Bayrischzell Workshop 2009

Noncommutativity and Physics: Quantum Geometries and Gravity

Constraining spacetime noncommutativity at high energies

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* Models based on the Seiberg-Witten mapping
* Expansion in power series in θ $\rightarrow$ new vertices
* Any gauge groups
* Arbitrary matter representation
* No charge quantization problem
* No UV/IR mixing due to θ expansion
* Unitarity is OK for: $\theta^{ij} \neq$, $\theta^{0i} = 0$;

carefull cannonical quantization produces always unitary theory: (*Bahns, Fredenhagen, Doplicher, Piaticelli: Time in S matrix treated in form of slices*)

* By covariant generalization of $\theta^{0i} = 0$ to:

$$\theta_{\mu\nu}\theta^{\mu\nu} \;=\; -\theta^2 = \frac{2}{\Lambda_{\rm NC}^4}\left(\vec{B}_\theta^2 - \vec{E}_\theta^2\right) > 0$$

known as *perturbative unitarity condition* one avoids potential difficulties with unitarity in noncommutative gauge field theories

* Covariant NCSM Yukawa couplings OK
* Direct neutrino-photon coupling in NC background
* Models 1 & 2: mNCSM & nmNCSM constructed as an effective, anomaly free, with 1-loop renormalizable gauge sector, GFT at first order in noncommutative parameter θ
* Model 3: SU(N) GFT constructed as an renormalizable theory via renormalization of θ $\rightarrow$ RGE for noncommutative deformation parameter h.
* In noncommutative chiral model for fermions there is NO typical 4ψ divergence, as for Dirac fermions.

NCSM ACTIONS

$$S_{\mathsf{NCSM}} = S_{\mathsf{fermions}} + S_{\mathsf{gauge}} + S_{\mathsf{Higgs}} + S_{\mathsf{Yukawa}}$$

$$\begin{aligned} S_{\mathsf{fermions}} &= \int d^4x \sum_{i=1}^{3} \Big(\overline{\widehat{L}}_L^{(i)} \star (i\widehat{\not{D}}\, \widehat{L}_L^{(i)}) + \overline{\widehat{Q}}_L^{(i)} \star (i\widehat{\not{D}}\, \widehat{Q}_L^{(i)}) \\ &+ \overline{\widehat{e}}_R^{(i)} \star (i\widehat{\not{D}}\, \widehat{e}_R^{(i)}) + \overline{\widehat{u}}_R^{(i)} \star (i\widehat{\not{D}}\, \widehat{u}_R^{(i)}) + \overline{\widehat{d}}_R^{(i)} \star (i\widehat{\not{D}}\, \widehat{d}_R^{(i)}) \Big) \end{aligned}$$

$$S_{\mathsf{gauge}} = -\frac{1}{2} \int d^4x \sum_{\mathcal{R}} c_{\mathcal{R}} \mathbf{Tr} \Big(\mathcal{R}(\widehat{F}_{\mu\nu}) \star \mathcal{R}(\widehat{F}^{\mu\nu}) \Big)$$

$\mathcal{R}$ – unitary, irreducible and inequivalent representations of a gauge group. Real coefficients $c_{\mathcal{R}}$ that are subject to the constraints.

$$\frac{1}{g_{SM}^2} = \sum_{\mathcal{R}} c_{\mathcal{R}} \mathbf{Tr} \Big(\mathcal{R}(T_{SM}^a) \mathcal{R}(T_{SM}^a) \Big).$$

$$\begin{aligned} S_{\mathsf{Higgs}} &= \int d^4x \Big(h_0^\dagger (\widehat{D}_\mu \widehat{\Phi}) \star h_0 (\widehat{D}^\mu \widehat{\Phi}) - \mu^2 h_0^\dagger (\widehat{\Phi}) \star h_0 (\widehat{\Phi}) \\ &- \lambda h_0^\dagger (\widehat{\Phi}) \star h_0 (\widehat{\Phi}) \star h_0^\dagger (\widehat{\Phi}) \star h_0 (\widehat{\Phi}) \Big) \end{aligned}$$

$$\begin{aligned} S_{\mathsf{Yukawa}} &= -\int d^4x \sum_{i,j=1}^{3} \\ &\times \Big(G_e^{(ij)} (\overline{\widehat{L}}_L^{(i)} \star h_e(\widehat{\Phi}) \star \widehat{e}_R^{(j)}) + G_e^{\dagger\,(ij)} (\overline{\widehat{e}}_R^{(i)} \star h_e(\widehat{\Phi})^\dagger \star \widehat{L}_L^{(j)}) \\ &+ G_u^{(ij)} (\overline{\widehat{Q}}_L^{(i)} \star h_u(\widehat{\Phi}_c) \star \widehat{u}_R^{(j)}) + G_u^{\dagger\,(ij)} (\overline{\widehat{u}}_R^{(i)} \star h_u(\widehat{\Phi}_c)^\dagger \star \widehat{Q}_L^{(j)}) \\ &+ G_d^{(ij)} (\overline{\widehat{Q}}_L^{(i)} \star h_d(\widehat{\Phi}) \star \widehat{d}_R^{(j)}) + G_d^{\dagger\,(ij)} (\overline{\widehat{d}}_R^{(i)} \star h_d(\widehat{\Phi})^\dagger \star \widehat{Q}_L^{(j)}) \Big) \end{aligned}$$

PURE GAUGE SECTOR

Renormalisable pure gauge sector action

1: Commutative GFT, that are renormalizable are extended to the NC space with deformed gauge transformations. These deformations are not unique. For instance deformed action S_g depends on the choice of representation. This derives from the fact that $\widehat{F}^{\mu\nu}$ is enveloping algebra, not Lee algebra valued.

$$S_g^{min} = -\frac{1}{2}\,\mathsf{Tr}\int d^4x \widehat{F}_{\mu\nu} \star \widehat{F}^{\mu\nu}.$$

The trace $\mathbf{Tr}$ is over all representations.
$\widehat{\varphi}$'s are the noncommutative Weyl spinors.
2: Seiberg-Witten map up to 1st order in θ.
Points 1: and 2: leads to:

$$\begin{aligned} S_g^{min} &= -\frac{1}{2}\mathsf{Tr}\int d^4x\, F_{\mu\nu}F^{\mu\nu} \\ &+ h\,\theta^{\rho\sigma}\mathsf{Tr}\int d^4x \left[\left(\frac{1}{4}F_{\rho\sigma}F_{\mu\nu} - F_{\rho\mu}F_{\sigma\nu}\right)F^{\mu\nu}\right]. \end{aligned}$$

3: Clearly we do not know the meaning of 'minimal coupling concept' for some NCGFT in the NC space. However, renormalization is the principle that help us to find such acceptable couplings. We learned that the renormalizability condition of some specific NCGFT requires introduction of the higher order NC gauge interaction by expanding general NC action in terms of NC field strengths. This lead us to the deformation of 'minimal' action S_g to higher order

$$\begin{aligned} S_g &= \mathsf{Tr}\int \mathrm{d}^4x \Big[-\frac{1}{2}\widehat{F}_{\mu\nu}(x)\star\widehat{F}^{\mu\nu}(x) \\ &+ \mathrm{i}(a-1)\,x^{\mu}\star x^{\nu}\star\widehat{F}_{\mu\nu}(x)\star\widehat{F}_{\rho\sigma}(x)\star\widehat{F}^{\rho\sigma}(x)\Big], \end{aligned}$$

with a being free parameter determining renormalizable deformation.

4: SW map for NC field strength up to the first order in $h\theta^{\mu\nu}$ gives

$$\begin{aligned} S_g &= \mathsf{Tr}\int \mathrm{d}^4x \,\Big[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} \\ &\qquad + h\theta^{\mu\nu}\left(\frac{a}{4}F_{\mu\nu}F_{\rho\sigma}-F_{\mu\rho}F_{\nu\sigma}\right)F^{\rho\sigma}\Big] \end{aligned}$$

MATTER SECTOR

Minimal fermion action

In both the mNCSM and nmNCSM models, the chosen minimal fermion action was

$$S_{\psi}^{min} \;=\; \int d^4x \overline{\widehat{\psi}} \star i\widehat{\not{D}} \star \widehat{\psi} + \widehat{S}_{\text{Yukawa}}, \;\; \widehat{D}_\mu = \partial_\mu - i\widehat{V}_\mu \star,$$

depends on the ordinary fields V_μ and ψ via SW

The Yukawa sector in the NCSM, when expanded in terms of ordinary fields, involves quite complicated interactions . However, if we consider the noncommutative contributions to the fermion field interactions in the QED sector only, (as turns out to be sufficient for our purpose of obtaining rate for the quarkonia decay into two photons), we can use the simplified fermion Lagrangian

$$\begin{aligned}
S_{\psi,A}^{min} &\sim \int d^4x\, \overline{\widehat{\psi}} \star (i\widehat{\not{D}}^A - m_f) \star \widehat{\psi} = S_{\psi,A}^{SM} + S_{\psi,A}^{\theta}\,, \\
S_{\psi,A}^{SM} &= \int d^4x \overline{\psi}\,(i\not{D}^A - m_f)\,\psi, \\
S_{\psi,A}^{\theta} &= -\frac{eh}{4}\int d^4x\, \overline{\psi}\, A_{\mu\nu}\,(i\theta^{\mu\nu\rho}\, D_\rho^A - m_f\,\theta^{\mu\nu})\psi,
\end{aligned}$$

$$D_\mu^A = \partial_\mu - ieA_\mu;\;\; \theta^{\mu\nu\rho} = \theta^{\mu\nu}\gamma^\rho + \theta^{\nu\rho}\gamma^\mu + \theta^{\rho\mu}\gamma^\nu\,.$$

Deformed fermion action

Possible deformations contributing to the fermion action, and satisfying the following conditions:

(a) They are real and include two fermions fields,

(b) they are invariant under noncommutative gauge transformations and thus involve star products, non-commutative gauge covariant derivatives and non-commutative field strengths,

(c) involve a contraction with a $\theta^{\mu\nu}$ tensor outside the star product —as in the $a-$dependent term—,

(d) do not alter the tree-level 2-point function of fermion propagator,

(e) include zero or positive powers of some mass parameter. antysimmetrised products of γ matrices: $\{I, \gamma^{\mu}, \sigma^{\mu\nu}, \gamma^{\mu\nu\rho}, \gamma^{5}\}$,

Terms satisfying the conditions stated above are given by sums of integrals of the following monomials t_i multiplied by real coefficients:

$$
\begin{aligned}
t_1 &= h\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\gamma^{\mu}(\widehat{\mathcal{D}}_{\mu}\widehat{F}_{\alpha\beta})\star\widehat{\psi},\\
t_2 &= h\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\gamma_{\beta}(\widehat{\mathcal{D}}^{\mu}\widehat{F}_{\mu\alpha})\star\widehat{\psi},\\
t_3 &= ih\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\gamma^{\rho}(2\widehat{F}_{\alpha\beta}\star\widehat{D}_{\rho}+(\widehat{\mathcal{D}}_{\rho}\widehat{F}_{\alpha\beta}))\star\widehat{\psi},\\
t_4 &= ih\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\gamma^{\rho}(2\widehat{F}_{\beta\rho}\star\widehat{D}_{\alpha}+(\widehat{\mathcal{D}}_{\alpha}\widehat{F}_{\beta\rho}))\star\widehat{\psi},\\
t_5 &= ih\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\gamma_{\beta}(2\widehat{F}_{\mu\alpha}\star\widehat{D}^{\mu}+(\widehat{\mathcal{D}}^{\mu}\widehat{F}_{\mu\alpha}))\star\widehat{\psi},\\
t_6 &= ih\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\gamma_{\alpha\beta}{}^{\rho}(\widehat{\mathcal{D}}^{\mu}\widehat{F}_{\mu\rho})\star\widehat{\psi},\\
t_7 &= ih\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\gamma_{\beta}{}^{\mu\nu}(\widehat{\mathcal{D}}_{\alpha}\widehat{F}_{\mu\nu})\star\widehat{\psi},\\
t_8 &= h\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\gamma_{\alpha\beta}{}^{\rho}(2\widehat{F}_{\mu\rho}\star\widehat{D}^{\mu}+(\widehat{\mathcal{D}}^{\mu}\widehat{F}_{\mu\rho}))\star\widehat{\psi},\\
t_9 &= h\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\gamma_{\beta}{}^{\mu\nu}(2\widehat{F}_{\mu\nu}\star\widehat{D}_{\alpha}+(\widehat{\mathcal{D}}_{\alpha}\widehat{F}_{\mu\nu}))\star\widehat{\psi},\\
t_{10} &= h\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\gamma_{\beta}{}^{\mu\nu}(2\widehat{F}_{\nu\alpha}\star\widehat{D}_{\mu}+(\widehat{\mathcal{D}}_{\mu}\widehat{F}_{\nu\alpha}))\star\widehat{\psi},\\
t_{11} &= m\,h\,\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\widehat{F}_{\alpha\beta}\star\widehat{\psi},\\
t_{12} &= im\,h\,\theta^{\alpha\beta}\bar{\widehat{\psi}}\star\sigma_{\beta}{}^{\rho}\widehat{F}_{\alpha\rho}\star\widehat{\psi},\\
t_{13} &= im\,h\,\tilde{\theta}^{\alpha\beta}\bar{\widehat{\psi}}\star\gamma^{5}\widehat{F}_{\alpha\beta}\star\widehat{\psi},
\end{aligned}
$$

$\tilde{\theta}^{\alpha\beta}\equiv\frac{1}{2}\epsilon^{\alpha\beta\rho\sigma}\theta_{\rho\sigma}$, $\widehat{\mathcal{D}}_{\mu}=\partial_{\mu}-i[\widehat{V}_{\mu}\stackrel{\star}{,}\quad]$.

$$
S_{\psi}^{H}(x_i)=\int d^4x\sum_i x_i t_i,\qquad x_i\in\mathbb{R}\,,
$$

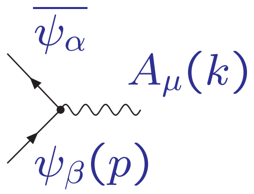
x_i - free deformation parameters, determind via renormalization and phenomenology.

Field redefinitions reduce above to

$$
\begin{aligned}
S_{\psi}^{H}(x_i) &= \int d^4x\Big[x_4t_4+x_5t_5+x_6t_6\\
&\qquad +x_8t_8+x_{10}t_{10}+x_{11}t_{11}\Big].
\end{aligned}
$$

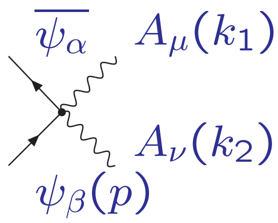
Feynman rules

$$S = S_g + S_\psi = S_g^{min} + S_g^H(a) + S_\psi^{min} + S_\psi^H(x_i)$$

- 

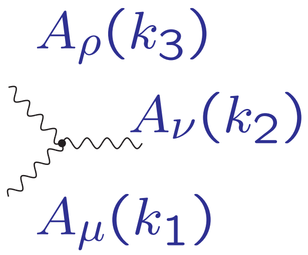
$$iee_q\left[\gamma^\mu - i\frac{h}{2}k_\nu\left(\theta_H^{\mu\nu\rho}p_{\text{in}}^\rho + \tilde{\theta}_H^{\mu\nu\rho}k_{\text{in}}^\rho - (1+4x_{11})m_f\theta_{\mu\nu}\right)\right]_{\alpha\beta}$$

$$\begin{aligned}
\theta_H^{\mu\nu\rho} &= \theta^{\mu\nu\rho} - 4x_4(\theta^{\rho\mu}\gamma^\nu + \theta^{\nu\rho}\gamma^\mu) - 4x_5(-g^{\nu\rho}\theta^{\mu\alpha}\gamma_\alpha + g^{\mu\rho}\theta^{\nu\alpha}\gamma_\alpha) \\
&\quad +8ix_8 g^{\mu\rho}\theta^{\sigma\eta}\gamma_{\sigma\eta}{}^\nu + 8ix_{10}\theta^{\nu\sigma}\gamma_\sigma{}^{\rho\mu}\,, \\
\tilde{\theta}_H^{\mu\nu\rho} &= \frac{1}{2}(\theta_H^{\mu\nu\rho} - \theta^{\mu\nu\rho}) - 4x_6 g^{\rho\mu}\theta^{\sigma\eta}\gamma_{\sigma\eta}{}^\nu \quad e_q = \frac{2}{3},\ -\frac{1}{3}
\end{aligned}$$

- 

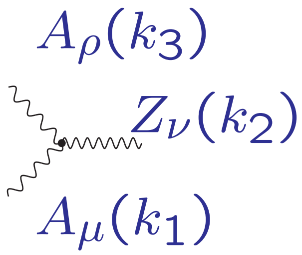
$$-\frac{h}{2}e^2e_q^2\left[\theta_H^{\mu\nu\rho}\right]_{\alpha\beta}(k_1^\rho - k_2^\rho)\,,$$

•



$$2\,e\,\sin 2\theta_W \mathsf{K}_{\gamma\gamma\gamma}\,\Theta_3^{\mu\nu\rho}(a;k_1,k_2,k_3)\,,$$

•

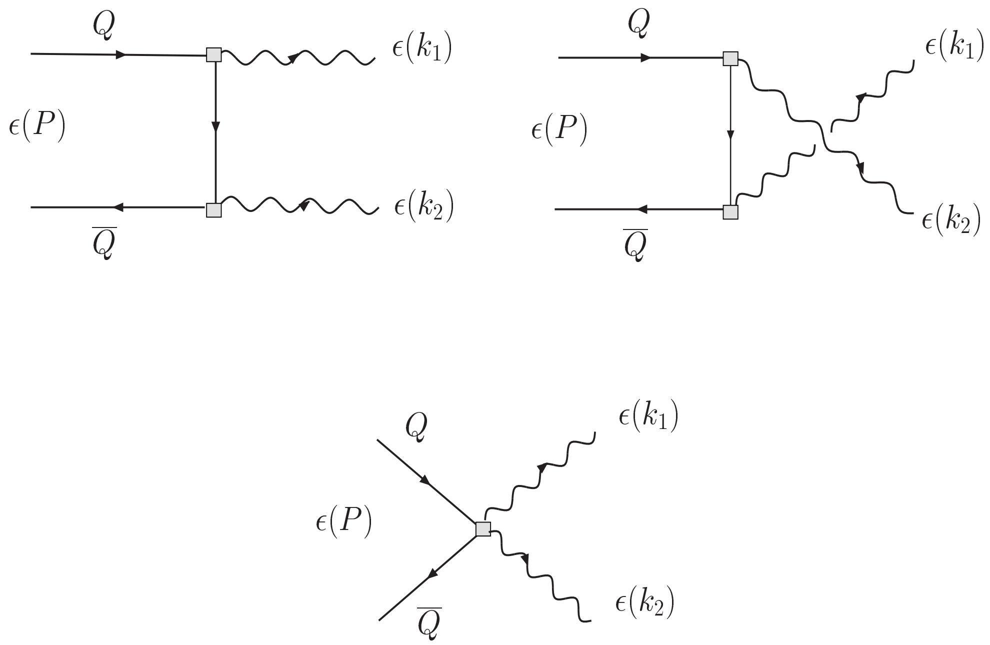


$$-2\,e\,\sin 2\theta_W \mathsf{K}_{Z\gamma\gamma}\,\Theta_3^{\mu\nu\rho}(a;k_1,k_2,k_3)\,,$$

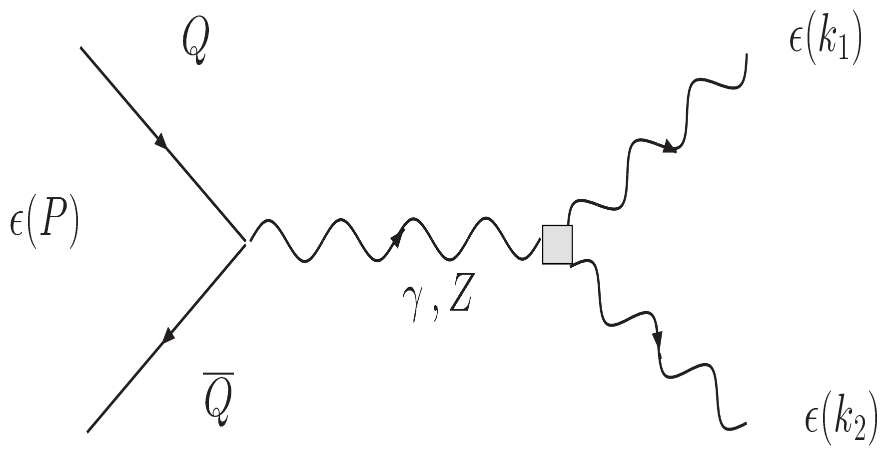
$$\begin{aligned}
\Theta_3^{\mu\nu\rho}(a;k_1,k_2,k_3) \;=\; & -(k_1\theta k_2) \\
& \times[(k_1-k_2)^\rho g^{\mu\nu}+(k_2-k_3)^\mu g^{\nu\rho}+(k_3-k_1)^\nu g^{\rho\mu}] \\
& -\theta^{\mu\nu}\,[k_1^\rho\,(k_2k_3)-k_2^\rho\,(k_1k_3)] \\
& -\theta^{\nu\rho}\,[k_2^\mu\,(k_3k_1)-k_3^\mu\,(k_2k_1)] \\
& -\theta^{\rho\mu}\,[k_3^\nu\,(k_1k_2)-k_1^\nu\,(k_3k_2)] \\
& +(\theta k_2)^\mu\,\left[g^{\nu\rho}\,k_3^2-k_3^\nu k_3^\rho\right]+(\theta k_3)^\mu\,\left[g^{\nu\rho}\,k_2^2-k_2^\nu k_2^\rho\right] \\
& +(\theta k_3)^\nu\,\left[g^{\mu\rho}\,k_1^2-k_1^\mu k_1^\rho\right]+(\theta k_1)^\nu\,\left[g^{\mu\rho}\,k_3^2-k_3^\mu k_3^\rho\right] \\
& +(\theta k_1)^\rho\,\left[g^{\mu\nu}\,k_2^2-k_2^\mu k_2^\nu\right]+(\theta k_2)^\rho\,\left[g^{\mu\nu}\,k_1^2-k_1^\mu k_1^\nu\right] \\
& +\theta^{\mu\alpha}(ak_1+k_2+k_3)_\alpha\,\left[g^{\nu\rho}\,(k_3k_2)-k_3^\nu k_2^\rho\right] \\
& +\theta^{\nu\alpha}(k_1+ak_2+k_3)_\alpha\,\left[g^{\mu\rho}\,(k_3k_1)-k_3^\mu k_1^\rho\right] \\
& +\theta^{\rho\alpha}(k_1+k_2+ak_3)_\alpha\,\left[g^{\mu\nu}\,(k_2k_1)-k_2^\mu k_1^\nu\right]\,.
\end{aligned}$$

$k_1+k_2+k_3=0$

QUARKONIA TO $\gamma\gamma$ DECAYS



$$
\begin{aligned}
\mathcal{A}_1(x_4, x_5, x_{11}) \quad &= \quad ih\pi 4\sqrt{3M}\alpha e_q^2 |\Psi_{\bar{q}q_1}(0)| \epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(P) \\
&\times \quad \left\{ (k_2 - k_1)^\rho \left[(1 - 4x_4) \left(\theta^{\mu\nu} - 2g^{\mu\nu} \frac{(k_1\theta k_2)}{M^2} \right) \right.\right. \\
&+ \quad \left.(-4x_5) \left(\theta^{\mu\nu} - \frac{k_2^\mu (k_1\theta)^\nu - k_1^\nu (k_2\theta)^\mu}{M^2} \right) \right] \\
&+ \quad (1 - 4x_4 - 2x_5) \left[2g^{\mu\rho} \left((k_1\theta)^\nu - 2k_1^\nu \frac{(k_1\theta k_2)}{M^2} \right) \right. \\
&+ \quad \left. 2g^{\nu\rho} \left((k_2\theta)^\mu + 2k_2^\mu \frac{(k_1\theta k_2)}{M^2} \right) \right] \\
&- \quad 4x_5 (P\theta)^\rho \left(g^{\mu\nu} - \frac{2}{M^2} k_2^\mu k_1^\nu \right) \\
&+ \quad \left.(8x_{11}) \left[(k_1\theta)^\mu \left(g^{\nu\rho} - \frac{2}{M^2} k_2^\rho k_1^\nu \right) + (k_2\theta)^\nu \left(g^{\mu\rho} - \frac{2}{M^2} k_1^\rho k_2^\mu \right) \right] \right\},
\end{aligned}
$$



$$
\begin{aligned}
\mathcal{A}_2(a) &= -ih\pi\frac{16\sqrt{3}}{M^{3/2}}\alpha|\Psi_{\bar{q}q_1}(0)|\,\epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(P) \\
&\times \left[e_q \sin 2\theta_W K_{\gamma\gamma\gamma} + \left(\frac{M}{M_Z}\right)^2 c_V^q K_{Z\gamma\gamma}\right]\Theta_3^{\mu\nu\rho}[a; P, -k_1, -k_2] \\
&= -ih\,\pi\,8\sqrt{3M}\,\alpha\,|\Psi_{\bar{q}q_1}(0)|\,\epsilon_\mu(k_1)\epsilon_\nu(k_2)\epsilon_\rho(P) \\
&\times \left[e_q \sin 2\theta_W K_{\gamma\gamma\gamma} + \left(\frac{M}{M_Z}\right)^2 c_V^q K_{Z\gamma\gamma}\right] \\
&\quad \times\Big\{(k_2-k_1)^\rho\left(\theta^{\mu\nu} - 2g^{\mu\nu}\frac{(k_1\theta k_2)}{M^2}\right) \\
&\quad +2g^{\mu\rho}\left((k_1\theta)^\nu - 2k_1^\nu\frac{(k_1\theta k_2)}{M^2}\right) \\
&\quad +2g^{\nu\rho}\left((k_2\theta)^\mu + 2k_2^\mu\frac{(k_1\theta k_2)}{M^2}\right) \\
&\quad -(a-1)\left[(P\theta)^\rho\left(g^{\mu\nu} - \frac{2}{M^2}k_2^\mu k_1^\nu\right)\right. \\
&\quad \left. -(k_1\theta)^\mu\left(g^{\nu\rho} - \frac{2}{M^2}k_2^\rho k_1^\nu\right) - (k_2\theta)^\nu\left(g^{\mu\rho} - \frac{2}{M^2}k_1^\rho k_2^\mu\right)\right]\Big\}\,.
\end{aligned}
$$

Quarkonia decays: $J/\psi \to \gamma\gamma$, $\Upsilon(1S) \to \gamma\gamma$

$1 \geq \Lambda_{\rm NC}/\text{TeV} \geq 0.25$

$\Gamma^{\rm exp.}(\Upsilon(1S) \to e^+e^-) = (1.314 \pm 0.029)\,\text{keV}$, $\Gamma^{\rm exp.}_{\rm tot}(\Upsilon(1S)) = (53.0 \pm 1.5)\,\text{keV}$
$\Gamma^{\rm exp.}(J/\psi \to e^+e^-) = (5.4 \pm 0.15 \pm 0.07)\,\text{keV}$, $\Gamma^{\rm exp.}_{\rm tot}(J/\psi) = (91.0 \pm 3.2)\,\text{keV}$

$$3.1 \cdot 10^{-12} \lesssim BR^{a=1,\, x_i=0}_{[J/\psi\to\gamma\gamma],max} \lesssim 7.8 \cdot 10^{-10},$$
$$1.7 \cdot 10^{-11} \lesssim BR^{a=1,\, x_i=0}_{[\Upsilon\to\gamma\gamma],max} \lesssim 4.3 \cdot 10^{-9},$$
$$2.3 \cdot 10^{-13} \lesssim BR^{a=1,\, x_i=0}_{[J/\psi\to\gamma\gamma],min} \lesssim 5.9 \cdot 10^{-11},$$
$$1.0 \cdot 10^{-26} \lesssim BR^{a=1,\, x_i=0}_{[\Upsilon\to\gamma\gamma],min} \lesssim 2.6 \cdot 10^{-24}\,,$$

$$2.4 \cdot 10^{-12} \lesssim BR^{a=3,\, x_i=0}_{[J/\psi\to\gamma\gamma],max} \lesssim 6.3 \cdot 10^{-10},$$
$$7.5 \cdot 10^{-11} \lesssim BR^{a=3,\, x_i=0}_{[\Upsilon\to\gamma\gamma],max} \lesssim 1.9 \cdot 10^{-8},$$
$$5.1 \cdot 10^{-13} \lesssim BR^{a=3,\, x_i=0}_{[J/\psi\to\gamma\gamma],min} \lesssim 1.3 \cdot 10^{-10},$$
$$4.6 \cdot 10^{-12} \lesssim BR^{a=3,\, x_i=0}_{[\Upsilon\to\gamma\gamma],min} \lesssim 1.2 \cdot 10^{-9}\,,$$

$$4.5 \cdot 10^{-11} \lesssim BR^{a=3,\, |x_i|=1}_{[J/\psi\to\gamma\gamma],max} \lesssim 1.2 \cdot 10^{-8},$$
$$5.6 \cdot 10^{-10} \lesssim BR^{a=3,\, |x_i|=1}_{[\Upsilon\to\gamma\gamma],max} \lesssim 1.4 \cdot 10^{-7},$$
$$BR^{a=3,\, |x_i|=1}_{[J/\psi\to\gamma\gamma],min} \simeq 0, \quad BR^{a=3,\, |x_i|=1}_{[\Upsilon\to\gamma\gamma],min} \simeq 0\,.$$

$J/\psi : x_4 = -1.00, x_5 = -1.00, x_{11} = 1.00, K_{Z\gamma\gamma} = -0.254, K_{\gamma\gamma\gamma} = 0.129,$
$\Upsilon : x_4 = -1.00, x_5 = -1.00, x_{11} = 1.00, K_{Z\gamma\gamma} = 0.00950, K_{\gamma\gamma\gamma} = -0.576,$

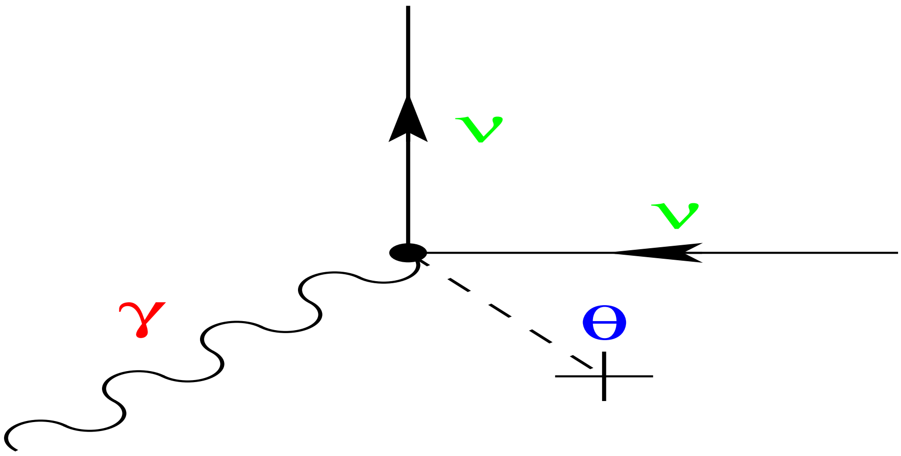
$J/\psi : x_4 = 0.250, x_5 = -2.85 \cdot 10^{-7}, x_{11} = 2.86 \cdot 10^{-7}, K_{Z\gamma\gamma} = 0.202, K_{\gamma\gamma\gamma} = 7.93 \cdot 10^{-5},$
$\Upsilon : x_4 = 0.250, x_5 = 2.05 \cdot 10^{-7}, x_{11} = -2.03 \cdot 10^{-7}, K_{Z\gamma\gamma} = -0.192, K_{\gamma\gamma\gamma} = 2.56 \cdot 10^{-3}.$

NEUTRINO SECTOR

ASTROPHYSICS

"Transverse plasmon" decay: $\gamma_{\rm pl} \to \nu\bar{\nu}$

[P. Schupp, J.Trampetic, J. Wess and G. Raffelt, "The photon neutrino interaction in non-commutative gauge field theory and astrophysical bounds," Eur. Phys. J. C **36** (2004) 405]



Neutrino-photon interaction introduced via:
$\star$–commutator with covariant derivative

$$\widehat{D}_\mu \widehat{\psi} = \partial_\mu \widehat{\psi} - i\kappa e \left[\widehat{A}_\mu \star \widehat{\psi} - \widehat{\psi} \star \widehat{A}_\mu\right]$$

The action for a neutral fermion that couples, in the adjoint of non-commutative U(1), to an Abelian gauge boson in the NC background is:

$$S = \int d^4x \left(\bar{\widehat{\psi}} \star i\gamma^\mu \widehat{D}_\mu \widehat{\psi} - m\bar{\widehat{\psi}} \star \widehat{\psi}\right)$$

SW map: $\widehat{\psi} = \psi + e\theta^{\nu\rho} A_\rho \partial_\nu \psi + \mathcal{O}(\theta^2)$
$\widehat{A}_\mu = A_\mu + e\theta^{\rho\nu} A_\nu \left[\partial_\rho A_\mu - \frac{1}{2}\partial_\mu A_\rho\right] + \mathcal{O}(\theta^2)$

The gauge invariant action of order θ^1 and $\kappa = 1$

$$\begin{aligned} S &= \int d^4x\,\bar{\psi}\,\Big[(i\gamma^\mu\partial\mu - m) \\ &- \frac{e}{2}F_{\mu\nu}(i\theta^{\mu\nu\rho}\partial\rho - \theta^{\mu\nu}m)\Big]\,\psi. \end{aligned}$$

Feynman rule for $\gamma(q) \to \nu(k')\bar{\nu}(k)$ vertex:

$$\Gamma^\mu_{\binom{\mathrm{L}}{\mathrm{R}}}(\nu\bar{\nu}\gamma) = ie\frac{1}{2}(1 \mp \gamma_5)\left[(q\theta k)\gamma^\mu + (\not{k} - m_\nu)(\theta q)^\mu - \not{q}(\theta k)^\mu\right].$$

For massless neutrinos the vertex becomes totally symmetric:

$$\Gamma^\mu_{\binom{\mathrm{L}}{\mathrm{R}}}(\nu\bar{\nu}\gamma) = ie\frac{1}{2}(1 \mp \gamma_5)\theta^{\mu\nu\tau}k_\nu q_\tau$$

In a stellar plasma, the dispersion relation of photons is identical with that of a massive particle

$$q^2 \equiv \mathrm{E}_\gamma^2 - \mathbf{q}_\gamma^2 = \omega_{\mathrm{pl}}^2$$

ω_{pl} – the plasma frequency.
The plasmon (off-shell photon) decay rate to the left and/or right massive neutrinos

$$\begin{aligned} \Gamma_{\mathrm{NC}}(\gamma_{\mathrm{pl}} \to \bar{\nu}_{\binom{\mathrm{L}}{\mathrm{R}}}\nu_{\binom{\mathrm{L}}{\mathrm{R}}}) &= \frac{\alpha}{48}\frac{\omega_{\mathrm{pl}}^6}{\mathrm{E}_\gamma\Lambda_{\mathrm{NC}}^4}\sqrt{1 - 4\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}} \\ &\times \left[\left(1 + 2\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2} - 12\frac{m_\nu^4}{\omega_{\mathrm{pl}}^4}\right)\sum_{i=1}^{3}(c^{0i})^2 + 2\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}\left(1 - 4\frac{m_\nu^2}{\omega_{\mathrm{pl}}^2}\right)\sum_{\substack{i,j=1\\ i<j}}^{3}(c^{ij})^2\right]. \end{aligned}$$

In the rest frame of the medium and for massless neutrinos the decay rate is

$$\Gamma_{\rm NC}(\gamma_{\rm pl} \to \nu_{\binom{\rm L}{\rm R}}\bar{\nu}_{\binom{\rm L}{\rm R}}) = \frac{\alpha}{48}\,\frac{1}{\Lambda_{\rm NC}^4}\,\frac{\omega_{\rm pl}^6}{E_\gamma}$$

The corresponding SM neutrino-penguin-loop rate:

$$\Gamma_{\rm SM}\left(\gamma_{\rm pl} \to \nu_{\rm L}\bar{\nu}_{\rm L}\right) = \frac{c_V^2 G_F^2}{48\pi^2\alpha}\,\frac{\omega_{\rm pl}^6}{E_\gamma}.$$

For ν_e: $\quad c_V = \frac{1}{2} + 2\sin^2\Theta_W$

$\nu_\mu,\ \nu_\tau$: $\quad c_V = -\frac{1}{2} + 2\sin^2\Theta_W$

For the SM: $c_V^2 = 0.79$.

$$\mathcal{R} \equiv \frac{\sum_{\rm flavours}\Gamma_{\rm NC}\left(\gamma_{\rm pl} \to \nu_{\rm L}\bar{\nu}_{\rm L} + \nu_{\rm R}\bar{\nu}_{\rm R}\right)}{\sum_{\rm flavours}\Gamma_{\rm SM}(\gamma_{\rm pl} \to \nu_{\rm L}\bar{\nu}_{\rm L})} = \frac{6\pi^2\alpha^2}{c_V^2 G_F^2 \Lambda_{\rm NC}^4},$$

$$\Lambda_{\rm NC} = \frac{80.8}{\mathcal{R}^{1/4}}\,(\rm GeV).$$

A standard globular cluster stars argument:

any new energy-loss mechanism must not exceed the standard neutrino losses by much:

$\longrightarrow$ approximate requirement: $\mathcal{R} < 1$ $\longrightarrow$

$$\longrightarrow \quad \Lambda_{\rm NC} > \left(\frac{6\pi^2\alpha^2}{c_V^2 G_F^2}\right)^{1/4} \cong 81\ \text{GeV}$$

COSMOLOGY

[R. Horvat and J.Trampetic, "Constraining space time noncommutativity with primoradial nucleosynthesis," Phys.Rev. D **79** (2009)]

*NC quantum corrections to photon and neutrino self-energies start to appear at θ^2.

*In our computations of S-matrix at θ^1 LSZ is OK and poles of Green functions are as usual!

*Coupling of sterile neutrinos ν_R to photons is generation -blind, see Feynman rule: $(1 \mp \gamma_5)\theta^{\mu\nu\tau}k_\nu q_\tau$.

*Energy density of 3 light ν_R at nucleosynthesis time ($T \sim 1\text{MeV}$) is equivalent to the effective additional number of doublet neutrino specis ΔN_ν:

$$3\left(\frac{T_{\nu_R}}{T_{\nu_L}}\right)^4 \lesssim \Delta N_{\nu,max} ,$$

$$\frac{T_{\nu_R}}{T_{\nu_L}} = \left[\frac{g_{*S}(T_{\nu_L})}{g_{*S}(T_{dec})}\right]^{1/3} ,$$

here g_{*S} are degrees of freedom specifying the entropy of the still interacting species

$$g_{*S}(T_{dec}) \gtrsim \frac{24.5}{(\Delta N_{\nu,max})^{3/4}} .$$

*Using above vertex we compute total cross section for scattering of ν_R on charged fermions

$$\sigma_{scatt}(f^{\pm}\,\nu_R \to f^{\pm}\,\nu_R) \simeq 36\,\alpha^2 \frac{E^2}{\Lambda_{\rm NC}^4}, \quad E \simeq 9T\,.$$

*ν_R decouple at T_C when thermaly averaged scattering rate Γ_{scatt} and H -expansion rate of the Universe in radiation-dominated epoch are about equal.

$$\begin{aligned}
\Gamma_{scatt}(T_{dec}) &\simeq H(T_{dec}) \\
\Gamma_{scatt}(T_{dec}) &= <n_{scatt}\,\sigma_{scatt}\,v>, \quad n_{scatt} \simeq 0.18T^3\,' \\
H(T_{dec}) &\simeq 1.66 g_*^{1/2} T^2/M_{Pl}, \quad g_* \simeq g_{*S}\,.
\end{aligned}$$

This and σ_{scatt} gives

$$T_{dec} \simeq 0.5\,\alpha^{-2/3} M_{Pl}^{-1/3} \Lambda_{\rm NC}^{4/3}\,.$$

*Imposing conservative bound $\Delta N_{\nu,max} = 1, (e,\mu,s)$ enforces constraint $T_{dec} > T_C$ - critical temperature for deconfinment restoration phase transition $T_C \sim 200$MeV from where we found $\Lambda_{\rm NC} > 3$TeV.
*For $\Delta N_{\nu,max} < 0.2$, (all charged lepton and quarks) we have $T_{dec} < 300$ GeV and $\Lambda_{\rm NC} > 10^3$TeV.
* Perturbative expansion in terms of $\Lambda_{\rm NC}$ is OK only if $E^2/\Lambda_{\rm NC}^2 < 1$, E-caracteristic energy of the process.
*For cosmological settings this translates into $T_{dec}^2/\Lambda_{\rm NC}^2 < 1$, and our bound always fulfill this constraint.

DISCUSION

Limits on Λ_{NC} from theory and experiment

DECAYS: 1$\rightarrow$ 2

* $Z \rightarrow \gamma\gamma$ $\Rightarrow \Lambda_{NC} > 1$ TeV, [Buric, Latas, Radovanovic, JT]

* $\gamma_{pl} \rightarrow \nu\bar{\nu}$ $\Rightarrow \Lambda_{NC} >$ 81 GeV, [Schupp, JT, Wess, Raffelt]

* $J/\psi \rightarrow \gamma\gamma$ $\Rightarrow \Lambda_{NC} >$ 9 GeV, [Melic, Passek, J.T.]

* $J/\psi \rightarrow \gamma\gamma$ $\Rightarrow \Lambda_{NC} > 250$ GeV, [Tamarit, J.T.]

* $K \rightarrow \pi\gamma$ $\Rightarrow \Lambda_{NC} >$ 43 GeV, [Melic, Passek, J.T.]

SCATTERINGS: 2$\rightarrow$ 2

* $e^+e^- \rightarrow \gamma\gamma \Rightarrow \Lambda_{NC} > 141$ GeV, [OPAL Coll.-2003]

* $\gamma\gamma \rightarrow \bar{f}f$ $\Rightarrow \Lambda_{NC} > 200$ GeV, [T. Ohl et al.]

* $\bar{f}f \rightarrow Z\gamma$ $\Rightarrow \Lambda_{NC} > 1000$ GeV, [T. Ohl et al.]

* $e^+e^- \rightarrow W^+W^- \Rightarrow \Lambda_{NC} \in [.1, 1]$ TeV, [Conley, Hewett]

* $WW \rightarrow WW \Rightarrow \Lambda_{NC} \in [.5, 5]$ TeV, [Conley, Hewett]

g-2:

* $\mu^+ - \mu^- \Rightarrow \Lambda_{NC} > 1000$ GeV, [A. Joseph]

ASTROPHYSICS

NEUTRINO DIPOLE MOMENTS:

* $(\mathrm{d_{mag}})^{\mathrm{Dirac}} \Rightarrow \Lambda_{\mathrm{NC}} >$ 1.8 TeV, [Minkowski et al.]

* $(\mathrm{d^{el}_{mag}})^{\mathrm{Majorana}} \Rightarrow \Lambda_{\mathrm{NC}} >$ 150 TeV, [Minkowski et al.]

* $\frac{dipole_{NC}}{anapole_{SM}} \Rightarrow \Lambda_{\mathrm{NC}} >$ 10 TeV, [Ettefaghi, Haghighat]

SCATTERINGS - Supernova SN1987A:

* $f\bar{f} \to \nu_R\bar{\nu}_R \Rightarrow \Lambda_{\mathrm{NC}} >$ 3.7 TeV, [Ettefaghi, Haghighat]

* $f\nu_R \to f\nu_R \Rightarrow \Lambda_{\mathrm{NC}} <$ 1.1 TeV, [Ettefaghi, Haghighat]

COSMOLOGY

* $\mathrm{BB}_{(e,\mu,s)} \Rightarrow \Lambda_{\mathrm{NC}} > 3$ TeV, [R. Horvat, J.T.]

* $\mathrm{BB}_{(all\ \ fermions)} \Rightarrow \Lambda_{\mathrm{NC}} > 10^3$ TeV, [R. Horvat, J.T.]

SUMMARY

$\star$ Principle of renormalizability implemented on our θ-expanded NCGFT led us to well defined deformations parametrized by free parameters a .
$\star$ Divergences cancel differently than in commutative GFT and this depends on the representations.
$\star$ NC Model: nmNCSM gauge sector is renormalizable and FINITE for $a = 3$. No renormalizaion of h.
$\star$ Deformed fermion lagrangian parametrized by free parameters x_i as an effective theory produces promising panorama. There is a hope that in the future renormalisability will fix x_i's.
$\star$ Complete NCSM (minimal and/or nonminimal) up to θ^2 and consequently deeper understanding of S-matrix and LSZ formalism still needed.
$\star$ Large number of heavy quark-antquark pairs harvested at LHCb; i.e. 10^{12} $B\bar{B}$ per year, and probably (10^{14} $D\bar{D}$; 10^{18} $K\bar{K}$), give us hope that experimental branching ratios $BR(\Upsilon(J/\psi) \to \gamma\gamma) \sim 10^{-9}(10^{-11})$ could be accessible, thus reaching our maximum predicted values for the rates.
$\star$ Certainly all discussed high energy experiments, from collider physics to neutrino-photon astrophysics and cosmology would produce at least reliable bounds.
$\star$ Perhaps some of these experiments could even measure some of these processes, depending on the scale of noncommutativity.