On the Localization of a Renormalizable Translation Invariant U(1) NCGM

Talk presented by René I. P. Sedmik

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in collaboration with: D. Blaschke, A. Rofner, M. Schweda

May 17, 2009

Introduction

Prerequisites and Assumptions:

- simple deformation, non-commuting space-time coordinates on $\mathbb{R}^4_\theta$:

$$[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}\theta^{\mu\nu}\,, \quad \text{with } \theta^{\mu\nu} = \theta \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad \text{and } \theta \in \mathbb{R}\,.$$

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- definition of the Groenewold-Moyal $\star$-product:

$$f(x) \star g(x) = e^{\frac{\mathrm{i}}{2}\theta^{\mu\nu}\partial^x_\mu \partial^y_\nu} f(x) g(y)\Big|_{x=y}$$
$$\neq g(x) \star f(x)$$

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- invariance under cyclic permutations of the integral

$$\int d^4x\, f(x) \star g(x) \star h(x) = \int d^4x\, h(x) \star f(x) \star g(x)$$
$$\Longrightarrow \int d^4x f(x) \star g(x) = \int d^4x f(x) g(x)$$

A first scalar approach

Naïve implementation of scalar Klein Gordon theory in $\mathbb{R}^4_\theta$:

$$S[\phi] = \int d^4x \, \frac{1}{2}\partial_\mu\phi \star \partial_\mu\phi - \frac{m^2}{2}\phi^{\star 2} - \frac{\lambda}{4!}\phi^{\star 4}$$

leads to a propagator

$$G_\phi(k) = \frac{1}{k^2 + m^2}$$

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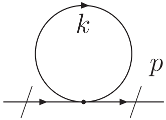

$\sim \; \frac{c_1}{p^2} + c_2 m^2 \ln p^2 + F(p) \Rightarrow$ UV/IR mixing

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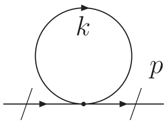

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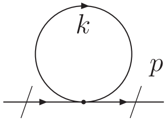

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...still the same result
Q: Where are the improvements?

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IR Damping

Q: Where are the improvements?

A: The new propagator 'damps' in higher loop insertions...

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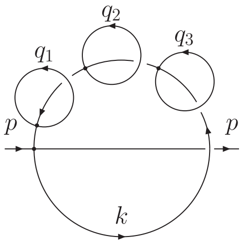

$$\sim c \sum_{\eta=\pm1} \int d^4k \, \underbrace{\frac{e^{i\eta k\theta p}}{(k^2)^n \left[k^2 + m^2 \quad\quad \right]^{n+1}}}$$

$$\xrightarrow[|k|\to 0]{} c' \times \begin{cases} \frac{1}{(k^2)^n} & \text{naïve model} \\ & \end{cases}$$

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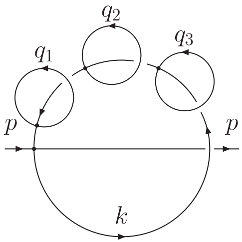

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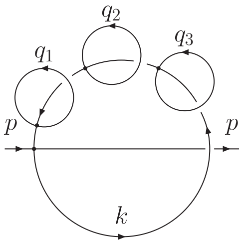

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This result is independent of the order![2]

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How to gauge this model?

$$\textbf{Ansatz:} \quad \int d^4x \phi(x) \star \frac{1}{\Box}\phi(x) \quad \Rightarrow \quad \int d^4x \frac{1}{4} F^{\mu\nu} \star \frac{1}{D^2 \widetilde{D}^2} \star F_{\mu\nu} \,,$$

$$U(1)_\theta : \; \delta_\varepsilon A_\mu = \partial_\mu \varepsilon + \mathrm{i}g \left[A_\mu \stackrel{\star}{,} \varepsilon \right] \,, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}g \left[A_\mu \stackrel{\star}{,} A_\nu \right] \,,$$

$D_\mu \bullet \equiv \partial_\mu \bullet - \mathrm{i}g \left[A_\mu \stackrel{\star}{,} \bullet \right]$ and $\widetilde{D}_\mu = \theta_{\mu\nu} D^\nu$.

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- Expression $\frac{1}{D^2} F \equiv Y$ transforms BRST covariantly ($sY = \mathrm{i}g \left[c \stackrel{\star}{,} Y\right]$)

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- Formal power series in the gauge field A_μ (by recursion):

$$F = D^2 \star \frac{1}{D^2} \star F \equiv D^2 Y = \Box Y - \mathrm{i}g \partial^\mu \left[A_\mu \stackrel{\star}{,} Y\right] - \mathrm{i}g \left[A^\mu \stackrel{\star}{,} \partial_\mu Y\right] + \ldots\,,$$

$$\Rightarrow Y = \frac{1}{\Box} F - f(Y)$$

Problem: Infinite series $\Rightarrow$ infinite number of gauge boson vertices![3]

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Alternative way

Rewrite $\int d^4x \frac{1}{4} F^{\mu\nu} \star \frac{a^2}{D^2 \widetilde{D}^2} F_{\mu\nu} \to \int d^4x \left[a B_{\mu\nu} F_{\mu\nu} - B_{\mu\nu} \star \widetilde{D}^2 D^2 B_{\mu\nu} \right]$

[4] D. N. Blaschke, A. Rofner, M. Schweda and R. I. P. Sedmik, *One-Loop Calculations for a Translation Invariant Non-Commutative Gauge Model*, [arXiv:0901.1681]

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- $B_{\mu\nu}$ has its own dynamics:

$$G^{BB}_{\rho\sigma,\tau\epsilon}(k) = \frac{-1}{4k^2\tilde{k}^2}\left[\delta_{\rho\tau}\delta_{\sigma\epsilon} - \delta_{\rho\epsilon}\delta_{\sigma\tau} + a^2 \frac{k_\sigma k_\tau \delta_{\rho\epsilon} + k_\rho k_\epsilon \delta_{\sigma\tau} - k_\sigma k_\epsilon \delta_{\rho\tau} - k_\rho k_\tau \delta_{\sigma\epsilon}}{k^2\tilde{k}^2 \left(k^2 + \frac{a'^2}{\tilde{k}^2}\right)}\right].$$

Q: How can $B_{\mu\nu}$ be interpreted?

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A: New degrees of freedom![4]

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Auxiliary fields and BRST doublets I

Idea (Vilar *et al.*[5]): Make the auxiliary field complex and assign appropriate ghosts to it

[5] L. C. Q. Vilar, O. S. Ventura, D. G. Tedesco and V. E. R. Lemes, *Renormalizable Noncommutative U(1) Gauge Theory Without IR/UV Mixing*, [arXiv:0902.2956]

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- Fields: $B_{\mu\nu} \in \mathbb{R}^4_\theta \to \{B_{\mu\nu}, \bar{B}_{\mu\nu}\}$, ghosts: $\{\xi_{\mu\nu}, \bar{\xi}_{\mu\nu}\}$
- BRST doublets(quartets):

$$
\begin{aligned}
s\bar{\xi}_{\mu\nu} &= \bar{B}_{\mu\nu} - \mathrm{i}g \left\{c \stackrel{\star}{,} \bar{\xi}_{\mu\nu}\right\}, & s\bar{B}_{\mu\nu} &= -\mathrm{i}g \left[c \stackrel{\star}{,} \bar{B}_{\mu\nu}\right] \\
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Note: already known from QED, where $s\bar{c} = b,\ sb = 0$.

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  Note: already known from QED, where $s\bar{c} = b, \; sb = 0$.
- Relevant part of the action (including an additional doublet $\{\chi_{\mu\nu}, \bar{\chi}_{\mu\nu}\}$ with ghosts $\{\psi_{\mu\nu}, \bar{\psi}_{\mu\nu}\}$):

$$\int d^4x \frac{1}{4} F^{\mu\nu} \star \frac{\gamma^2}{(D^2)^2} F_{\mu\nu} \to$$

$$\int d^4x \, s\left[\bar{\psi}_{\mu\nu} \star D^2 B^{\mu\nu} + \bar{\xi}_{\mu\nu} \star D^2 \chi^{\mu\nu} + \gamma^2 \bar{\psi}_{\mu\nu} \star \chi^{\mu\nu}\right] - \frac{\mathrm{i}}{2}\gamma B_{\mu\nu} F^{\mu\nu} + \frac{\mathrm{i}}{2}\gamma \bar{B}_{\mu\nu} F^{\mu\nu}$$

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Targets achieved:

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- ✓ BRST invariance (almost).The only remaining physical field (not BRST exact) is $A_\mu \Rightarrow$ no additional dof.
- ? Renormalizability? $\Rightarrow$ Algebraic renormalization (UV)$+$ explicit computation of coefficients
  Explicit renormalization in the IR?

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Drawback: complicated! 12 fields, 4 BRST sources, 14 Slavnov sources (yielding 9 propagators and 13 vertices)

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Just a glimpse:

$$\Sigma = \int d^4x \Big\{ \tfrac{1}{4} F_{\mu\nu} \star F^{\mu\nu} + ib \star \partial_\mu A^\mu + \bar{c} \star \partial^\mu D_\mu c + \bar{\chi}_{\mu\nu} \star D^2 B^{\mu\nu} + \bar{B}_{\mu\nu} \star D^2 \chi^{\mu\nu}$$
$$+ \gamma^2 \bar{\chi}_{\mu\nu} \star \chi^{\mu\nu} + \bar{J}_{\mu\nu\alpha\beta} \star \{B^{\mu\nu} \overset{\star}{,} F^{\alpha\beta}\} + J_{\mu\nu\alpha\beta} \star \{\bar{B}^{\mu\nu} \overset{\star}{,} F^{\alpha\beta}\} - \bar{\psi}_{\mu\nu} \star D^2 \xi^{\mu\nu} - \bar{\xi}_{\mu\nu} \star D^2 \psi^{\mu\nu}$$
$$- \gamma^2 \bar{\psi}_{\mu\nu} \star \psi^{\mu\nu} - \bar{Q}_{\mu\nu\alpha\beta} \star \{\xi^{\mu\nu} \overset{\star}{,} F^{\alpha\beta}\} - \Omega_\mu \star D^\mu c - \tfrac{i}{2} L \star g \{c \overset{\star}{,} c\}$$
$$- i\bar{u}^{\mu\nu} \star g\{c \overset{\star}{,} \xi_{\mu\nu}\} + u^{\mu\nu} \star (\bar{B}_{\mu\nu} - ig\{c \overset{\star}{,} \bar{\xi}_{\mu\nu}\})$$
$$+ \bar{v}^{\mu\nu} \star (\xi_{\mu\nu} - ig[c \overset{\star}{,} B_{\mu\nu}]) - iv^{\mu\nu} \star g[c \overset{\star}{,} \bar{B}_{\mu\nu}]$$
$$- i\bar{P}^{\mu\nu} \star g\{c \overset{\star}{,} \psi_{\mu\nu}\} + P^{\mu\nu} \star (\bar{\chi}_{\mu\nu} - ig\{c \overset{\star}{,} \bar{\psi}_{\mu\nu}\})$$
$$+ \bar{R}^{\mu\nu} \star (\psi_{\mu\nu} - ig[c \overset{\star}{,} \chi_{\mu\nu}]) - iR^{\mu\nu} \star g[c \overset{\star}{,} \bar{\chi}_{\mu\nu}]$$
$$+ \bar{M}^{\mu\nu\alpha\beta} \star (J_{\mu\nu\alpha\beta} - ig\{c \overset{\star}{,} Q_{\mu\nu\alpha\beta}\}) + M^{\mu\nu\alpha\beta} \star (\bar{J}_{\mu\nu\alpha\beta} - ig\{c \overset{\star}{,} \bar{Q}_{\mu\nu\alpha\beta}\})$$
$$- i\bar{N}^{\mu\nu\alpha\beta} \star g[c \overset{\star}{,} J_{\mu\nu\alpha\beta}] - iN^{\mu\nu\alpha\beta} \star g[c \overset{\star}{,} \bar{J}_{\mu\nu\alpha\beta}] \Big\},$$

Proposition of a new model

Thinking ...

Q: Can't we simplify this model in any way?

Proposition of a new model

Thinking ...

Q: Can't we simplify this model in any way?

A: Yes, we can. $\Rightarrow$ Reduce the quartet to a doublet!

Construction of the model I

Localization by fields $\{B_{\mu\nu},\, \bar{B}_{\mu\nu}\}$, with ghosts $\{\psi_{\mu\nu},\, \bar{\psi}_{\mu\nu}\}$

$$\int d^4x \frac{1}{4} F^{\mu\nu} \star \frac{\lambda^2}{\mu^2 D^2 \widetilde{D}^2} F_{\mu\nu} \rightarrow$$

$$S_{\mathsf{loc}} = \int d^4x \left[\frac{\lambda}{2} \left(B_{\mu\nu} + \bar{B}_{\mu\nu} \right) F^{\mu\nu} - \mu^2 \bar{B}_{\mu\nu} D^2 \widetilde{D}^2 B^{\mu\nu} + \mu^2 \bar{\psi}_{\mu\nu} D^2 \widetilde{D}^2 \psi^{\mu\nu} \right],$$

Landau **gauge fixing**

$$S_{\phi\pi} = \int d^4x \left(b \partial^\mu A_\mu - \bar{c} \partial^\mu D_\mu c \right)$$

(Note: All products are $\star$ products, even if not denoted explicitely!)

Construction of the model II

BRST transformations with doublet structures:

$$\begin{aligned} sA_\mu &= D_\mu c\,, & sc &= \mathrm{i}gcc\,,\\ s\bar{c} &= b\,, & sb &= 0\,,\\ sF_{\mu\nu} &= \mathrm{i}g\left[c, F_{\mu\nu}\right]\,, & & \\ s\bar{\psi}_{\mu\nu} &= \bar{B}_{\mu\nu} + \mathrm{i}g\left\{c, \bar{\psi}_{\mu\nu}\right\}\,, & s\bar{B}_{\mu\nu} &= \mathrm{i}g\left[c, \bar{B}_{\mu\nu}\right]\,,\\ sB_{\mu\nu} &= \psi_{\mu\nu} + \mathrm{i}g\left[c, B_{\mu\nu}\right]\,, & s\psi_{\mu\nu} &= \mathrm{i}g\left\{c, \psi_{\mu\nu}\right\}\,. \end{aligned}$$

$$S_{\mathrm{loc}} = \int d^4x \left[\frac{\lambda}{2} B_{\mu\nu} F^{\mu\nu} + s\left(\frac{\lambda}{2}\bar{\psi}_{\mu\nu}F^{\mu\nu} - \mu^2\bar{\psi}_{\mu\nu}D^2\widetilde{D}^2B^{\mu\nu}\right)\right],$$

⇒ The action is almost BRST invariant (up to a 'soft breaking').

Construction of the model II

BRST transformations with doublet structures:

$$
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sA_\mu &= D_\mu c\,, & sc &= \mathrm{i}gcc\,,\\
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sF_{\mu\nu} &= \mathrm{i}g\left[c, F_{\mu\nu}\right]\,,\\
s\bar{\psi}_{\mu\nu} &= \bar{B}_{\mu\nu} + \mathrm{i}g\left\{c, \bar{\psi}_{\mu\nu}\right\}\,, & s\bar{B}_{\mu\nu} &= \mathrm{i}g\left[c, \bar{B}_{\mu\nu}\right]\,,\\
sB_{\mu\nu} &= \psi_{\mu\nu} + \mathrm{i}g\left[c, B_{\mu\nu}\right]\,, & s\psi_{\mu\nu} &= \mathrm{i}g\left\{c, \psi_{\mu\nu}\right\}\,.
\end{aligned}
$$

$$
S_{\text{loc}} = \int d^4x \left[\frac{\lambda}{2} B_{\mu\nu} F^{\mu\nu} + s\left(\frac{\lambda}{2}\bar{\psi}_{\mu\nu}F^{\mu\nu} - \mu^2 \bar{\psi}_{\mu\nu} D^2 \widetilde{D}^2 B^{\mu\nu}\right)\right],
$$

$\Rightarrow$ The action is almost BRST invariant (up to a 'soft breaking').
$\Rightarrow$ Ready for algebraic renormalization?

Breaking term

Non-BRST-invariant term in the action:

$$sS_{\text{break}} = s\int d^4x \frac{\lambda}{2} B_{\mu\nu} F^{\mu\nu} = \int d^4x \frac{\lambda}{2} \psi_{\mu\nu} F^{\mu\nu}$$

[6] L. Baulieu and S. P. Sorella, *Soft breaking of BRST invariance for introducing non-perturbative infrared effects in a local and renormalizable way*, *Physics Letters B* **671** (2009) 481, [arXiv:0808.1356]

Breaking term

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has to be eliminated to enable UV renormalization.[6]
$\Rightarrow$ introduction of pairs of 'BRST sources' $\{Q_{\mu\nu\alpha\beta}, \bar{Q}_{\mu\nu\alpha\beta}\}$ and $\{J_{\mu\nu\alpha\beta}, \bar{J}_{\mu\nu\alpha\beta}\}$,

$$s\bar{Q}_{\mu\nu\alpha\beta} = \bar{J}_{\mu\nu\alpha\beta} + \mathrm{i}g\left\{c, \bar{Q}_{\mu\nu\alpha\beta}\right\}, \qquad s\bar{J}_{\mu\nu\alpha\beta} = \mathrm{i}g\left[c, \bar{J}_{\mu\nu\alpha\beta}\right],$$
$$sQ_{\mu\nu\alpha\beta} = J_{\mu\nu\alpha\beta} + \mathrm{i}g\left\{c, Q_{\mu\nu\alpha\beta}\right\}, \qquad sJ_{\mu\nu\alpha\beta} = \mathrm{i}g\left[c, J_{\mu\nu\alpha\beta}\right],$$

$$S_{\mathsf{break}} \to s\int d^4x \bar{Q}_{\mu\nu\alpha\beta} B^{\mu\nu} F^{\alpha\beta} = \int d^4x \bar{J}_{\mu\nu\alpha\beta} B^{\mu\nu} F^{\alpha\beta} - \bar{Q}_{\mu\nu\alpha\beta} \psi^{\mu\nu} F^{\alpha\beta}$$

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Soft breaking mechanism

But: Breaking term implements the actual IR damping of the $\frac{1}{p^2}$ model.

[7] D. Zwanziger, *Local and Renormalizable Action from the Gribov Horizon*, *Nucl. Phys.* **B323** (1989) 513–544.,

D. Zwanziger, *Renormalizability of the critical limit of lattice gauge theory by BRS invariance*, *Nucl. Phys.* **B399** (1993) 477–513.

Soft breaking mechanism

But: Breaking term implements the actual IR damping of the $\frac{1}{p^2}$ model.
Trick by Zwanziger[7]: Assign 'physical values' in the IR, recovering the breaking:

$$\bar{Q}_{\mu\nu\alpha\beta}\big|_{\text{phys}} = 0\,, \quad \bar{J}_{\mu\nu\alpha\beta}\big|_{\text{phys}} = \frac{\lambda}{4}\left(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}\right)\,,$$
$$Q_{\mu\nu\alpha\beta}\big|_{\text{phys}} = 0\,, \quad J_{\mu\nu\alpha\beta}\big|_{\text{phys}} = \frac{\lambda}{4}\left(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}\right)\,.$$

IR	UV
$S_{\text{break}} = \frac{\lambda}{2} B_{\mu\nu} F^{\mu\nu}$	$S_{\text{break}} = s \int d^4x \bar{Q}_{\mu\nu\alpha\beta} B^{\mu\nu} F^{\alpha\beta}$

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Complete action

$$S_{\text{inv}} = \int d^4x \frac{1}{4} F_{\mu\nu} F^{\mu\nu} ,$$

$$S_{\phi\pi} = \int d^4x \, s \left(\bar{c} \, \partial^\mu A_\mu \right) = \int d^4x \left(b \, \partial^\mu A_\mu - \bar{c} \, \partial^\mu D_\mu c \right) ,$$

$$S_{\text{new}} = \int d^4x \, s \left(J_{\mu\nu\alpha\beta} \bar{\psi}^{\mu\nu} F^{\alpha\beta} - \mu^2 \bar{\psi}_{\mu\nu} D^2 \widetilde{D}^2 B^{\mu\nu} \right)$$

$$= \int d^4x \left(J_{\mu\nu\alpha\beta} \bar{B}^{\mu\nu} F^{\alpha\beta} - \mu^2 \bar{B}_{\mu\nu} D^2 \widetilde{D}^2 B^{\mu\nu} + \mu^2 \bar{\psi}_{\mu\nu} D^2 \widetilde{D}^2 \psi^{\mu\nu} \right) ,$$

$$S_{\text{break}} = \int d^4x \, s \left(\bar{Q}_{\mu\nu\alpha\beta} B^{\mu\nu} F^{\alpha\beta} \right) = \int d^4x \left(\bar{J}_{\mu\nu\alpha\beta} B^{\mu\nu} F^{\alpha\beta} - \bar{Q}_{\mu\nu\alpha\beta} \psi^{\mu\nu} F^{\alpha\beta} \right) ,$$

$$S_{\text{ext}} = \int d^4x \Big(\Omega^A_\mu D^\mu c + \mathrm{i}g \, \Omega^c cc + \Omega^B_{\mu\nu} \left(\psi^{\mu\nu} + \mathrm{i}g \left[c, B^{\mu\nu} \right] \right) + \mathrm{i}g \, \Omega^{\bar{B}}_{\mu\nu} \left[c, \bar{B}^{\mu\nu} \right]$$

$$+ \mathrm{i}g \, \Omega^\psi_{\mu\nu} \left\{ c, \psi^{\mu\nu} \right\} + \Omega^{\bar{\psi}}_{\mu\nu} \left(\bar{B}^{\mu\nu} + \mathrm{i}g \left\{ c, \bar{\psi}^{\mu\nu} \right\} \right) + \Omega^Q_{\mu\nu\alpha\beta} \left(J^{\mu\nu\alpha\beta} + \mathrm{i}g \left\{ c, Q^{\mu\nu\alpha\beta} \right\} \right)$$

$$+ \mathrm{i}g \, \Omega^J_{\mu\nu\alpha\beta} \left[c, J^{\mu\nu\alpha\beta} \right] + \Omega^{\bar{Q}}_{\mu\nu\alpha\beta} \left(\bar{J}^{\mu\nu\alpha\beta} + \mathrm{i}g \left\{ c, \bar{Q}^{\mu\nu\alpha\beta} \right\} \right) + \mathrm{i}g \, \Omega^{\bar{J}}_{\mu\nu\alpha\beta} \left[c, \bar{J}^{\mu\nu\alpha\beta} \right] \Big)$$

Propagators

$$G^{\bar{c}c}(k) = -\frac{1}{k^2}\,, \qquad a' \equiv \frac{\lambda}{\mu}\,,$$

$$G^{\bar{\psi}\psi}_{\mu\nu,\rho\sigma}(k) = \frac{(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})}{2\mu^2 k^2 \tilde{k}^2}\,,$$

$$G^{AA}_{\mu\nu}(k) = \frac{1}{\left(k^2 + \frac{a'^2}{\tilde{k}^2}\right)}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\,,$$

$$G^{AB}_{\mu,\rho\sigma}(k) = \frac{\mathrm{i}a'}{2\mu}\frac{(k_\rho\delta_{\mu\sigma} - k_\sigma\delta_{\mu\rho})}{k^2\tilde{k}^2\left(k^2 + \frac{a'^2}{\tilde{k}^2}\right)} = G^{A\bar{B}}_{\mu,\rho\sigma}(k) = -G^{\bar{B}A}_{\rho\sigma,\mu}(k)\,,$$

$$G^{\bar{B}B}_{\mu\nu,\rho\sigma}(k) = \frac{-1}{2\mu^2 k^2\tilde{k}^2}\left[\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho} - a'^2\frac{k_\mu k_\rho\delta_{\nu\sigma} + k_\nu k_\sigma\delta_{\mu\rho} - k_\mu k_\sigma\delta_{\nu\rho} - k_\nu k_\rho\delta_{\mu\sigma}}{2k^2\tilde{k}^2\left(k^2 + \frac{a'^2}{\tilde{k}^2}\right)}\right]\,,$$

$$G^{BB}_{\mu\nu,\rho\sigma}(k) = \frac{a'^2}{4k^2\tilde{k}^2}\left[\frac{k_\mu k_\rho\delta_{\nu\sigma} + k_\nu k_\sigma\delta_{\mu\rho} - k_\mu k_\sigma\delta_{\nu\rho} - k_\nu k_\rho\delta_{\mu\sigma}}{\mu^2 k^2\tilde{k}^2\left(k^2 + \frac{a'^2}{\tilde{k}^2}\right)}\right] = G^{\bar{B}\bar{B}}_{\mu\nu,\rho\sigma}(k)\,,$$

Power counting

Superficial degree of (UV) divergence in 4 dimensions:

$$d_\gamma = 4 - E_A - E_{c/\bar{c}} - 2E_B - 2E_{\bar{B}} - 2E_{\psi\bar{\psi}} - 2E_\theta \,,$$

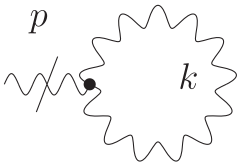
and with $E_\lambda = E_B + E_{\bar{B}} + E_{\psi/\bar{\psi}} + E_\theta$,

$$d_\gamma = 4 - E_A - E_{c/\bar{c}} - 2E_\lambda \,,$$

E_x counts the number of external legs of field x,
E_λ and E_θ count the overall powers of λ and θ in the graph.

One loop computations

Tadpoles
with one external boson vanish, as $\int d^4k \sin \frac{k\tilde{p}}{2}\Big|_{p\to 0} \to 0$



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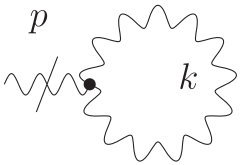
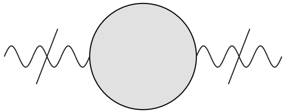


Vacuum Polarization

- Leading terms for $|p| \to 0$ are $\propto \frac{2g^2}{\pi^2}\frac{\tilde{p}_\mu\tilde{p}_\nu}{(\tilde{p}^2)^2}$ + finite.
- Transversality as expected

18 graphs, 7 thereof are divergent



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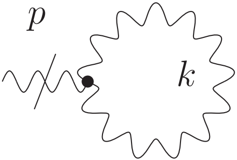

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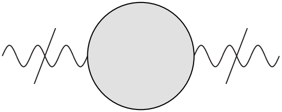
- Transversality as expected
- Sectors $B_{\mu\nu}$ and $\psi_{\mu\nu}$ cancel as expected

18 graphs, 7 thereof are divergent



Conclusion

What has been achieved?

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- Non-commutative $U(1)_\theta$ gauge theory

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Conclusion

What has been achieved?

- Non-commutative $U(1)_\theta$ gauge theory
- Translation invariant and local
- Potentially renormalizable

Outlook

What are the next steps?

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- Next step: Completition of the algebraic renormalization procedure in the UV

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- Next step: Completition of the algebraic renormalization procedure in the UV
- IR renormalization (as was achieved for the scalar model)
- Transition to Minkowski? - we'll try

Thank you

Thank you for your attention!

Thanks to my collaborators:
Daniel Blaschke, Arnold Rofner, Manfred Schweda

Equivalence of local and nonlocal terms

$$
\begin{aligned}
Z &= \int \mathcal{D}(\bar{\psi}\psi\bar{B}BA)\ \exp\left\{-\left(\int d^4x \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + S_{\text{loc}}\right)\right\} \\
&= \int \mathcal{D}(\bar{B}BA)\det{}^4\left(\mu^2 D^2 \widetilde{D}^2\right)\exp\left\{-\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda}{2}\left(B_{\mu\nu}+\bar{B}_{\mu\nu}\right)F^{\mu\nu}\right.\right. \\
&\qquad\qquad \left.\left. - \mu^2 \bar{B}_{\mu\nu} D^2 \widetilde{D}^2 B^{\mu\nu}\right]\right\} \\
&= \int \mathcal{D}(\bar{B}BA)\det{}^4\left(\mu^2 D^2 \widetilde{D}^2\right)\exp\left\{-\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda^2}{4\mu^2}F_{\mu\nu}\frac{1}{\widetilde{D}^2 D^2}F^{\mu\nu} - \right.\right. \\
&\qquad \left.\left. - \left(\bar{B}_{\mu\nu} - \frac{\lambda}{2\mu^2}\frac{1}{\widetilde{D}^2 D^2}F_{\mu\nu}\right)\mu^2 D^2 \widetilde{D}^2\left(B^{\mu\nu} - \frac{\lambda}{2\mu^2}\frac{1}{\widetilde{D}^2 D^2}F^{\mu\nu}\right)\right]\right\} \\
&= \int \mathcal{D}A\,\det{}^{4-4}\left(D^2 \widetilde{D}^2\right)\exp\left\{-\int d^4x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\lambda^2}{4\mu^2}F_{\mu\nu}\frac{1}{\widetilde{D}^2 D^2}F^{\mu\nu}\right]\right\}.
\end{aligned}
$$

Properties of fields, ghosts and sources

Table: Properties of fields and sources.

Field	A_μ	c	$\bar{c}$	$B_{\mu\nu}$	$\bar{B}_{\mu\nu}$	$\psi_{\mu\nu}$	$\bar{\psi}_{\mu\nu}$	$J_{\alpha\beta\mu\nu}$	$\bar{J}_{\alpha\beta\mu\nu}$	$Q_{\alpha\beta\mu\nu}$	$\bar{Q}_{\alpha\beta\mu\nu}$
$g_\sharp$	0	1	-1	0	0	1	-1	0	0	-1	-1
Mass dim.	1	0	2	1	1	1	1	1	1	1	1
Statistics	b	f	f	b	b	f	f	b	b	f	f
Source	Ω^A_μ	Ω^c	b	$\Omega^B_{\mu\nu}$	$\Omega^{\bar{B}}_{\mu\nu}$	$\Omega^\psi_{\mu\nu}$	$\Omega^{\bar{\psi}}_{\mu\nu}$	$\Omega^J_{\alpha\beta\mu\nu}$	$\Omega^{\bar{J}}_{\alpha\beta\mu\nu}$	$\Omega^Q_{\alpha\beta\mu\nu}$	$\Omega^{\bar{Q}}_{\alpha\beta\mu\nu}$
$g_\sharp$	-1	-2	0	-1	-1	-2	0	-1	-1	0	0
Mass dim.	3	4	2	3	3	3	3	3	3	3	3
Statistics	f	b	b	f	f	b	b	f	f	b	b

1-loop vacuum polarization graphs