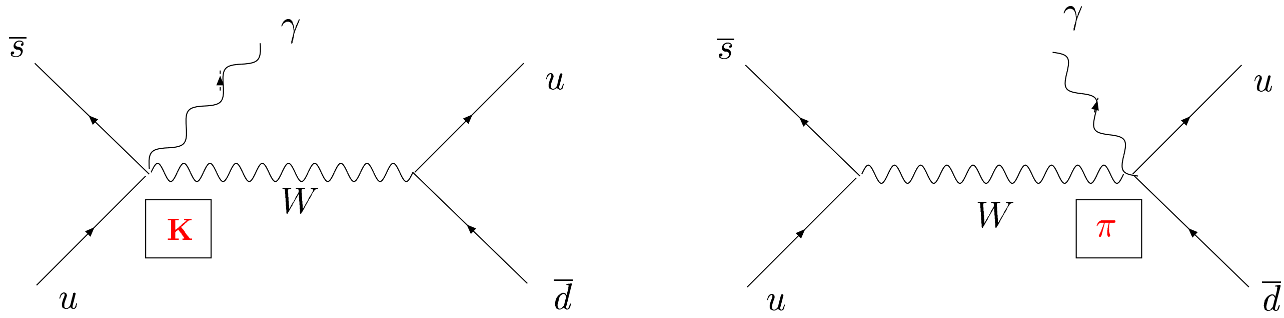
[B. Melic, K. Passek-Kumericki and J.T.; K → pi gamma decay and space-time noncommutativity, Phys. Rev. D **72** (2005) 057502]



(a)



(b)



(c)

Total free quark amplitude : $\mathcal{M} = (\mathcal{M}^{\rm SM}_{(a+b)} + \mathcal{M}^{\theta}_{(a+b+c)})_\mu\, \varepsilon^\mu(q)\,.$

Hadronization: collinear quarks, the VSA and the PCAC:

$$\langle\pi|J^\dagger_\mu J^\mu|K\rangle = \langle\pi|J^\dagger_\mu|0\rangle\langle 0|J^\mu|K\rangle,\ \ \langle\pi^+(p)|\bar u\gamma_\mu\gamma_5 d|0\rangle = -ip_\mu f_\pi$$

Total amplitude for the $K^+ \to \pi^+\gamma$ decay:

$$\begin{aligned}
\mathcal{A}(K^+\to\pi^+\gamma) &= \langle\pi^+(p)|(\mathcal{M}^{\rm SM}_{(a+b)} + \mathcal{M}^{\theta}_{(a+b+c)})_\mu|K^+(k)\rangle\,\varepsilon^\mu(q) \\
&= \frac{e\,G_F}{4\sqrt{2}}V_{ud}V^\dagger_{us}f_\pi f_K\left(\mathcal{A}^{\rm SM}_{(a+b)} + i\mathcal{A}^{\theta}_{(a+b+c)}\right)_\mu \varepsilon^\mu(q) \\
\mathcal{A}^{\rm SM}(K^+\to\pi^+\gamma) &= \frac{e\,G_F}{4\sqrt{2}}V_{ud}V^\dagger_{us}f_\pi f_K\left(\mathcal{A}^{\rm SM}_{(a+b)}\right)_\mu \varepsilon^\mu(q) = 0
\end{aligned}$$

Gauge and Lorentz invariance in SM satisfied!

$$\begin{aligned}
\left(\mathcal{A}^\theta_{(a)}\right)_\mu &= 2\left[k^2(\theta p)_\mu - p^2(\theta k)_\mu - 2(q\theta k)k_\mu\right] \\
\left(\mathcal{A}^\theta_{(b)}\right)_\mu &= \frac{kp}{kq}(Q_u+Q_s)\Big((q\theta k)k_\mu - (kq)(\theta k)_\mu\Big) \\
&\quad -\frac{kp}{kq}(Q_u+Q_d)\Big((q\theta k)p_\mu - (kq)(\theta p)_\mu\Big) \\
&\quad - R_\pi\,(Q_u-Q_s)(kq)(\theta p)_\mu + i\,R_\pi\,(Q_u+Q_s)\epsilon_{\mu\nu\rho\tau}q^\nu(\theta p)^\rho k^\tau \\
&\quad + R_K\,(Q_u-Q_d)(kq)(\theta k)_\mu + i\,R_K\,(Q_u+Q_d)\epsilon_{\mu\nu\rho\tau}q^\nu(\theta k)^\rho p^\tau \\
\left(\mathcal{A}^\theta_{(c)}\right)_\mu &= (Q_u+Q_d)\Big(p^2(\theta k)_\mu - (kp)(\theta p)_\mu + (q\theta k)p_\mu\Big) \\
&\quad - (Q_u+Q_s)\Big(k^2(\theta p)_\mu - (kp)(\theta k)_\mu - (q\theta k)k_\mu\Big) \\
&\quad + 2(m_dQ_u+m_uQ_d)\frac{p^2(\theta k)_\mu}{m_d+m_u} - 2(m_sQ_u+m_uQ_s)\frac{k^2(\theta p)_\mu}{m_s+m_u}
\end{aligned}$$

$$R_\pi = \frac{p^2}{kq}\frac{m_d-m_u}{m_d+m_u},\quad R_K = \frac{k^2}{kq}\frac{m_s-m_u}{m_s+m_u}$$

VERY IMPORTANT : $k = p+q\,,\ \ Q_u = \frac{2}{3},\ \ Q_d = Q_s = -\frac{1}{3}$

Gauge invariance for $\mathcal{A}^\theta = \left(\mathcal{A}^\theta_{(a+b+c)}\right)_\mu \varepsilon^\mu(q)$ in NCSM satisfied!

$$\left(\mathcal{A}_W^\theta\right)_\mu = 2\left[k^2(\theta p)_\mu - p^2(\theta k)_\mu - 2(q\theta k)k_\mu\right]$$
$$\left(\mathcal{A}_1^\theta\right)_\mu = -\left(Q_u + Q_d\right)(kp)(\theta q)_\mu$$
$$\left(\mathcal{A}_2^\theta\right)_\mu = 2\left(Q_u + Q_d\right)(q\theta k)k_\mu$$
$$\left(\mathcal{A}_3^\theta\right)_\mu = -R_\pi\left(Q_u - Q_d\right)(kq)(\theta p)_\mu + i\,R_\pi(Q_u + Q_d)\epsilon_{\mu\nu\rho\tau}q^\nu(\theta p)^\rho k^\tau$$
$$+R_K\left(Q_u - Q_d\right)(kq)(\theta k)_\mu + iR_K\left(Q_u + Q_d\right)\epsilon_{\mu\nu\rho\tau}q^\nu(\theta k)^\rho p^\tau$$
$$\left(\mathcal{A}_\pi^\theta\right)_\mu = \left(Q_u + Q_d\right)\left(p^2(\theta k)_\mu - (kp)(\theta p)_\mu + (q\theta k)p_\mu\right)$$
$$+2(m_d Q_u + m_u Q_d)\frac{p^2(\theta k)_\mu}{m_d + m_u}$$
$$\left(\mathcal{A}_K^\theta\right)_\mu = -\left(Q_u + Q_d\right)\left(k^2(\theta p)_\mu - (kp)(\theta k)_\mu - (q\theta k)k_\mu\right)$$
$$-2(m_s Q_u + m_u Q_d)\frac{k^2(\theta p)_\mu}{m_s + m_u}$$

Total amplitude for the $K^+ \to \pi^+\gamma$ decay in the NCSM:

$$\mathcal{A}^\theta(K^+ \to \pi^+\gamma) = \frac{ie\,G_F}{4\sqrt{2}}V_{ud}V_{us}^\dagger f_\pi f_K$$
$$\times\Big[2\left(1 - (Q_u + Q_d)\right)\left(k^2(\theta p)_\mu - p^2(\theta k)_\mu - (q\theta k)k_\mu\right)$$
$$+(Q_u - Q_d)(R_\pi + R_K)(kq)(\theta q)_\mu$$
$$+i\left(Q_u + Q_d\right)\epsilon_{\mu\nu\rho\tau}q^\nu\left(R_\pi(\theta p)^\rho + R_K(\theta k)^\rho\right)k^\tau\Big]\varepsilon^\mu(q)$$

Maximal noncommutativity for $\vec{E}_\theta^2 \simeq \vec{B}_\theta^2 \simeq 1$:

$$BR(K^+ \to \pi^+\gamma) = \tau_{K^+}\Gamma(K^+ \to \pi^+\gamma)$$
$$\simeq \frac{\alpha\tau_{K^+}}{128}G_F^2 f_\pi^2 f_K^2 |V_{ud}V_{us}^\dagger|^2\frac{m_K^5}{\Lambda_{\rm NC}^4}\left(1 - \frac{m_\pi^2}{m_K^2}\right)\left[1 - \frac{50}{27}\frac{m_\pi^2}{m_K^2} + \frac{25}{27}\frac{m_\pi^4}{m_K^4}\right]$$
$$\simeq 0.8 \times 10^{-16}\ \left(1\ \text{TeV}/\Lambda_{\rm NC}\right)^4,$$

τ_{K^+} is the K^+ meson mean life. The other interesting modes could easily be found from:

$$BR(K^+ \to \pi^+\gamma) : BR(D_s^+ \to \pi^+\gamma)$$
$$: BR(D^+ \to \pi^+\gamma) : BR(B^+ \to \pi^+\gamma)$$
$$\simeq 1 : 2.40 : 0.20 : 0.01.$$

Discusion: HADRON SECTOR

The range of the scale of non-commutativity:

$$1 \geq \Lambda_{NC}/\text{TeV} \geq 0.25$$

Quarkonia decays: $J/\psi \to \gamma\gamma$, $\Upsilon(1S) \to \gamma\gamma$

$\Gamma^{\text{exp.}}(\Upsilon(1S) \to e^+e^-) = (1.314 \pm 0.029)\,\text{keV}$
$\Gamma^{\text{exp.}}_{\text{tot}}(\Upsilon(1S)) = (53.0 \pm 1.5)\,\text{keV}$
$\Gamma^{\text{exp.}}(J/\psi \to e^+e^-) = (5.4 \pm 0.15 \pm 0.07)\,\text{keV}$
$\Gamma^{\text{exp.}}_{\text{tot}}(J/\psi) = (91.0 \pm 3.2)\,\text{keV}$

$$5 \times 10^{-12} \lesssim BR^{\text{mNCSM}}(\Upsilon(1S) \to \gamma\gamma) \lesssim 10^{-9}$$
$$5 \times 10^{-13} \lesssim BR^{\text{mNCSM}}(J/\psi \to \gamma\gamma) \lesssim 10^{-10}$$
$$2 \times 10^{-11} \lesssim BR^{\text{nmNCSM}}(\Upsilon(1S) \to \gamma\gamma) \lesssim 4 \times 10^{-9}$$
$$3 \times 10^{-12} \lesssim BR^{\text{nmNCSM}}(J/\psi \to \gamma\gamma) \lesssim 8 \times 10^{-10}$$

Flavor changing decays: $K^+ \to \pi^+\gamma$

$$0.8 \times 10^{-16} \lesssim BR(K^+ \to \pi^+\gamma) \lesssim 2.0 \times 10^{-14}$$

Note the experimental result for similar process

$$BR(K^+ \to \pi^+\nu\bar{\nu}) \simeq 1.6 \times 10^{-10}$$

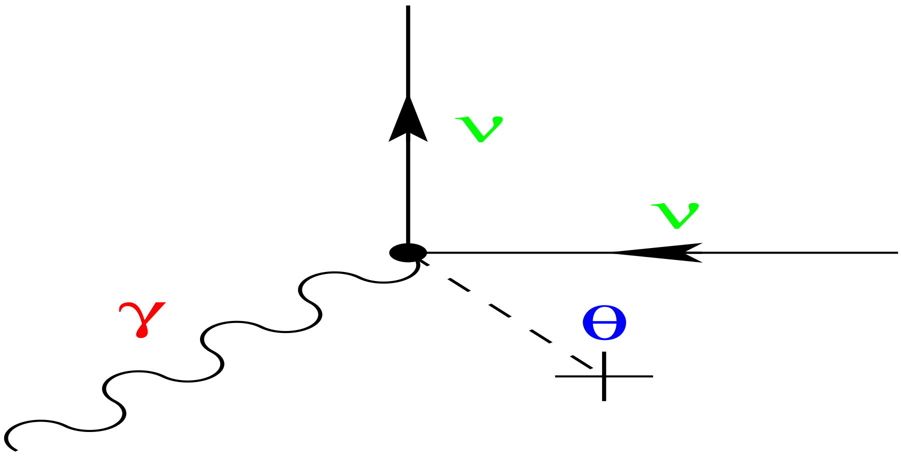
Brookhaven experiments (at 90%CL):

E787 (2002) $\longrightarrow$ $BR(K^+ \to \pi^+\gamma) < 3.6 \times 10^{-7}$
E949 (2005) $\longrightarrow$ $BR(K^+ \to \pi^+\gamma) < 2.3 \times 10^{-9}$

NEUTRINO SECTOR

“Transverse plasmon” decay: $\gamma_{\rm pl} \to \nu\bar{\nu}$

[P. Schupp, J.Trampetic, J. Wess and G. Raffelt, “The photon neutrino interaction in non-commutative gauge field theory and astrophysical bounds,” Eur. Phys. J. C **36** (2004) 405]



Neutrino-photon interaction introduced via:
$\star$–commutator with covariant derivative

$$\widehat{D}_\mu\widehat{\psi} = \partial_\mu\widehat{\psi} - i\kappa e\left[\widehat{A}_\mu \star \widehat{\psi} - \widehat{\psi} \star \widehat{A}_\mu\right]$$

The action for a neutral fermion that couples, in the adjoint of non-commutative U(1), to an Abelian gauge boson in the NC background is:

$$S = \int d^4x \left(\bar{\widehat{\psi}} \star i\gamma^\mu \widehat{D}_\mu\widehat{\psi} - m\bar{\widehat{\psi}} \star \widehat{\psi}\right)$$

SW map: $\widehat{\psi} = \psi + e\theta^{\nu\rho}A_\rho\partial_\nu\psi + \mathcal{O}(\theta^2)$
$\widehat{A}_\mu = A_\mu + e\theta^{\rho\nu}A_\nu\left[\partial_\rho A_\mu - \frac{1}{2}\partial_\mu A_\rho\right] + \mathcal{O}(\theta^2)$

The gauge invariant action of order θ^1 and $\kappa = 1$

$$S = \int d^4x\, \bar{\psi}\, \Big[(i\gamma^\mu \partial\mu - m) - \frac{e}{2} F_{\mu\nu}(i\theta^{\mu\nu\rho}\partial\rho - \theta^{\mu\nu} m)\Big]\, \psi .$$

Feynman rule for $\gamma(q) \to \nu(k')\bar{\nu}(k)$ vertex:

$$\Gamma^\mu_{\binom{\mathrm{L}}{\mathrm{R}}}(\nu\bar{\nu}\gamma) = ie\frac{1}{2}(1 \mp \gamma_5)\left[(q\theta k)\gamma^\mu + (\not{k} - m_\nu)(\theta q)^\mu - \not{q}(\theta k)^\mu\right].$$

For massless neutrinos the vertex becomes totally symmetric:

$$\Gamma^\mu_{\binom{\mathrm{L}}{\mathrm{R}}}(\nu\bar{\nu}\gamma) = ie\frac{1}{2}(1 \mp \gamma_5)\left(\theta^{\mu\nu}\gamma^\tau + \theta^{\nu\tau}\gamma^\mu + \theta^{\tau\mu}\gamma^\nu\right) k_\nu q_\tau$$

In a stellar plasma, the dispersion relation of photons is identical with that of a massive particle

$$q^2 \equiv \mathrm{E}_\gamma^2 - \mathbf{q}_\gamma^2 = \omega_{\mathrm{pl}}^2$$

ω_{pl} – the plasma frequency.
The plasmon (off-shell photon) decay rate to the left and/or right massive neutrinos

$$\Gamma_{\mathrm{NC}}(\gamma_{\mathrm{pl}} \to \bar{\nu}_{\binom{\mathrm{L}}{\mathrm{R}}}\nu_{\binom{\mathrm{L}}{\mathrm{R}}}) = \frac{\alpha}{48}\frac{\omega_{\mathrm{pl}}^6}{\mathrm{E}_\gamma \Lambda_{\mathrm{NC}}^4}\sqrt{1 - 4\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}} \times \left[\left(1 + 2\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2} - 12\frac{m_\nu^4}{\omega_{\mathrm{pl}}^4}\right)\sum_{i=1}^{3}(c^{0i})^2 + 2\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}\left(1 - 4\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}\right)\sum_{\substack{i,j=1\\ i<j}}^{3}(c^{ij})^2\right].$$

In the rest frame of the medium and for massless neutrinos the decay rate is

$$\Gamma_{\mathrm{NC}}(\gamma_{\mathrm{pl}} \to \nu_{\binom{\mathrm{L}}{\mathrm{R}}}\bar{\nu}_{\binom{\mathrm{L}}{\mathrm{R}}}) = \frac{\alpha}{48}\frac{1}{\Lambda_{\mathrm{NC}}^4}\frac{\omega_{\mathrm{pl}}^6}{\mathrm{E}_\gamma}$$

The corresponding SM neutrino-penguin-loop rate:

$$\Gamma_{\mathrm{SM}}\left(\gamma_{\mathrm{pl}} \to \nu_{\mathrm{L}}\bar{\nu}_{\mathrm{L}}\right) = \frac{c_{\mathrm{V}}^2 G_{\mathrm{F}}^2}{48\pi^2\alpha}\frac{\omega_{\mathrm{pl}}^6}{\mathrm{E}_\gamma}.$$

For ν_e: $\quad c_{\mathrm{V}} = \frac{1}{2} + 2\sin^2\Theta_{\mathrm{W}}$

$\nu_\mu,\ \nu_\tau$: $\quad c_{\mathrm{V}} = -\frac{1}{2} + 2\sin^2\Theta_{\mathrm{W}}$

For the SM: $c_{\mathrm{V}}^2 = 0.79$.

$$\mathcal{R} \equiv \frac{\sum_{\mathrm{flavours}}\Gamma_{\mathrm{NC}}\left(\gamma_{\mathrm{pl}} \to \nu_{\mathrm{L}}\bar{\nu}_{\mathrm{L}} + \nu_{\mathrm{R}}\bar{\nu}_{\mathrm{R}}\right)}{\sum_{\mathrm{flavours}}\Gamma_{\mathrm{SM}}(\gamma_{\mathrm{pl}} \to \nu_{\mathrm{L}}\bar{\nu}_{\mathrm{L}})} = \frac{6\pi^2\alpha^2}{c_{\mathrm{V}}^2 G_{\mathrm{F}}^2\Lambda_{\mathrm{NC}}^4},$$

$$\Lambda_{\mathrm{NC}} = \frac{80.8}{\mathcal{R}^{1/4}}\,(\mathrm{GeV}).$$

A standard globular cluster stars argument:

any new energy-loss mechanism must not exceed the standard neutrino losses by much:

$\longrightarrow$ approximate requirement: $\mathcal{R} < 1$ $\longrightarrow$

$$\longrightarrow \quad \Lambda_{\mathrm{NC}} > \left(\frac{6\pi^2\alpha^2}{c_{\mathrm{V}}^2 G_{\mathrm{F}}^2}\right)^{1/4} \cong 81\ \mathrm{GeV}$$

Neutrino $\star$-dipole moments and $\star$-charge radii: $(\mathrm{d}^{\mathrm{el}}_{\mathrm{mag}})_{\mathrm{NC}}$, $\langle r^2_\nu \rangle_{\mathrm{NC}}$

[Peter Minkowski, Peter Schupp and Josip Trampetic, "Neutrino dipole moments and charge radii in noncommutative space-time," Eur. Phys. J. C **37** (2004) 123]

$\nu_i \longrightarrow \nu_j + \gamma$ transitions, generated through 1-loop electroweak "neutrino–penguin" diagrams: the exchange of $\ell = e, \mu, \tau$ and $W^\pm, Z$:

$$
\begin{aligned}
J^{\mathrm{eff}}_\mu(\gamma\nu\bar{\nu})\epsilon^\mu(q) &= \Big\{F_1(q^2)\bar{\nu}_j(p')(\gamma_\mu q^2 - q_\mu \not{q})\nu_i(p)_L \\
&- iF_2(q^2)\Big[m_{\nu_j}\bar{\nu}_j(p')\sigma_{\mu\nu}q^\nu \nu_i(p)_L \\
&+ m_{\nu_i}\bar{\nu}_j(p')\sigma_{\mu\nu}q^\nu\nu_i(p)_R\Big]\Big\}\epsilon^\mu(q).
\end{aligned}
$$

General decomposition of the second term

$$
\mathrm{T} = -i\epsilon^\mu(q)\bar{\nu}(p')\left[A(q^2) - B(q^2)\gamma_5\right]\sigma_{\mu\nu}q^\nu\nu(p),
$$

gives the electric and magnetic dipole moments

$$
d^{\mathrm{el}}_{ji} \equiv B(0) = \frac{-e}{M^{*2}}\left(m_{\nu_i} - m_{\nu_j}\right)\sum_{\ell=e,\mu,\tau} \mathrm{U}^\dagger_{jk}\mathrm{U}_{ki}F_2\left(\frac{m^2_{\ell_k}}{m^2_W}\right),
$$

$$
\mu_{ji} \equiv A(0) = \frac{-e}{M^{*2}}\left(m_{\nu_i} + m_{\nu_j}\right)\sum_{\ell=e,\mu,\tau} \mathrm{U}^\dagger_{jk}\mathrm{U}_{ki}F_2\left(\frac{m^2_{\ell_k}}{m^2_W}\right),
$$

where $i, j, k = 1, 2, 3$ denotes neutrino species, and

$$
F_2\left(\frac{m^2_{\ell_k}}{m^2_W}\right) \simeq -\frac{3}{2} + \frac{3}{4}\frac{m^2_{\ell_k}}{m^2_W}, \quad \frac{m^2_{\ell_k}}{m^2_W} \ll 1,
$$

$M^* = 4\pi\, v = 3.1$ TeV;
$v = (\sqrt{2}\, G_F)^{-1/2} = 246$ GeV, VEV for Higgs field.

Dirac neutrino: $i = j$; $\mu_{ii} \equiv \mu_{\nu_i}$. ($m_\nu = 0.05\,\text{eV}$):

$$
\begin{aligned}
\mu_{\nu_i} &= \frac{3e}{M^{*2}} m_{\nu_i} \left[1 - \frac{1}{2} \sum_{\ell=e,\mu,\tau} \frac{m_\ell^2}{m_W^2} |\mathsf{U}_{\ell i}|^2 \right] \\
&\simeq 1.56 \times 10^{-26} [\text{e/eV}] = 0.29 \times 10^{-30}\, [\text{e cm}] \\
&= 1.60 \times 10^{-20} \mu_B.
\end{aligned}
$$

in units of [e cm] and Bohr magneton [μ_B], Chirality flip arises only from the neutrino masses.

Majorana neutrino: No particle–antiparticle distinction ($\psi_i = \psi_i^c$); one has to use both charged lepton and antilepton propagators in the loop calculation of neutrino-penguin diagrams $\to$ the first term in F_2 vanishes in the summation over ℓ due to the orthogonality condition of U (GIM cancellation)

$$
\begin{aligned}
d^{\text{el}}_{\bar{\nu}_j \nu_i} &= \frac{3e}{2M^{*2}} \left(m_{\nu_i} - m_{\nu_j} \right) \sum_{\ell=e,\mu,\tau} \frac{m_{\ell_k}^2}{m_W^2} \mathsf{U}^\dagger_{jk} \mathsf{U}_{ki}, \\
\mu_{\bar{\nu}_j \nu_i} &= \frac{3e}{2M^{*2}} \left(m_{\nu_i} + m_{\nu_j} \right) \sum_{\ell=e,\mu,\tau} \frac{m_{\ell_k}^2}{m_W^2} \mathsf{U}^\dagger_{jk} \mathsf{U}_{ki}.
\end{aligned}
$$

Transition matrix element T is a complex antisymmetric quantity in lepton-flavor space:

$$\begin{aligned} \mathsf{T}_{\mathsf{ji}} &= -i\epsilon^{\mu}\bar{\nu}_j\left[(A_{ji}-A_{ij})-(B_{ji}-B_{ij})\gamma_5\right]\sigma_{\mu\nu}q^{\nu}\nu_i \\ &= -i\epsilon^{\mu}\bar{\nu}_j\left[2\mathrm{iIm}A_{ji}-2\mathrm{Re}B_{ji}\gamma_5\right]\sigma_{\mu\nu}q^{\nu}\nu_i. \end{aligned}$$

Explicitly clear that for $i=j$, $d^{\mathrm{el}}_{\nu_i}=\mu_{\nu_i}=0$.
The above first (second) term vanishes if the relative CP of ν_i and ν_j is even (odd).

$$\begin{aligned} d^{\mathrm{el}}_{\nu_i\nu_j} &= \frac{3e}{2M^{*2}}\left(m_{\nu_i}-m_{\nu_j}\right)\sum_{\ell=e,\mu,\tau}\frac{m^2_{\ell_k}}{m^2_W}\mathrm{Re}\mathsf{U}^{\dagger}_{jk}\mathsf{U}_{ki}, \\ \mu_{\nu_i\nu_j} &= \frac{3e}{2M^{*2}}\left(m_{\nu_i}+m_{\nu_j}\right)\sum_{\ell=e,\mu,\tau}\frac{m^2_{\ell_k}}{m^2_W}\mathrm{i\,Im}\mathsf{U}^{\dagger}_{jk}\mathsf{U}_{ki}, \end{aligned}$$

Majorana case: mixing matrix U is approximatively unitary

$$\sum_{i=1}^{3}\mathsf{U}^{\dagger}_{jk}\mathsf{U}_{ki}=\delta_{ji}-\varepsilon_{ji},$$

where ε is a hermitian nonnegative matrix

$$\begin{aligned} |\varepsilon|=\sqrt{\mathrm{Tr}\ \varepsilon^2} &= \mathcal{O}\left(m_{\nu_{\mathrm{light}}}/m_{\nu_{\mathrm{heavy}}}\right), \\ &\sim 10^{-22}\ \text{to}\ 10^{-21}. \end{aligned}$$

The case $|\varepsilon|=0$ is excluded by the very existence of oscillation effects.

The neutrino dipole moments violate lepton number by ± 2 and for a general neutrino mass matrix, they independently violate CP.

The corresponding analytic structure is quite definite, globally referred to as the see–saw mechanism.

Assuming hierarchical structure:
$|m_3 + m_2| \simeq |m_3 - m_2| \simeq |\Delta m^2_{32}|^{1/2} = 0.05$ eV.
Seting: $|\mathrm{Re}U^\dagger_{3\tau}U_{\tau 2}| \simeq |\mathrm{Im}U^\dagger_{3\tau}U_{\tau 2}| \leq 0.5$.
The electric and magnetic transition dipole moments denoted as $\left(d^{\mathrm{el}}_{\mathrm{mag}}\right)_{23}$ are

$$\begin{aligned}
\left|\left(d^{\mathrm{el}}_{\mathrm{mag}}\right)_{23}\right| &= \frac{3e}{2M^{*2}}\frac{m_\tau^2}{m_W^2}\sqrt{|\Delta m^2_{32}|}\binom{|\mathrm{Re}U^\dagger_{3\tau}U_{\tau 2}|}{|\mathrm{Im}U^\dagger_{3\tau}U_{\tau 2}|}, \\
&\lesssim 2.03\times 10^{-30}[\mathrm{e/eV}] = 0.38\times 10^{-34}\,[\mathrm{e\ cm}] \\
&= 2.07\times 10^{-24}\,\mu_B < \left|\left(d^{\mathrm{el}}_{\mathrm{mag}}\right)_{\mathrm{d-quark}}\right|
\end{aligned}$$

To extract an upper limit on the $\star$-gradient interaction we compare the strength $|m_\nu\, e\, \kappa\, \theta\, F|$ with the dipole transition interactions $|F\, d^{\mathrm{el}}_{\mathrm{mag}}|$ for Dirac/Majorana cases. Assuming that contributions from the neutrino-mass extended SM are at least as large as those from noncommutativity, for $\kappa = 1$ we derive the following bound:

$$|\Lambda_{\mathrm{NC}}|^{\mathrm{Dirac}}_{\mathrm{Majorana}} \gtrsim \left|\frac{e\,\kappa\, m_\nu}{\left(d^{\mathrm{el}}_{\mathrm{mag}}\right)^{\mathrm{Dirac}}_{\mathrm{Majorana}}}\right|^{1/2} \simeq \binom{1.80}{150}\ \mathrm{TeV}.$$

This is the main result of our considerations which on the scale of noncommutativity involves only the basic properties of neutrinos and photons.

Radius of the photon–neutrino interaction: Neutrino $\star$-charge radii $r^{*2} = \langle r_\nu^2 \rangle_{\rm NC}$

Noncommutativity can be a source of "transvers plasmon" decay into neutrino–antineutrino pairs. This is to be compared with the same process induced by the neutrino charge radii defined by the axial electromagnetic interaction form factor in the neutrino-mass extended SM:

$$\langle r_\nu^2 \rangle = 6 \left[\frac{\partial F_1(q^2)}{\partial q^2} \right]_{q^2=0} ; \left[F_1(q^2) \right]_{q^2 \to 0} \longrightarrow \frac{q^2}{6} \langle r_\nu^2 \rangle ,$$

which in the limit of massless neutrinos corresponds to

$$\langle r_{\nu_\ell}^2 \rangle \cong \frac{2}{M^{*2}} \left(3 - 2 \log \frac{m_\ell^2}{m_W^2} \right) = \frac{G_F}{\sqrt{2}\,\pi^2} \left(\frac{3}{4} + \log \frac{m_W}{m_\ell} \right).$$

We estimate the charge radii in the SM from by taking $\ell = e$: $\sqrt{|\langle r_{\nu_e}^2 \rangle|_{\rm SM}} \simeq 6.4 \times 10^{-17}$ [cm]. Here we remark that astrophysical estimates give interesting bounds. These calculations should implement all neutrino flavor properties. The so derived bounds may also help in establishing the Majorana nature of light neutrinos.

To estimate the $\star$-charge radii we first evaluate the SM rate induced by the charge radii:

$$\sum_{\ell=e,\mu,\tau} \Gamma_{\rm SM}(\gamma \to \bar\nu_\ell^{\rm L} \nu_\ell^{\rm L}) = \frac{\alpha}{144} \frac{q^6}{E_\gamma} \sum_{\ell=e,\mu,\tau} \left| \langle r_{\nu_\ell}^2 \rangle \right|^2 ,$$

Plasmon at rest $q^2 = E_\gamma^2 = \omega_{\rm pl}^2$.
Average of the plasmon frequencies of red-giant and white-dwarf stars $\omega_{\rm pl} = 15$ keV gives

$$\Gamma^{-1}_{\rm SM}(\gamma \to \bar\nu\nu) = \left(\frac{1\,{\rm keV}}{\omega_{\rm pl}}\right)^5 \times 0.25 \times 10^{13}\ {\rm years}$$
$$\simeq 3 \times 10^6\ {\rm years};$$

Compare this with astrophysical observations.

Off-shell photon to massless Majorana neutrinos NC decay rate in the rest system of medium:

$$\sum_{\ell=e,\mu,\tau} \Gamma_{\rm NC}(\gamma \to \bar\nu_\ell^{\rm L}\nu_\ell^{\rm L}) = \frac{\alpha}{16}\frac{\kappa^2 q^6}{{\rm E}_\gamma \Lambda^4_{\rm NC}} \sum_{i=1}^{3} (c^{0i})^2$$

and $\Gamma_{\rm SM}(\gamma \to \bar\nu_\ell^{\rm L}\nu_\ell^{\rm L}) \gtrsim \Gamma_{\rm NC}(\gamma \to \bar\nu_\ell^{\rm L}\nu_\ell^{\rm L})$ gives the range of noncommutativity via the $\star$–charge radii:

$$r^* = \sqrt{|\langle r_\nu^2\rangle_{\rm NC}|} = \frac{\sqrt{\sqrt{3}\,\kappa}}{\Lambda_{\rm NC}}.$$

For $|\Lambda_{\rm NC}|^{\rm Dirac}_{\rm Majorana} = \binom{1.8}{150}$ TeV and $\kappa = 1$

$$r^*|^{\rm Dirac}_{\rm Majorana} \lesssim \binom{1.4 \times 10^{-17}}{1.6 \times 10^{-19}}[{\rm cm}]; \quad \binom{\simeq \frac{1}{5}\sqrt{|\langle r_{\nu_e}^2\rangle|_{\rm SM}}}{\rm unobservable}.$$

The $\star$–induced charge radii r^* at the $\Lambda_{\rm NC} \gtrsim 150$ TeV, is dominated by the neutrino-mass extended standard model physics, and is practically unobservable.

Discusion: NEUTRINO SECTOR

* From the energy loss in the globular stellar clusters, requirement $\longrightarrow \mathcal{R} < 1 \quad \longrightarrow$
– The constraint $\Lambda_{NC} > 80$ GeV, represents the lower bound on the NC scale – Λ_{NC}
* By comparing SM and NCSM neutrino electric and magnetic moments we found

$$|\Lambda_{NC}|^{\text{Dirac}}_{\text{Majorana}} \gtrsim \left| \frac{e\,\kappa\, m_\nu}{\left(d^{\text{el}}_{\text{mag}}\right)^{\text{Dirac}}_{\text{Majorana}}} \right|^{1/2} \simeq \binom{1.80}{150}\ \text{TeV}.$$

– Above bounds depend on the NC coupling κ and are based on different laboratory.

– In this way we can “understand” neutrinos as particles which manifest themselfs as Majorana objects at the very short distances and/or very high energies.

– We believe that the difference between Dirac and Majorana, produced by standard model physics, points toward the right direction for the determination of the real nature of neutrinos.

DISCUSION

Limits on Λ_{NC} from theory and experiment

DECAYS: $1 \to 2$

* $Z \to \gamma\gamma \quad \Rightarrow \Lambda_{NC} > \binom{110}{1000}$ GeV, ([Duplančić,...] [Burić,...])
* $\gamma_{pl} \to \nu\bar{\nu} \quad \Rightarrow \Lambda_{NC} >$ 81 GeV, [Schupp, JT, Wess, Raffelt]
* $J/\psi \to \gamma\gamma \quad \Rightarrow \Lambda_{NC} >$ 9 GeV, [Melic, Passek, J.T.]
* $K \to \pi\gamma \quad \Rightarrow \Lambda_{NC} >$ 43 GeV, [Melic, Passek, J.T.]

SCATTERINGS: $2 \to 2$

* $e^+e^- \to \gamma\gamma \Rightarrow \Lambda_{NC} >$ 141 GeV, [OPAL Coll. (2003)]
* $\gamma\gamma \to \bar{f}f \quad \Rightarrow \Lambda_{NC} >$ 200 GeV, [T. Ohl et al.]
* $\bar{f}f \to Z\gamma \quad \Rightarrow \Lambda_{NC} >$ 1000 GeV, [T. Ohl et al.]

NEUTRINO DIPOLE MOMENTS:

* $(d_{mag})^{Dirac} \quad \Rightarrow \Lambda_{NC} >$ 1.8 TeV, [Minkowski et al.]
* $(d^{el}_{mag})^{Majorana} \Rightarrow \Lambda_{NC} >$ 150 TeV, [Minkowski et al.]

SUMMARY

⋆ Principle of renormalizability implemented on our θ-expanded NCGFT led us to well defined deformations via introduction of higher order NC gauge action class for mNCSM, nmNCSM and NC SU(N) models. This extension was parametrized by generically free parameter a.

⋆ Divergences cancel differently than in commutative GFT and this depends on the representations.

⋆ Model 1: mNCSM gauge sector is renormalizable for $a = 1$. No renormalizaion of h. Only renormalization of fields and couplings necessary.

⋆ Model 2: nmNCSM gauge sector is renormalizable and FINITE for $a = 3$. No renormalizaion of h.

⋆ Model 3: NC SU(N) theory is renormalizable only for $a = 1, 3$. See above.

− Case $a = 3$, requires renormalization of NC deformation parameter h, which becomes *the running deformation parameter and vanishes for large* μ.

− Λ_{NC} runs too and it is very smooth, ⇒ small change when μ increases ⇒ large degree of stability of NC SU(N) theory within a wide range of μ.

⋆ Our computations shows that for NC chiral electrodynamics the (U(1) case with Majorana spinors), the 4ψ divergent part vanishes identically.

⋆ For NC chiral fermions in the fundamental representation of SU(2) with Majorana spinors we break the SU(2) symmetry. However, the 4ψ divergent parts vanishes identically, too.

CONCLUSION

⋆ Renormalization principle is fixing the freedom parameter $a = 1, 3$ for our θ-expanded NC GFT :

$$S_g = -\frac{1}{2}\mathrm{Tr}\int d^4x \left(1 + i(a-1)\,\widehat{x}^{\mu}\star\widehat{x}^{\nu}\star\widehat{F}_{\mu\nu}\right)\star\widehat{F}_{\rho\sigma}\star\widehat{F}^{\rho\sigma}.$$

This way principle of renormalization determines NC renormalizable deformation.

⋆ The solution $a = 3$, while shifting the model to the higher order, hints into the discovery of the key role of the higher NC gauge interaction in 1-loop renormalizability of classes of NCGFT at θ^1.

⋆ Hence, the nmNCSM gauge sector, which produces SM forbidden $Z \to \gamma\gamma$ decay, is renormalizable and FINITE $\to$ no renormalizaion of h needed.

⋆ Hence, in the case of NC SU(N) the noncommutativity deformation parameter h had to be renormalized and it is asymptotically free.

⋆ Adding to ϕ^4 NCGFT $\Omega\int d^4x\,\widehat{x}\star\widehat{x}\star\widehat{\phi}\star\widehat{\phi}$, renormalization determines NC deformation up to all orders.

⋆ NC chiral fermions: for SU(2) and U(1) models NO typical 4ψ divergence, as for Dirac fermions.

⋆ We beleive that all above could be of paramounth importance in modifying fermion and Higgs matter sectors so that it becomes renormalizable.

⋆ Phenomenological results as $Z \to \gamma\gamma$ are ROBUST due to the 1-loop renormalizability and FINITNESS of the nmNCSM gauge sector.