Renormalizable nc QFT

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Introduction

- Wightman QFT
- Euclidean formulation
- Space-Time

Deformed Minkowski Space-Time

- model independent: Wedge Locality H G + Lechner

Deformed Euclidean Space-Time Matter Fields

- Scalar fields, modified actions perturbative renormalization H G + Wulkenhaar, Vignes-Tourneret
- Fermions: spectral triple

Conclusions

Wightman QFT

- H separable Hilbert space
- strongly continous unitary rep. U of Poincaré group on H
  joint spectrum $\sigma(P_\mu)$ in closed forward light cone
- $\Omega \in H$ invariant under U
- $S(R^d) \ni f \longmapsto \phi(f)$ op.-valued distr. with domain D,
- Covariance:

$$U(y,\Lambda)\phi(f)U(y,\Lambda)^{-1} = \phi(f^{(y,\Lambda)}) \, , \qquad f^{(y,\Lambda)}(x) = f(\Lambda^{-1}(x-y))$$

- Locality

$$[\phi(f),\phi(g)]_\pm \Psi = 0 \qquad \text{for} \qquad suppf \subset (suppg)'$$

- Define Wighman functions

$$W_n(f_1 \otimes ... \otimes f_n) :=< \Omega\phi(f_1)\cdots\phi(f_n)\Omega >$$

Euclidean QFT

- Define Schwinger functions
  $S_N(z_1, ..., z_N) = \int \Phi(z_1)...\Phi(z_N) d\nu(\Phi)$
  $d\nu = \frac{1}{Z} e^{-\int L_{int}(\Phi)} d\mu(\Phi)$, $d\mu$ is reg. Gauss measure
- deduce Feynman rules,
  needs regularization - renormalization
- IR, UV AND convergence problem use RG FLOW
- Renormalons, Landau ghost, trivial Higgs model?
- add "Gravity" or deform Space-Time

Project

merge general relativity with quantum physics through noncommutative geometry

Space-Time Concepts

in Newton gr., QM, ED, GR, QFT and QG ?
Limited localisation of events in space-time

$$D \geq R_{ss} = \frac{G}{c^4}\frac{hc}{\lambda} \geq \frac{G}{c^4}\frac{hc}{D} \tag{1}$$

- gives Planck lenght as a lower bound: Doplicher, Fredenhagen, Roberts 1994

- Connes: Noncommutative Geometry
- Replace manifold by algebra deform it
- keep differential calculus derivations....
- Replace fields by projective modules
- Replace integrals by traces
- use renormalized perturbation expansion

Deform Minkowski

H G Gandalf Lechner, AQFT: Buchholz and Summers
DFR Quantum conditions:

$$[\hat{x}^\mu, \hat{x}^\nu] = iQ^{\mu\nu}, \ [\hat{x}^\mu, Q^{\sigma\tau}] = 0$$
$$Q_{\mu\nu}Q^{\mu\nu} = c_1 \ \epsilon^{\mu\nu\sigma\tau}Q_{\mu\nu}Q_{\sigma\tau} = c_2$$

Field operators are defined as tensor product:

$$\Phi^{\otimes}(x) = \int d\mu_p (e^{ipx} e^{ip\hat{x}} \otimes \Phi_p)$$

Vacuum states: take: $\omega_\theta = \nu \otimes < \Omega, .\Omega >$
ω_θ is independent on ν
GNS rep. of pol algebra build from $\Phi^{\otimes}(f)$
wrt ω_θ yields

$$H_\theta = H, \qquad \Omega_\theta = \Omega\,, \qquad \pi_\theta(\phi^{\otimes}(f)) =: \phi^\theta(f)$$

$$\Psi_n(f_1 \otimes ... \otimes f_n) := \phi(f_1)...\phi_n(f_n)\Omega \phi^\theta(f)\Psi_n(G) = \Psi_{n+1}(f \otimes_\theta G)$$

Deform Wightman QFT

Definition (Moyal tensor product)

$f_n \in S(R^{nd})$, $g_m \in S(R^{md})$

$$(f_n \widetilde{\otimes_\theta g}_m)(p,q) := \prod_{l=1}^{n} \prod_{r=1}^{m} e^{-\frac{i}{2} p_l \theta q_r} \tilde{f}_n(p)\, \tilde{g}_m(q)$$

$$\Leftrightarrow (f_n \otimes_\theta g_m)(x,y) = \pi^{-d} \int d^d\xi \int d^d k\, f_n^{\xi}(x)\, g_m^{\theta k}(y)\, e^{-2i\xi\cdot k}$$

Deformation similar to that of Balachandran et.al., Chaichian et.al, Soloviev

$$\tilde{\omega}_\theta(p_1,\dots,p_n) := \langle \Omega, \tilde{\phi}^\theta(p_1)\cdots\tilde{\phi}^\theta(p_n)\Omega\rangle = \tilde{\omega}_0(p_1,\dots,p_n)\cdot \prod_{l<r} e^{-\frac{i}{2} p_l \theta p_r}$$

strongly continuous commutative limit $\theta \to 0$

algebraic formulation of deformation $\phi \to \phi^\theta$ possible for quite general operators

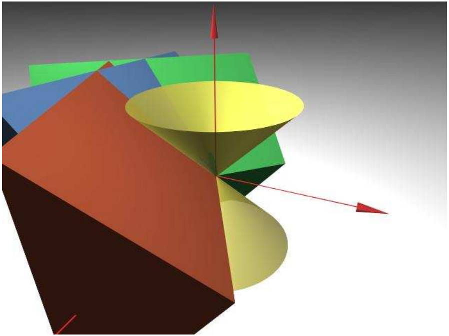
Transformation behavior of deformed quantum fields

- ϕ^θ acts on Hilbert space H of the undeformed theory $\Rightarrow$ consider cov. prop. of ϕ^θ w.r.t. undeformed rep. U
- $$U(y,\Lambda)\phi^\theta(f)U(y,\Lambda)^{-1} = \phi^{\pm\Lambda\theta\Lambda^T}(f^{(y,\Lambda)})$$
- Consider family of fields
- Choose θ in $d = 4$ dimensions

$$\theta_1 = \begin{pmatrix} 0 & \kappa_e & 0 & 0 \\ -\kappa_e & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa_m \\ 0 & 0 & -\kappa_m & 0 \end{pmatrix}$$

Wedges

We relate the antisymmetric matrices to Wedges:
$W_1 = \left\{ x \in \mathbb{R}^D | x_1 > |x_0| \right\}$ act on standard wedge by proper Lorentz transformations $i_\Lambda(W) = \Lambda W$. Stabilizer group is $SO(1,1)xSO(2)$, which corresponds to boosts and rotations.



Wedges and Wedge local QF

$\mathcal{A} = \{\gamma_\Lambda(\theta_1) | \Lambda \in \mathcal{L}_+\}$, $\theta(\Lambda W_1) := \gamma_\Lambda(\theta_1) = \Lambda\theta_1\Lambda^\dagger$
We define wedge local fields through: $\phi = \{\phi_W | W \subset \mathcal{W}_0\}$ get family of fields, covariance and localization in wedges.

$$U_{y,\Lambda}\Phi_W(x)U^\dagger_{y,\Lambda} = \Phi_{\gamma_\Lambda(Q(W))}(\Lambda x + y)$$

Theorem

Let $\kappa_e \geq 0$ the family $\Phi_W(x)$ is a wedge local quantum field:

$$[\phi_{W_1}(f), \phi_{-W_1}(g)](\psi) = 0,$$

for $supp(f) \subset W_1$, $supp(g) \subset -W_1$.

Proof relies on spectrum condition and support properties

DeformEuclideanΦ^4_θ

Formulation

ϕ^4 on nc $\mathbb{R}^4$, $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$ antisymmetric,
or equivalently star product

$$(a * b)(x) = \int dy \int dk a(x + \frac{\theta k}{2}) b(x + y) e^{iky}$$

differential calculus,....

ϕ^4 action

$$S = \int dp (p^2 + m^2) \phi_p \phi_{-p} + \lambda \int \prod_{j=1}^{4} (dp_j \phi_{p_j}) \, \delta(\sum_{j=1}^{4} p_j) e^{-i \sum_{i<j} p_i \theta p_j}$$

Feynman rules

cyclic order of momenta leads to ribbon graphs

Model is not renormalizable

One possible solution: modify action

Theorem

H. G. and R. Wulkenhaar ϕ^4 model modified,
IR/UV mixing: short and long distances related
Theorem: Action

$$S=\int d^4x\Big(\frac{1}{2}\partial_\mu\phi\star\partial^\mu\phi+\frac{\Omega^2}{2}(\tilde{x}_\mu\phi)\star(\tilde{x}^\mu\phi)+\frac{\mu_0^2}{2}\phi\star\phi+\lambda\phi\star\phi\star\phi\star\phi\Big)(x)$$

for $\tilde{x}_\mu := 2(\theta^{-1})_{\mu\nu}x^\nu$
is perturbativly renormalizable to all orders in λ, 3 proofs,

- Rivasseau et al: Multiscale analysis: matrix base and position space
- Action has Langmann-Szabo position-momentum duality
  $S[\phi;\mu_0,\lambda,\Omega]\mapsto\Omega^2 S[\phi;\frac{\mu_0}{\Omega},\frac{\lambda}{\Omega^2},\frac{1}{\Omega}]$
- oscillator term is curvature !...... Buric, Wohlgenannt

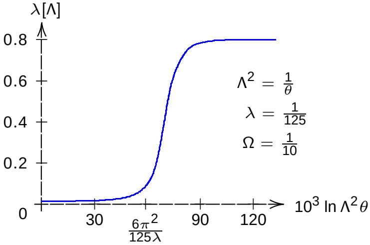
Taming Landau Ghost

evaluate β function, H. G. and R. Wulkenhaar,

$$\beta_\lambda = c\lambda^2 \frac{(1-\Omega^2)}{(1+\Omega^2)^3} + \mathcal{O}(\lambda^3)$$

flow bounded, L. ghost killed!
Due to wave fct. renormalization
$\Omega = 1$ betafunction vanishes
to all orders!

$\lambda[\Lambda]$ diverges in comm. case



- perturbation theory remains valid at all scales!
- non-perturbative construction of the model possible!

Gurau, Magnen, Rivasseau, Tanasa new ren.m: add $\int \Phi^2 \frac{\alpha}{p^2}$

Degenerate Deformation

H G + F Vignes-Tourneret, $rank(\theta) = 2$, action:

$$S_0[\Phi] = \int d^4x \Phi(x)(-\Delta + \Omega^2 \tilde{x}^2_{3,4} + m^2)\Phi(x) + \lambda \int d^4x \Phi^{*4}(x)$$

$$S_\kappa[\Phi] = S_0[\Phi] + \frac{\kappa^2}{\theta^2} \int d^2p \tilde{\Phi}(p,0)\tilde{\Phi}(-p,0)$$

This QFT is ren. to all orders in pertubation theory
Topology of Ribbon graphs: genus g, B broken faces
$B = 1$ regular graph, $B > 1$ nonregular
use multiscale analysis, slice propagator ....
Power counting: The degree of convergence of a Feynman graph is given by $N(G) - 4 + 4g + 2B - 2$
Conclusion: Planar regular and irregular 2 point fcts have to be renormalized
Four point fct: Planar regular graphs are divergent.

A spectral triple

H. G. and Raimar Wulkenhaar,
Take Dirac operator on Hilbert space $L^2(R^4) \otimes C^{16}$

$$D_8 = (i\Gamma^\mu \partial_\mu + \Omega \Gamma^{\mu+4} \tilde{x}_\mu)$$

$\mu = 1, ...4$, Γ_k generate 8-dim Clifford algebra $\{\Gamma_k \Gamma_l\} = 2\delta_{kl}$

$$D_8^2 = (-\Delta + \Omega^2 ||\tilde{x}||^2) 1 - i\Omega \theta^{-1}_{\mu\nu} [\Gamma^\mu, \Gamma^{\nu+4}]$$

compute action of Dirac operator on sections of spinor bundle

$$[D_8, f] * \psi = i[\Gamma^\mu + \Omega \Gamma^{\mu+4}](\partial \mu f) * \psi$$

only 4 dim. differential appears
leads to spectral triple
configuration space dimension 4, phase space dim. 8

- nc Gross-Neveu model ren. by F Vignes-Tourneret

Conclusions

- There exists a general deformation scheme
- Fields on deformed Minkowski space-time show wedge locality
- principles?

Matter Fields

- Graphs couple internal and external momenta
  gives IR divergence not renormalizable
- modified actions yield renormalizable models
- RG flow save
- fermions give spectral triple
- further models: add nonlocal term
- generalization to gauge models? Gravity???