Emergent Geometry

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Based on work with:
Rodrigo Delgadillo-Blando (Cinvestav, Mexico) and
Badis Ydri (Algeria)
PRL 100,201601 (2008) (arXiv:0712.3011) and JHEP 2009 (arXiv:0806.0558)

Introduction

It now seems possible to address questions such as

- Is the dimensionality of spacetime fixed or dynamical?
- Are spacetime geometry and topology inputs or outputs of the dynamics?

One can at least make models where spacetime emerges from more primitive structures.

Older Ideas

An old idea is that Einstein gravity and the Einstein Hilbert action were induced effects of matter propagating on a predetermined background. But here the metric is already prescribed. It is the dynamics that is induced.

Causal Sets

One very appealing idea is that a discrete causal structure is sufficient to determine the geometry. This is essentially true for classical Minkowski signature geometry. However the quantization is difficult and natrually leads one to search for extensions of quantum mechanics. So at a quantal level progress has been slow.

See X. Martin, D. O'Connor and R.D. Sorkin, *Phys. Rev.* **D71** 024029 2005.

Geometry from Random matrices.

This idea is that discrete triangulations of random surfaces can be mapped to random matrices. The random matrices then describe the surgace and its gravitational fluctuations. Unfortunately, this appears to be a very Euclidean approach. But it is based on random matrix theory and so falls into the same circle of ideas as I will discuss.

In the simplest example, the idea here is that $\mathcal{N} = 4$ supersymmetric Yang-Mills in four dimensional Minkowski space at weak coupling behaves like a 4-dimensional Yang-Mills theory. However, at strong coupling it behaves as a 10-dimensional supergravity theory. Therefore effectively growing 6 extra dimensions, with gravitational fluctuations.
I hope to shed a little more light on how these extra dimensions emerge at the end of the lecture.

The Simplest Matrix Model

Consider the Gaussian probability distribution

$$\mathcal{P}(\Phi) = \frac{e^{-bTr(\Phi^2)}}{Z}$$

where $Z = \int [d\Phi] e^{-bTr(\Phi^2)}$.
This distribution splits into the normalized Riemannian measure on $SU(N)/U(1)^N$ and a probability distribution for the eigenvalues of Φ. The latter converges in the large N limit to the Wigner semi-circle distribution

$$\rho(\lambda) = \frac{b}{\pi}\sqrt{\frac{2N}{b} - \lambda^2}.$$

Unitary invariant random matrix models are typically characterised by the eigenvalue distribution of the eigenvalues of the random matrix.

- The eigenvalues repell one another.
- The eigenvalue distribution consists of a series of cuts.
- The spread in eigenvalues grows as $\sqrt{N}$.
- Phase transitions occur when cuts merge or separate.

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A pure matrix model

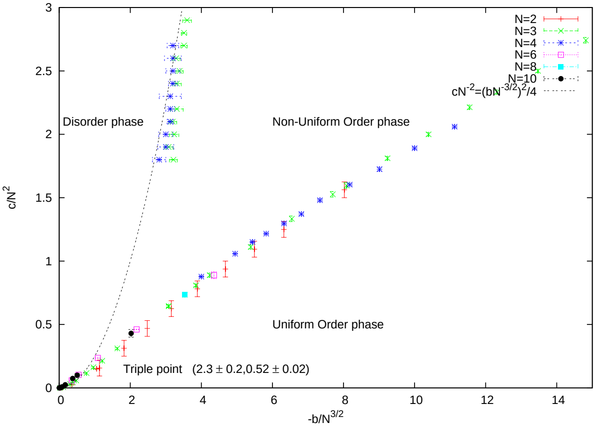
$V(\phi) = Tr(b\phi^2 + c\phi^4)$ with ϕ an $N \times N$ matrix.

- The model is characterized by the distribution of the eigenvalues of ϕ.
- For $c = 0$ the eigenvalues have a Wigner semi-circle distribution.
- For $c > 0$ and $b << 0$ the eigenvalues fall into two disconnected regions, i.e. they have a "two cut" distribution.
- The transition is 3rd order and occurs at $b = -2\sqrt{Nc}$.
- The random matrix gravity transition occurs for $c < 0$ and $b > 0$.

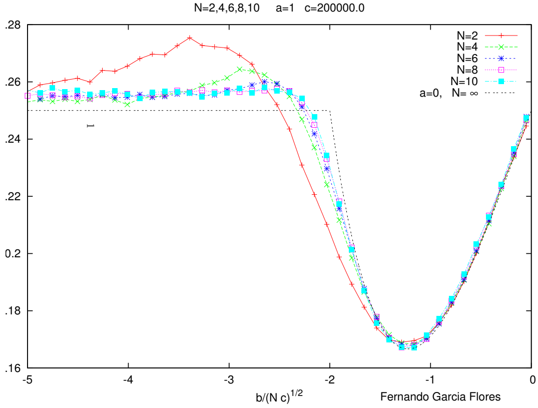
A fuzzy field theory model

$$S(\phi) = Tr(-a[L_a, \phi]^2 + b\phi^2 + c\phi^4)$$

L_a are the generators of $su(2)$ in the N dimensional representation.
again with ϕ an $N \times N$ matrix.



The specific heat $C_v = < S^2 > - < S >^2$ $S(\phi) = bTr(\phi^2) + cTr(\phi^4)$



The model for the background geometry.

Let us consider

the most general quartic polynomial single trace matrix model with global $SO(3)$ symmetry.

Matrix Energy

$E = \frac{Tr}{N}(-\frac{1}{4}[D_a, D_b]^2 + \frac{2i}{3}\epsilon_{abc}D_aD_bD_c + V(D))$

The Potential

$V(D) = bD_a^2 + c(D_a^2)^2$

breaks $D_a \to D_a + d_a\mathbf{1}$ symmetry.

Partition Function

$Z(\beta, g, b, c) = \int [dD_a] e^{-S(D)}$ where $S(D) = -\beta E(D)$

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Ground State

The model with $V = 0$.

The minimum energy configuration is

$D_a = L_a$ with $E_0 = -\frac{N^2-1}{48}$.

The L_a satisfy

$[L_a, L_b] = i\epsilon_{abc}L_c$ and $L_aL_a = \frac{N^2-1}{4}\mathbf{1}$.

These are the familiar commutation relations of angular momentum.

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A sphere from matrices

Let $N_a = \frac{2}{\sqrt{N^2-1}} L_a$

We get a sphere

$N_1^2 + N_2^2 + N_3^2 = \mathbf{1}$. A nice round sphere.

But it is non-commutative.

$[N_1, N_2] = \frac{2i}{\sqrt{N^2-1}} N_3$

There is an uncertainty principal for spatial position!
But for $N \to \infty$ we recover a commutative sphere.

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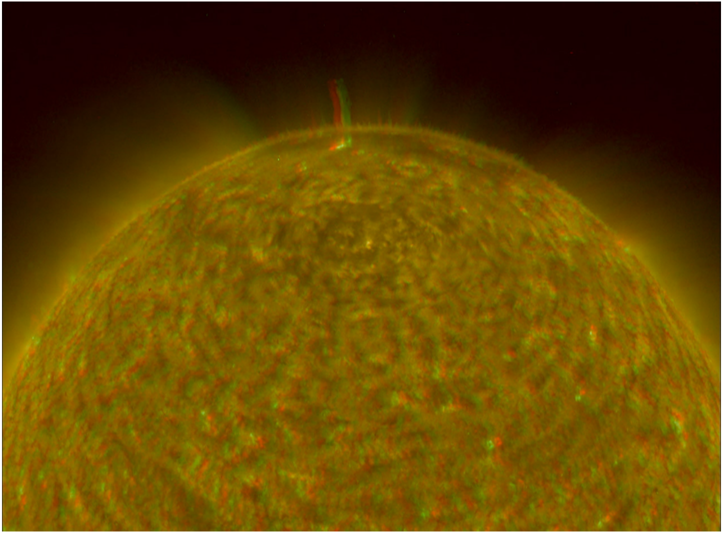
Our “fuzzy” sphere



Small fluctuations

Expanding around the minimum solution, $D_a = L_a + A_a$ yields a noncommutative Yang-Mills action with field strength

$$F_{ab} = i[L_a, A_b] - i[L_b, A_a] + \epsilon_{abc} A_c + i[A_a, A_b]$$

As written the gauge field includes a scalar field,

$$\Phi = \frac{1}{\sqrt{N^2 - 1}}(D_a - L_a)^2 = \frac{1}{2}(N_a A_a + A_a N_a + \frac{A_a^2}{\sqrt{c_2}}) ,$$

as the component of the gauge field normal to the sphere when viewed as imbeded in $\mathbf{R}^3$ with $N_a = \frac{L_a}{\sqrt{c_2}}$ and $c_2 = \sum_a L_a^2 = (N^2 - 1)/4$.

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Variations of the model have been proposed by H.Steinacker, *Nucl.Phys.B*679,66 (2004) and Presnajder *Mod.Phys.Lett.* A18 (2003) 2415. And a close relative (without the scalar field) has been solved exactly by H.Steinacker, R.J. Szabo, *hep-th/0701041*.

The model with $V(D) = 0$ arises as the low energy limit of a boundary $SU(2)$ WZW model at level k.

It can be thought of as the low energy dynamics of open strings moving on S^3. The minimum energy configuration corresponds to a stack of N $D0$ branes wrapping a fuzzy sphere centered at the origin.
A. Y. Alekseev, A. Recknagel, V. Schomerus, *JHEP* **010** 0005 (2000).

Larger context

The model can be obtained by reduction of $\mathcal{N} = 4$ SUSY Yang-Mills or equivalently from the ADS/CFT corresponding situation. Or from $d = 11$ supergravity.

An intermediate model in all of these reductions is the Berenstein, Maldacena, Nastase matrix model.

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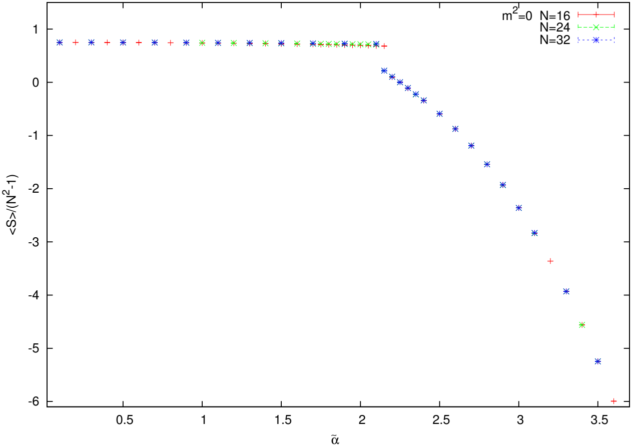
Monte Carlo Simulations

The singular part of the entropy is given by $\mathcal{S}/N^2$ where $\mathcal{S} =< S >$ and $\beta = \tilde{\alpha}^4$



The entropy jump

$\mathcal{S} = \frac{5}{12}$ as the transition is approached from the fuzzy sphere side,

and jumps to $\mathcal{S} = \frac{3}{4}$ in the high temperature phase.

Note

The infinite temperature entropy does not contribute $\frac{1}{2}$ per degree of freedom.

The model is highly interacting.

In fact the contribution is $\frac{1}{4}$ per degree of freedom.

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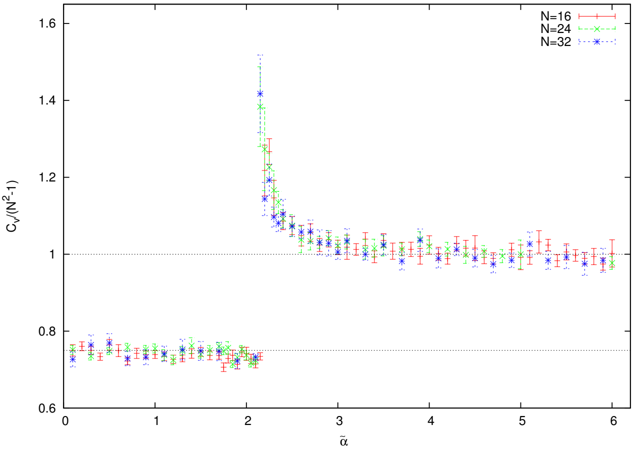
Specific Heat

The specific heat C_v/N^2 where $C_v =< S^2 > - < S >^2$ and



$\beta = \tilde{\alpha}^4$

Specific Heat Exponent

Entropy Jump

The transition is unusual in that it has a jump in the entropy. $\Delta\mathcal{S} = \frac{1}{3}$ indicating a 1st order transition.

Divergent Specific Heat

But it has a divergent specific heat $C = A_{-}(T_c - T)^{-\alpha}$ typical of a continuous (or second order) transition. We find the specific heat exponent $\alpha = \frac{1}{2}$.

Similar Transitions Occur in Dimer and 6-Vertex Models

The dimer and 6-vertex models also have asymmetric transitions with $\alpha = \frac{1}{2}$.

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Including $V(D)$

For the matrix potential $V(D)$ we focus on

$$\beta \frac{Tr}{N}(V(D)) = \frac{m^2 \tilde{\alpha}^4}{N}(-TrD_a^2 + \frac{2}{N^2-1} Tr(D_a^2)^2) \quad .$$

- This introduces just one new parameter m.
- For m large, it gives a deep well around $N_a^2 = \mathbf{1}$.

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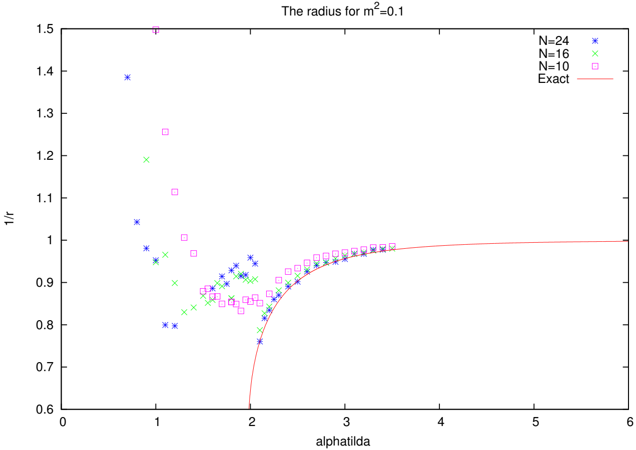
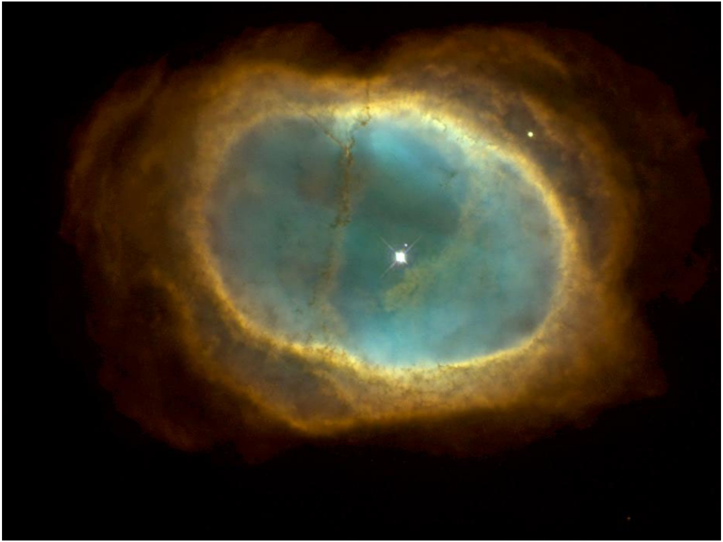
Measuring the radius of the sphere

$$\frac{1}{r} = \frac{2}{N(N^2-1)} Tr(D_a^2)$$

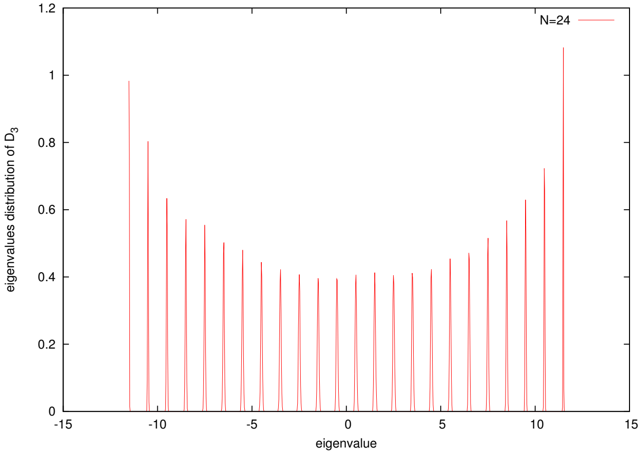
The fuzzy sphere expands and evaporates





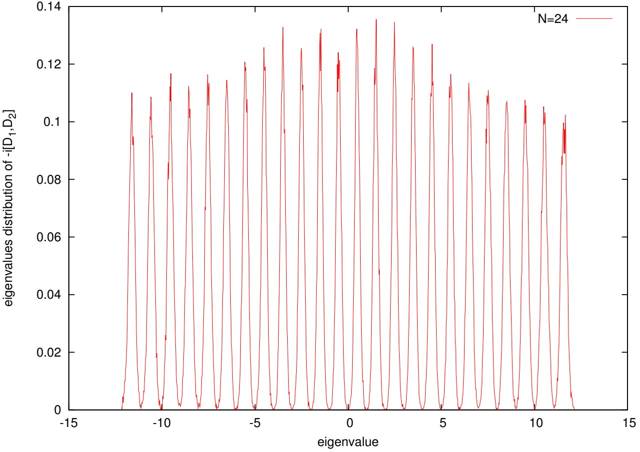
Eigenvalues in the low temperature phase

Eigenvalue distribution of D_3 for $N = 24$.



Eigenvalues in the low temperature phase

Eigenvalue distribution of $[D1, D_2]$ for $N = 24$.



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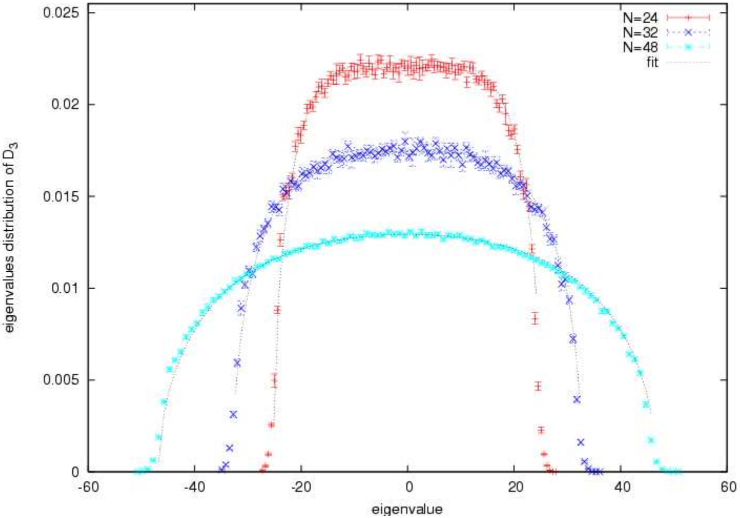
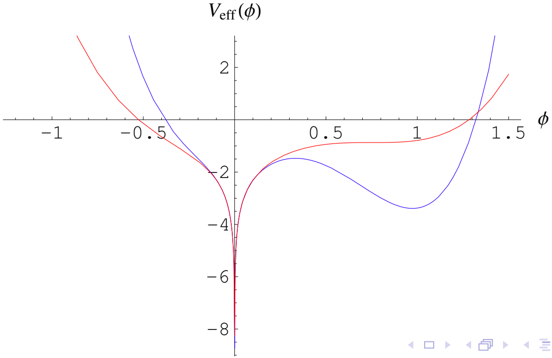
Eigenvalue distribution of D_3 for $N = 24$.



Effective potential

The effective potential, $V_{eff}(\phi)$, for ϕ where $D_a = \phi L_a$.

$$V_{eff} = \beta(\frac{1}{4}\phi^4 - \frac{1}{3}\phi^3) + \ln \phi^2$$



For the full model

$$V_{eff} = \tilde{\alpha}^4 \left(\frac{\phi^4}{4} - \frac{\phi^3}{3} + m^2 (\frac{\phi^4}{4} - \frac{\phi^2}{2}) \right) + \log \phi^2$$

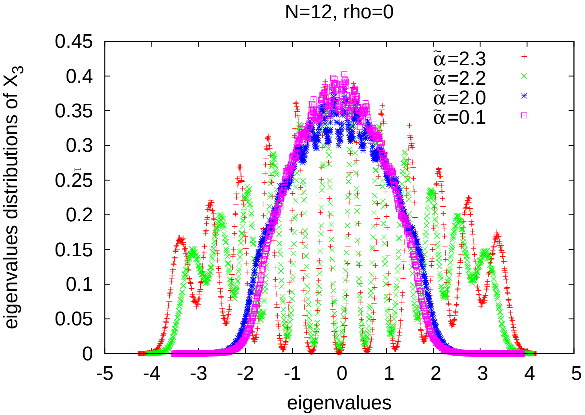
The location of the minimum gives predictions in excellent agreement with numerical data for the entropy and specific heat. It predicts the critical point as $\beta_c = (\frac{8}{3})^3$ for $m = 0$ and a critical exponent $\alpha = \frac{1}{2}$ for the divergence of the specific heat.

A closer look at the transition

Defining

$$X_a = (\frac{\beta}{N})^{1/4} D_a = \frac{\tilde{\alpha}}{N^{1/4}} D_a$$

And examining the eigenvalue distribution again:


N=12, rho=0
eigenvalues distributions of X_3
eigenvalues
$\tilde{\alpha}$=2.3
$\tilde{\alpha}$=2.2
$\tilde{\alpha}$=2.0
$\tilde{\alpha}$=0.1
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-5
-4
-3
-2
-1
0
1
2
3
4
5

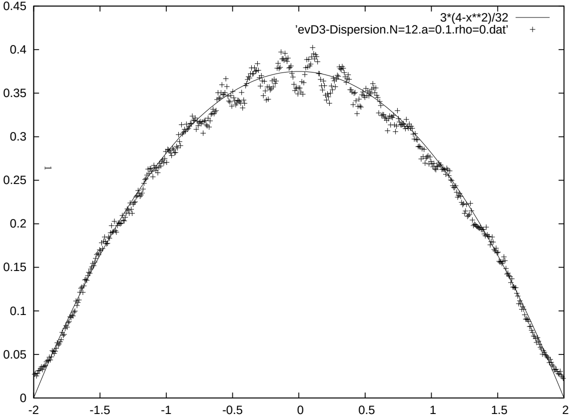
- In the fuzzy sphere phase the eigenvalues fluctuate around the discrete values corresponding to $D_a = L_a$, the irreducible representation of $SU(2)$.
- In the matrix phase, the distribution is largely independent of N and fluctuations are around commuting matrices with

$$X_a^2 = N$$

E.g for $N = 12$, the distribution for X_3 ranges from -2 to 2. Following Berenstein et al. (arXiv:0805.4658) one can expand small fluctuations around commuting diagonal matrices. To obtain that the distribution of such diagonal elements is S^2.

The distribution of eigenvalues of X_3 is then:

$$\rho(x) = \frac{9}{4N}(\frac{N}{3} - x^2)$$



3*(4-x**2)/32
'evD3-Dispersion.N=12.a=0.1.rho=0.dat'
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

A commutative two sphere has emerged but with much smaller radius than the fuzzy sphere. Thinking dynamically and suggestively:

A

s the system cools it goes through a phase of rapid expansion.

This is precisely the same phenomenon as happens in the AdS/CFT correspondence!

Conclusions

- We have we believe a good understanding of the 3-matrix model. It provides a concrete model where one can track the geometry as it passes through a phase transition and dissapears.

  Such transitions belong to a new universality class of topological phase transitions.
- The transition is from one where the underlying geometry at a microscopic level is non-commutative, and described by a fuzzy sphere with matter fluctuations to one a commutative sphere of much smaller radius.
- The geometrical phase emerges as the system cools. This is suggestive of a geometrical phase emerging as the universe cools, or perhaps as the relevant coupling runs to a larger scale.
- The fluctuations around the fuzzy sphere phase are consistent with being $U(1)$ gauge fields in the large mass limit.
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- The geometrical phase emerges as the system cools. This is suggestive of a geometrical phase emerging as the universe cools, or perhaps as the relevant coupling runs to a larger scale.
- The fluctuations around the fuzzy sphere phase are consistent with being $U(1)$ gauge fields in the large mass limit.
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Conclusions

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