State sum models, induced gravity and the spectral action

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Outline

Background & previous work

New proposals

Detailed remarks

Spectral action

- Classical action, spacetime M
- Non-commutative geometry F
- Kaluza-Klein unification on $M \times F$
- D, Ψ on $M \times F \longrightarrow$ SM + gravity fields on M
- Bosonic action spectral at unification $\longrightarrow$ predictions

Connes-Chamseddine action (Euclidean)

$$\mathrm{Tr}\big(f(D)\big) + \big(\overline{\Psi}, JD\Psi\big)$$

JWB Fermionic action (Lorentzian)

$$\big(\overline{\Psi}, D\Psi\big)$$

State sum models

- Discrete functional integrals on a triangulated manifold
- Input: gauge group (or category $\mathcal{C}$)
- Optional input: data for a local observable
- Output: partition function $Z \in \mathbb{C}$

Examples from physics

- lattice gauge theory
- 2d Yang-Mills
- 3d quantum gravity
- 4d quantum gravity models

Induced gravity (Sakharov)

$-+++$ metric and N fermion fields. Cutoff $= c$.

If

$$I = \int \left(\overline{\psi} D_c \psi - 2\Lambda_0\right) \mathrm{d}V$$

Integrating over matter modes gives

$$S_{\mathrm{eff}} = \int \frac{-c^2 N}{32\pi^2} - 2\Lambda_0 + \frac{cN}{192\pi^2} R + \text{etc.} \quad \mathrm{d}V$$

- Correct sign for R (MTW signs)
- Cosmological constant $\Lambda = \Lambda_0 + \frac{c^2 N}{64\pi^2}$
- Needs cutoff (from QG)

Spectral action proposal

D, Ψ on $M \times F$, $F =$ internal space.

Suggest spectral action

$$I = (\Psi, D\Psi)_c \; - \; 2\Lambda_0 \, \mathrm{vol}$$

- Bosonic action induced
- Renormalisability not an issue
- SM exactly correct? Lorentzian vs Euclidean

State sum models - wish list

Constructed from $\mathcal{C} \in$ tricat

Reasonable wish list:

- Diffeomorphism-invariant QFT on 4-manifolds
- Sum over geometries
- Discrete geometry at Planck scale
- Matter couplings
- Matter modes cut off at Planck energy c.
- Cosmological constant ($\Lambda_0 < 0$)
- Continuum limit: SM spectral action

- No Einstein-Hilbert action

Tricategory

A tricategory $\mathcal{C}$ contains

- Objects (0-morphisms)
- 1-morphisms mapping objects
- 2-morphisms mapping 1-morphisms (between the same objects)
- 3-morphisms mapping 2-morphisms (between the same 1-morphisms)
- 3 composition laws
- Structural maps for associativity, compatibility, units

Examples:

- A braided monoidal category (0- 1-morphisms trivial)
- Rep(crossed module) (0-morphisms trivial)
- Bicat

Compositions: 3d diagrams (e.g. knot invariants)

State sum model

Require tricategory $\mathcal{C}$:

- Diagrams have diffeomorphism invariance on S^3
- Linearity, $\mathrm{End}(1_{1_1}) = \mathbb{C}$. (Quantum Mechanics)
- Finiteness, semisimplicity

Labelling: k-simplexes $\longrightarrow$ k-morphisms   k=0,1,2,3

Weight of a simplex: closed diagram $\in \mathbb{C}$

State sum: $Z = \sum_{\text{labellings}} \prod_{\text{simplexes}} \text{weights}$

Triangulation independence

Summing over a suitable basis of all morphisms $\longrightarrow$ triangulation independence.

Examples:

- 3d quantum gravity
- Crane-Yetter model for $\mathcal{C} = \mathrm{Rep}(\mathrm{U}_q G)$

Constraints or extra data destroy triangulation independence:

- 4d quantum gravity models
- Lattice gauge theory

'Diffeomorphism' invariance

M, N triangulated manifolds, $d \leq 4$.

$$\begin{aligned} \text{'Diffeomorphism'} &= \text{PL homeomorphism}\colon M \to N \\ &= \text{Simplicial isomorphism}\colon M' \to N' \end{aligned}$$

for some subdivisions M', N'.

For $Z(M)$:

triangulation independence $\leftrightarrow$ 'diffeomorphism' invariance

Proof: subdivide.

Geometry

Models with a geometric interpretation:

- $\mathcal{C} = \text{Rep}(\mathrm{SU}(2))$ with constraints: Plebanski gravity
- $\mathcal{C} = \text{Rep}(\mathrm{SU}(2) \times \mathrm{SU}(2))$ with constraints: 4d Euclidean gravity models
- $\mathcal{C} = \text{Rep}(\mathrm{SO}(3,1))$ with constraints: 4d Lorenzian gravity models
- $\mathcal{C} = \text{Rep}(\mathrm{SO}(5))$, etc., with constraints: MacDowell-Mansouri gravity
- $\mathcal{C} = \text{Rep(Poincare 2-group)}$: quantum flat space

Cosmological constant: $G \to \mathrm{U}_q G$, or built in for $\mathrm{SO}(5)$.

More models needed. SM?

Matter couplings +C. Meusburger

- Use "tricat". (Functors, Natural transformations . . .)
- In 4d, highest two levels $\longrightarrow$ symmetric monoidal category
- . . . fermions and bosons.
- Other levels: strings, membranes??

Fermion functional determinant+J. Louko

Can do this in $d = 1$

- Background gauge field
- Functional integral = state sum model
- Fermionic integral on S^1 induces Wilson loop expectation value

$d > 1$: open problem

Relation to 4d quantum gravity models (maybe)

- Asymptotic expansion of fermion integral $\longrightarrow$ series in R^k
- Each term R^k involves k-nearest neighbours
- Full series is non-local
- Truncation to EH term is nearest-neighbour only
- ... but triangulation independence lost
- Relates to current 4d quantum gravity models?