Matrix Models, Gauge Theory and Emergent Gravity

Harold Steinacker

Department of Physics, University of Vienna

Bayrischzell 2009

Motivation

Gravity $\leftrightarrow$ Quantum Mechanics

- strongest impact:
  cosmological constant problem: huge discrepancy

$$\frac{\Lambda_{\rm QM}}{\Lambda_{\rm concordance}} \geq 10^{60}$$

  need to fix that!!
- Question: how do geometrical structures (metric, ...) arise in a quantum world?
  fascinating Answer: arise automatically ("emerge") in

(Yang-Mills) Matrix Models

$\Rightarrow$ quantum structure of space-time
$\Rightarrow$ modified, emergent gravity on NC spaces

main features of (Yang-Mills-) Matrix Models:

- M. M. known to describe NC gauge theory
- M. M. also contain gravity
  intrinsically NC mechanism
  dynamical quantum (NC) space-time
- extremely simple
- promise good behavior under quantization
  may solve cosm.const. problem, inflation
- $\left\{ \begin{array}{l} \text{space-time} \\ \text{metric} \end{array} \right\}$ not fundamental, emerge

relation NC gauge thy $\leftrightarrow$ gravity Rivelles 2002, Yang 2006, ...

IKKT (Matrix) Model 1996

Geometry from Matrix Models

consider D –dim. M.M.

$$S_{YM} = -Tr[X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'}, \qquad a, b = 1, ..., D$$

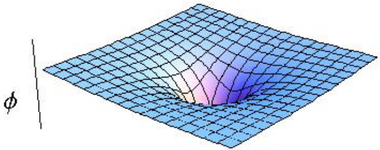
dynamical objects: $X^a \in Mat(\infty, \mathbb{C})$ ... hermitian matrices

$D = 10$ required by consistency at quantum level (maximal SUSY)

space-time as 3+1-dimensional brane solution: split matrices

$$\begin{aligned} X^a &= (X^\mu, \Phi^i), \qquad \mu = 1, ..., 4; \qquad X^\mu \sim x^\mu \\ \Phi^i &= \Phi^i(x^\mu) \qquad \text{... 4D brane } \mathcal{M}^4_\theta \subset \mathbb{R}^D \end{aligned}$$

$\Longleftrightarrow$ embedding through scalar fields $\phi^i(x)$



NC brane solutions:

equation of motion $[X^a, [X^{a'}, X^{b'}]]\eta_{aa'} = 0$

solutions:

- $[X^a, X^b] = 0$ ... ignore here
- $[X^a, X^b] = i\bar{\theta}^{ab}\,\mathbf{1}$, "quantum plane"

  where $\bar{\theta}^{ab}$ ... antisymmetric, nondegenerate
- many more, of type $[X^a, X^b] \sim i\{x^a, x^b\} = i\theta^{ab}(x)$

M.M. describes dynamical quantum (NC) space-time

interpretation:

- $X^\mu \sim x^\mu$ ... quantization of coordinate function on $\mathcal{M}^4_\theta$
- $\phi^i(x)$ ... embedding of $\mathcal{M}^4_\theta \hookrightarrow \mathbb{R}^D$
- $[X^\mu, X^\nu] \sim i\theta^{\mu\nu}(x)$ ... Poisson structure on $\mathcal{M}^4_\theta$

NC brane solutions:

equation of motion $[X^a, [X^{a'}, X^{b'}]]\eta_{aa'} = 0$

solutions:

- $[X^a, X^b] = 0$ ... ignore here
- $[X^a, X^b] = i\bar{\theta}^{ab}\, \mathbf{1}$, "quantum plane"

  where $\bar{\theta}^{ab}$ ... antisymmetric, nondegenerate
- many more, of type $[X^a, X^b] \sim i\{x^a, x^b\} = i\theta^{ab}(x)$

M.M. describes dynamical quantum (NC) space-time

interpretation:

- $X^\mu \sim x^\mu$ ... quantization of coordinate function on $\mathcal{M}^4_\theta$
- $\phi^i(x)$ ... embedding of $\mathcal{M}^4_\theta \hookrightarrow \mathbb{R}^D$
- $[X^\mu, X^\nu] \sim i\theta^{\mu\nu}(x)$ ... Poisson structure on $\mathcal{M}^4_\theta$

Noncommutative spaces and Poisson structure

$(\mathcal{M}, \theta^{\mu\nu}(x))$ ... $2n$-dimensional manifold with Poisson structure

Its quantization $\mathcal{M}_\theta$ is NC algebra such that

$$\begin{aligned} \mathcal{C}(\mathcal{M}) &\rightarrow \mathcal{A} \subset L(\mathcal{H}) \cong Mat(\infty, \mathbb{C}) \\ f(x) &\mapsto \hat{f}(X) \\ x^i &\mapsto X^i \end{aligned}$$

such that $$[\hat{f}(X), \hat{g}(X)] = i\{f(x), g(x)\} + O(\theta^2)$$

(cf. phase space in Quantum Mechanics)

furthermore:

$$\begin{aligned} [X^\mu, \phi(X)] &\sim i\theta^{\mu\nu}(x)\partial_\nu\phi(x) \\ (2\pi)^2\, Tr(\phi(X)) &\sim \int d^4x\, \rho(x)\, \phi(x) \\ \rho(x) &= \mathrm{Pfaff}\,(\theta^{-1}_{\mu\nu}) \ \ldots \end{aligned}$$ symplectic volume

(cf. Bohr-Sommerfeld quantization)

Effective geometry:

consider scalar field coupled to Matrix Model ("test particle")
use $[X^\mu, \varphi] \sim i\theta^{\mu\nu}(x)\partial_\nu\varphi \qquad \Rightarrow$

$$\begin{aligned} S[\varphi] &= Tr\,[X^a,\varphi][X^b,\varphi]\,\eta_{ab} \qquad (U(\mathcal{H}) \quad \text{gauge inv.!}) \\ &\sim \int d^4x\,\sqrt{|G_{\mu\nu}|}\,G^{\mu\nu}(x)\,\partial_\mu\varphi\partial_\nu\varphi \end{aligned}$$

$$\begin{aligned} G^{\mu\nu}(x) &= e^{-\sigma}\theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\; g_{\mu'\nu'}(x) \qquad \text{effective metric} \\ g_{\mu\nu}(x) &= \eta_{\mu\nu} + \partial_\mu\Phi^i\partial_\nu\Phi^j\delta_{ij} \qquad \text{induced metric on } \mathcal{M}^4_\theta \end{aligned}$$

$$e^{-2\sigma} = \frac{|\theta^{-1}_{\mu\nu}|}{|g_{\mu\nu}|}, \qquad |G_{\mu\nu}| = |g_{\mu\nu}|$$

φ couples to metric $G^{\mu\nu}(x)$, determined by $\theta^{\mu\nu}(x)$ & embedding ϕ^i

same for gauge fields, fermions

... quantized Poisson manifold with metric $(\mathcal{M}, \theta^{\mu\nu}(x), G_{\mu\nu}(x))$

Equations of motion: can show

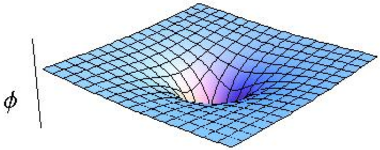
matrix e.o.m: $[X^a,[X^b,X^{a'}]]\eta_{aa'} = 0 \quad \Longleftrightarrow$ (H.S., NPB 810 (2009))

$$
\begin{aligned}
\Delta_G \Phi^i &= 0, \\
\nabla^\mu (e^\sigma \theta^{-1}_{\mu\nu}) &= e^{-\sigma}\, G_{\rho\nu}\theta^{\rho\mu}\partial_\mu \eta \\
\eta &= e^\sigma\, G^{\mu\nu} g_{\mu\nu}
\end{aligned}
$$

covariant formulation in semi-classical limit

furthermore: $\Delta_G x^\mu = 0 \quad \Rightarrow$

$\mathcal{M}^4 \hookrightarrow \mathbb{R}^D$ is harmonic embedding (w.r.t. $G_{\mu\nu}$)
minimal surface



Equations of motion: can show

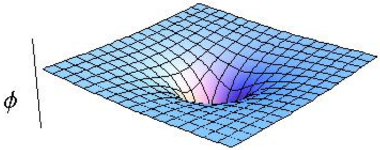
matrix e.o.m: $[X^a,[X^b,X^{a'}]]\eta_{aa'}=0 \quad \Longleftrightarrow$ (H.S., NPB 810 (2009))

$$
\begin{aligned}
\Delta_G\Phi^i &= 0,\\
\nabla^\mu(e^\sigma\theta^{-1}_{\mu\nu}) &= e^{-\sigma}\,G_{\rho\nu}\theta^{\rho\mu}\partial_\mu\eta\\
\eta &= e^\sigma\,G^{\mu\nu}g_{\mu\nu}
\end{aligned}
$$

covariant formulation in semi-classical limit

furthermore: $\Delta_G x^\mu = 0 \quad \Rightarrow$

$\mathcal{M}^4 \hookrightarrow \mathbb{R}^D$ is harmonic embedding (w.r.t. $G_{\mu\nu}$)
minimal surface



$su(n)$ gauge fields: same model, new vacuum

$$Y^a = \begin{pmatrix} Y^\mu \\ Y^i \end{pmatrix} = \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n \end{pmatrix}$$

include fluctuations:

$$Y^a = (1 + \mathcal{A}^\rho \partial_\rho) \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n \end{pmatrix}$$

where

$$\begin{aligned} \mathcal{A}^\mu &= -\theta^{\mu\nu} A_{\nu,\alpha} \otimes \lambda^\alpha, \qquad \lambda^\alpha \in \mathfrak{su}(n) \\ \Phi^i &= \Phi^i_\alpha \otimes \lambda^\alpha \end{aligned}$$

$\Rightarrow$ effective action:

$$\boxed{S_{YM} = \int d^4x \sqrt{G}\, e^\sigma\, G^{\mu\mu'} G^{\nu\nu'} \, tr\, F_{\mu\nu} F_{\mu'\nu'} + 2 \int \eta(x)\, tr\, F \wedge F}$$

(H.S., JHEP 0712:049 (2007), JHEP 0902:044,(2009))

... $\mathfrak{su}(n)$ Yang-Mills coupled to metric $G^{\mu\nu}(x)$

Symmetries & Noether theorem

Matrix model: translational symmetry $X^a \to X^a + c^a \mathbb{1}$
$\Rightarrow$ conserved "matrix current"

$$\boxed{[X^a, T^{bc}]\,\eta_{ab} = 0}$$

$$T^{ab} = [X^a, X^c][X^b, X^{c'}]\eta_{cc'} - \frac{1}{4}\eta^{ab}[X^d, X^c][X^{d'}, X^{c'}]\eta_{dd'}\eta_{cc'} + (a \leftrightarrow b)$$

semi-classical limit:

- $U(1)$ component:

$$\boxed{\nabla^\mu(e^\sigma \theta^{-1}_{\mu\nu}) = e^{-\sigma}\, G_{\rho\nu}\theta^{\rho\mu}\partial_\mu \eta}\quad \ldots \text{NC} \leftrightarrow \text{gravity}$$

- $SU(n)$ component: H.S., JHEP 0902:044,2009.

$$\boxed{0 = -\sqrt{G}(\nabla_\rho + i[A_\rho, .])(e^\sigma F^{\rho\nu}) - 2F_{\alpha\beta}\varepsilon^{\nu\alpha\beta\rho}\partial_\rho \eta}$$

... e.o.m. for Yang-Mills + "would-be top." coupled to $G_{\mu\nu}$

$\Rightarrow$ expect to be valid at quantum level !

Symmetries & Noether theorem

Matrix model: translational symmetry $X^a \to X^a + c^a\,\mathbb{1}$

$\Rightarrow$ conserved "matrix current"

$$[X^a, T^{bc}]\,\eta_{ab} = 0$$

$$T^{ab} = [X^a, X^c][X^b, X^{c'}]\eta_{cc'} - \frac{1}{4}\eta^{ab}[X^d, X^c][X^{d'}, X^{c'}]\eta_{dd'}\eta_{cc'} + (a \leftrightarrow b)$$

semi-classical limit:

- $U(1)$ component:

$$\nabla^\mu(e^\sigma\theta^{-1}_{\mu\nu}) = e^{-\sigma}\, G_{\rho\nu}\theta^{\rho\mu}\partial_\mu\eta$$ ... NC $\leftrightarrow$ gravity

- $SU(n)$ component: H.S., JHEP 0902:044,2009.

$$0 = -\sqrt{G}\,(\nabla_\rho + i[A_\rho, .])(e^\sigma F^{\rho\nu}) - 2F_{\alpha\beta}\varepsilon^{\nu\alpha\beta\rho}\partial_\rho\eta$$

... e.o.m. for Yang-Mills + "would-be top." coupled to $G_{\mu\nu}$

$\Rightarrow$ expect to be valid at quantum level !

Quantization and E-H action

Quantization of matrix model:

$$Z = \int dX^a d\Psi \, e^{-S[X]-S[\Psi]}$$

note:

- bosonic M. M.: "measure" for integral over geometry
- expect: well-defined for IKKT model $\equiv$ $D = 10$ model
  (6 scalars + 4 Weyl fermions, $N = 4$ SUSY)
- one-loop: fields couple to $G_{\mu\nu}$
  $\Rightarrow$ induced Einstein-Hilbert action:

$$S_{1-loop} \sim \int d^4x \sqrt{|G_{\mu\nu}|} \left(c_1 \Lambda_1^4 + c_2 \Lambda_4^2 R[G] + O(\log(\Lambda_{UV}))\right)$$

consider “would-be cosmological constant:

note: $\det G_{\mu\nu} \equiv \det g_{\mu\nu} \quad \Rightarrow$

$$\delta \int d^4x\sqrt{G} \sim \int d^4x\sqrt{g}g^{\mu\nu}\delta g_{\mu\nu} \sim \int d^4x\sqrt{g}\delta\phi^i\Delta_g\phi^j\delta_{ij}$$

vanishes for harmonic embeddings

$$\Delta_g\phi^i = 0$$

term is huge but irrelevant $\quad \Rightarrow$

$c_1\Lambda_1^4$ not cosmological constant,
harmonically embedded (e.g. flat) spaces protected from cosm. const. problem

consider “would-be cosmological constant:

note: $\det G_{\mu\nu} \equiv \det g_{\mu\nu} \quad \Rightarrow$

$$\delta \int d^4x \sqrt{G} \sim \int d^4x \sqrt{g} g^{\mu\nu} \delta g_{\mu\nu} \sim \int d^4x \sqrt{g} \delta\phi^i \Delta_g \phi^j \delta_{ij}$$

vanishes for harmonic embeddings

$$\Delta_g \phi^i = 0$$

term is huge but irrelevant $\Rightarrow$

$c_1 \Lambda_1^4$ not cosmological constant,
harmonically embedded (e.g. flat) spaces protected from cosm. const. problem

look for configurations with

$$\begin{aligned} g_{\mu\nu} &= G_{\mu\nu}, \\ \Delta_G \phi^i &= 0 = \Delta_g \phi^i \end{aligned}$$

holds for symplectic structure $\theta^{-1}_{\mu\nu}$ with

$$\begin{aligned} \star(\theta^{-1}) &= \pm\theta^{-1} && \text{Euclidean} \\ \star(\theta^{-1}) &= \pm i\theta^{-1} && \text{Minkowski (Wick rotation } X^0 \to iT \text{)} \end{aligned}$$

implies $\eta = e^\sigma$ and

$$\nabla^\mu \theta^{-1}_{\mu\nu} = 0$$

... solves e.o.m. for bare M. M. & term $\int d^4x \sqrt{G}\Lambda^4$

cosmological solutions (D. Klammer, H.S., arXiv:0903.0986, 2nd talk) next steps:

- search for Schwarzschild-like solution
  (perturbation of cosmolog. solution)
- connection with particle physics:
  compact extra dimensions $\mathcal{M}^4 \times S^2$
  (more restrictive, simpler than string thy)
  $\rightarrow$ effective gauge theories close to standard model (?)
  naturally $SU(3) \times SU(2) \times U(1) \times U(1)^*$,
  ... ongoing work
  T. Grammatikopoulos, P. Aschieri, H.S., G. Zoupanos 2006 ff
  (H. Grosse, F. Lizzi, H.S. in progress)

Summary:

- matrix-model $Tr[X^a, X^b][X^{a'}, X^{b'}]\,\eta_{aa'}\eta_{bb'}$

  $\rightarrow$

  dynamical NC spaces $\leftrightarrow$ emergent gravity
- *not* same as G.R., E-H action induced
- solves problem how to define NC $\mathfrak{su}(n)$ gauge theory
- suitable for quantizing gravity

  (IKKT model, $N = 4$ SUSY in $D = 4$)

Next:

- intriguing cosmological solution, no fine-tuning of cosm. const. (next talk, D. Klammer)
- Schwarzschild ... ??