

NCGUTS: Anomalies, Renormalisability and SUSY, where do we stand?

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PLAN

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10 years of the Enveloping-algebra formalism

- It is already **10 years** since the publishing of
 - ◇ "Gauge theory on noncommutative spaces",
Madore, Schraml, Schupp & Wess, EPJC16(2000)16,
 - ◇ "Noncommutative gauge theory for Poisson manifolds",
Jurco, Schupp & Wess, NPB584(2000)784,
 - ◇ "Enveloping algebra-valued gauge transformations for non-abelian gauge groups on non-commutative spaces",
Jurco, Schraml, Schupp & Wess, EPJC17(2000)521,
- where it was put forward a formalism –**THE ENVELOPING-ALGEBRA FORMALISM**–, which led to
 - ◇ "Non-commutative standard model",
Calmet, Jurco, Schupp, Wess & Wohlgemant, EPJC23(2002)363,
 - ◇ "Noncommutative GUTs, standard model and C,P,T",
Aschieri, Jurco, Schupp & Wess, NPB 651(2003)45.

There is an excellent **recent review** by **Blaschke, Kronberger, Sedmik & Wohlgemant** [Arxiv:1004.2127](https://arxiv.org/abs/1004.2127)

NC fields as ordinary-field SW map images

In the **ENVELOPING-ALGEBRA FORMALISM**:

- The noncommutative fields are functions of the ordinary fields –**no change in the no. of d.o.f.**– such that ordinary gauge orbits are mapped into noncommutative gauge orbits:

$$A_\mu[a_\mu, \psi, \theta] + s_{NC} A_\mu[a_\mu, \psi, \theta]A = A_\mu[a_\mu + s a_\mu, \psi + s \psi, \theta],$$

$$\Psi[a_\mu, \psi, \theta] + s_{NC} \Psi[a_\mu, \psi, \theta] = \Psi[a_\mu + s a_\mu, \psi + s \psi, \theta],$$

$$s_{NC} \Lambda[\lambda, \lambda, \psi, \theta] = s \Lambda[\lambda, \lambda, \psi, \theta],$$

$$A_\mu[a_\mu, \psi, \theta = 0] = a_\mu, \Psi[a_\mu, \psi, \theta = 0] = \psi, \Lambda[\lambda, \lambda, \psi, \theta = 0] = \lambda$$

$$s_{NC} A_\mu = \partial_\mu \Lambda - i[A_\mu, \Lambda]_\star, s_{NC} \Psi = i\Lambda \star \Psi, s_{NC} \Lambda = i\Lambda \star \Lambda,$$

$$s a_\mu = \partial_\mu \lambda - i[a_\mu, \lambda], s \psi = i\lambda \psi, s \lambda = i\lambda \lambda,$$

- a_μ and λ take values on the Lie algebra, \mathfrak{g} , of a compact Lie group, G
 $\implies A_\mu$ and Λ take values on the universal enveloping algebra of \mathfrak{g} .

NC gauge theories for any compact Lie group

- The action, S , for a (nonsusy) NC GUT (-inspired) theory for a compact Lie group, G , reads

$$S = S_{gauge} + S_{fermionic} + S_{Higgs} + S_{Yukawa},$$

$$S_{gauge} = \int d^4x -\frac{1}{2} \sum_{\mathcal{R}} c_{\mathcal{R}} \text{Tr}_{\mathcal{R}} F_{\mu\nu}[\mathcal{R}(A)] \star F^{\mu\nu}[\mathcal{R}(A)],$$

$$S_{fermionic} = \int d^4x \bar{\Psi}_L i \not{D}[\rho_{\psi}(A)] \Psi_L,$$

S_{Higgs} and S_{Yukawa} not to be considered below,

$$F_{\mu\nu}[\mathcal{R}(A)] = \partial_{\mu} \mathcal{R}(A)_{\nu} - \partial_{\nu} \mathcal{R}(A)_{\mu} - i[\mathcal{R}(A)_{\mu}, \mathcal{R}(A)_{\nu}] \star,$$

$$D_{\mu}[\rho_{\psi}(A)] \psi_L = \partial_{\mu} \psi_L - i \rho_{\psi}(A_{\mu}) \star \psi_L,$$

- $\Psi[\rho_{\psi}(a), \psi_L]$ is the NC **left-handed** spinor multiplet which is the NC counterpart of the ordinary **left-handed** spinor multiplet ψ_L . ψ_L carries an arbitrary unitary representation, ρ_{ψ} , of \mathfrak{g} .
- \mathcal{R} labels the unitary IRREPS –typically the adjoint and matter irreps– of \mathfrak{g} and $\sum_{\mathcal{R}} c_{\mathcal{R}} \text{Tr}_{\mathcal{R}} \mathcal{R}(T_i^a) \mathcal{R}(T_i^a) = \frac{1}{g^2}$, $G = \otimes_1 G_i$.
- Hybrid SW maps (right and left NC gauge trans.) needed for S_{Yukawa}

Quantising

- The QUANTUM version of the classical field theory defined above is obtained by integrating in the path-integral over the ordinary fields with Boltzmann factor

$$e^{iS}.$$

S is the action above, which we shall understand as a formal power expansion in $\theta^{\mu\nu}$.

- Caveat: This expansion in θ will not yield the right Physics at

$$\text{Energies} > 1/\sqrt{\theta}.$$

Pause and look back

- After those 10 years, it is advisable that we pause to look back and assess what has been achieved as regards the quantum properties of those theories.
- I will not cover all that has been done so far, but focus on
 - gauge anomalies,
 - renormalisability (when there are no Higgs and no Yukawa sectors),
 - existence Supersymmetric versions.

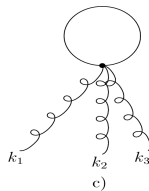
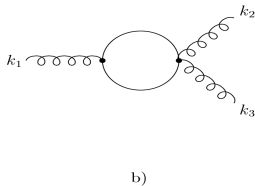
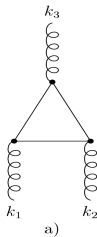
Gauge anomalies

- When quantizing a chiral gauge theory the first problem one has to face is that of gauge anomalies.
- The chiral vertices acquire θ -dependent terms, which can give rise to new θ -dependent anomalous contributions to the famous, already anomalous, triangle diagrams:

$$S_{fermionic} = \int d^4x \bar{\psi} i \not{\partial} \psi + \bar{\psi} \{ \not{a} - \frac{1}{2} \theta^{\alpha\beta} [\frac{1}{2} f_{\alpha\beta} i \not{D} (a) + \gamma^{\rho} f_{\rho\alpha} i D_{\beta} (a)] \} P_L \psi + o(\theta^2).$$

- So, I started the computation of the following three types of one-loop 3point diagrams giving would-be θ -dependent anomalies: $\rightarrow\rightarrow\rightarrow$

Would-be anomalous 3pt diagrams



Wrong guess!

- Actually, I was completely sure that they would give rise to new θ -dependent anomalous terms, which would lead to extra anomaly cancelation conditions, which in turn would make most –NC SM, NC GUTS..– of these theories meaningless at the quantum level.
- Couldn't be more wrong! I was very surprised to find that the θ -dependent anomalous contributions to the effective action, Γ , were BRS-exact, i.e., they were not truly anomalous terms:

$$s\Gamma[A[\mathbf{a}, \theta], \theta] = -\frac{i}{24\pi^2} \int d^4x \varepsilon^{\mu_1\mu_2\mu_3\mu_4} \text{Tr}(\partial_{\mu_1} \lambda \mathbf{a}_{\mu_2} \partial_{\mu_3} \mathbf{a}_{\mu_4}) \\ + s \left[\frac{1}{48\pi^2} \int d^4x \varepsilon^{\mu_1\mu_2\mu_3\mu_4} \theta^{\alpha\beta} \text{Tr}(\partial_\alpha \partial_{\mu_1} \mathbf{a}_{\mu_2} \partial_{\mu_3} \mathbf{a}_{\mu_4} \mathbf{a}_\beta) \right] + o(\mathbf{a}^3) + o(\theta^2).$$

- The computations were carried out by using DIM. REG. with a nonanticommuting γ_5 . More details in [CPM, NPB 652\(2003\)72](#).

Would-be anomaly at any order in θ

- The next **challenge** was to show -at one-loop- that there were **no θ -dependent gauge anomalies at any $O(\theta)$ and for any number of a_μ 's.**
- One can do so [F.Brandt, CPM & F. Ruiz, JHEP 07(2003)068] by using a mixture of explicit DIM. REG. computations, brute force solution of BRS equations and BRS techniques:

- By taking advantage of the fact that in DIM REG. the Jacobian of $\mathbb{I} + M$ -an operator which enters the SW map for fermions

$$\Psi_{\alpha I} = (\delta_{IJ} \delta_{\alpha\beta} + M[a, \partial, \gamma, \gamma_5; \theta]_{\alpha\beta IJ}) \psi_{\beta J} -$$

is **TRIVIAL**, we were able to obtain the **complete gauge anomaly candidate**:

$$\mathcal{A}[A, \Lambda, \theta] = -\frac{i}{24\pi^2} \int d^4x \epsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \text{Tr} \Lambda \star \partial_{\mu_1} \left(A_{\mu_2} \star \partial_{\mu_3} A_{\mu_4} + \frac{1}{2} A_{\mu_2} \star A_{\mu_3} \star A_{\mu_4} \right)$$

$$A_\mu = A[a, \theta]_\mu, \quad \Lambda = \Lambda[\lambda, \theta]$$

- Then,



Would-be anomaly at any order in θ , cont'

- by carrying out brute force computations and by using cohomological techniques, we obtained $\mathcal{B}[A^{(a,t\theta)}, t\theta]$ such that

$$t \frac{d}{dt} \mathcal{A}[A(a, t\theta), \Lambda(\lambda, t\theta), t\theta] = s_{NC} \mathcal{B}[A^{(a,t\theta)}, t\theta].$$

- and, hence,

$$\mathcal{A}[A(a, \theta), \Lambda(\lambda, \theta), \theta] = \mathcal{A}^{\text{Bardeen}} - s \int_0^1 \frac{dt}{t} \mathcal{B}[A(a, t\theta), t\theta]$$

- THE θ -DEPENDENT TERMS ARE COHOMOLOGICALLY TRIVIAL \implies THEY ARE NOT ANOMALOUS CONTRIBUTIONS!**

FUJIKAWA'S METHOD

FUJIKAWA'S METHOD

- Another way to obtain the gauge anomaly is Fujikawa's method: the gauge anomaly shows that the fermionic measure is not invariant under chiral gauge transformations. Fujikawa's method helps establish a connection with index theorems.
- As yet, we lack a derivation of the absence of θ -dependent anomalous terms by using Fujikawa's method.
- Within Fujikawa's formalism, the gauge anomaly comes in two guises, related by local redefinitions of the corresponding currents: the consistent form, \mathcal{A}_{con} , and the covariant form, \mathcal{A}_{cov}
 - \mathcal{A}_{con} verifies the WZ consistency conditions and involves lengthy and tedious algebra. It is not gauge covariant.
 - \mathcal{A}_{cov} does not verify the WZ conditions, it is gauge covariant and, as a result, the algebraic computations that lead to it are simpler than in the "consistent" case.

The covariant form of the gauge anomaly. I

- When I was preparing this talk, I decided to work out the covariant form of the gauge anomaly in the U(1) case –non-trivial from the Cohomological viewpoint: Barnich, Brandt & Henneaux, Phys. Rept. 338 (2000) 439—, up to first order in θ :

$$\begin{aligned}
 Z[a, \theta] &\equiv \int d\bar{\psi} d\psi \ e^{-\int d^4x \bar{\psi} i\mathcal{D}\psi} \\
 \mathcal{D} &= \hat{\mathcal{D}} + \hat{\mathcal{R}}, \quad \hat{\mathcal{D}} = \not{\partial} - i\not{a}P_L \\
 \hat{\mathcal{R}} &= -\left[\frac{1}{4}\theta^{\alpha\beta} f_{\alpha\beta}\gamma^\mu D_\mu + \frac{1}{2}\theta^{\alpha\beta} \gamma^\rho f_{\rho\alpha} D_\beta\right]P_L
 \end{aligned}$$

Then, following Fujikawa, one introduces two bases of orthonormal eigenfunctions $\{\varphi_m\}$ & $\{\phi_m\}$,

$$\left(i\mathcal{D}(a)\right)^\dagger i\mathcal{D}(a)\varphi_m = \lambda_m^2 \varphi_m, \quad i\mathcal{D}(a)\left(i\mathcal{D}(a)\right)^\dagger \phi_m = \lambda_m^2 \phi_m,$$

and expands

$$\psi = \sum_m a_m \varphi_m, \quad \bar{\psi} = \sum_m \bar{b}_m \phi_m^\dagger \implies d\bar{\psi} d\psi \equiv \prod_m d\bar{b}_m da_m.$$

The covariant form of the gauge anomaly. II

- The gauge anomaly equation in covariant disguise reads

$$\int d^4x \operatorname{Tr} \omega(x) (D^\mu [a] \mathcal{J}_\mu^{(\text{cov})})(x) = -\delta J \equiv \mathcal{A}[\omega, \mathbf{a}, \theta]_{\text{cov}},$$

where

$$\begin{aligned} \delta J &= d\bar{\psi}' d\psi' - d\bar{\psi} d\psi \quad \psi' = \psi + i\omega P_L \psi, \bar{\psi}' = \bar{\psi} - i\bar{\psi} P_R \omega \\ \delta J &= \lim_{\Lambda \rightarrow \infty} \int d^4x \sum_m \{ \phi_m^\dagger \omega e^{-\lambda_m^2/\Lambda^2} P_R \phi_m - \varphi_m^\dagger \omega e^{-\lambda_m^2/\Lambda^2} P_L \varphi_m \} \\ \mathcal{J}_\mu^{a, (\text{cov})}(x) &= \frac{1}{Z[\mathbf{a}, \theta]} \int d\bar{\psi} d\psi \frac{\delta S_{\text{fermionic}}}{\delta a^\mu(x)} e^{-S_{\text{fermionic}}}, \quad S_{\text{fermionic}} = \int d^4x \bar{\psi} i \mathcal{D} \psi \end{aligned}$$

- By changing to a plane wave basis, one gets

$$\begin{aligned} \mathcal{A}[\omega, \mathbf{a}, \theta]_{\text{cov}} &= \lim_{\Lambda \rightarrow \infty} - \int d^4x \operatorname{Tr} \omega(x) \int \frac{d^4p}{(2\pi)^4} \operatorname{tr} \left\{ \left(\gamma_5 e^{-ipx} e^{-\frac{(i\mathcal{D}^{(\theta)})(\mathbf{a})^2}{\Lambda^2}} e^{ipx} \right) \right\}, \\ \mathcal{D}^{(\theta)}(\mathbf{a}) &= \mathcal{D} + \mathcal{R}, \quad \mathcal{R} = -\left[\frac{1}{4} \theta^{\alpha\beta} f_{\alpha\beta} \gamma^\mu D_\mu + \frac{1}{2} \theta^{\alpha\beta} \gamma^\rho f_{\rho\alpha} D_\beta \right] \end{aligned}$$

The covariant form of the gauge anomaly. III

- By expanding in powers of θ and removing the terms that vanish as $\Lambda \rightarrow \infty$, one gets

$$\mathcal{A}[\omega, \mathbf{a}, \theta]_{\text{cov}} = \lim_{\Lambda \rightarrow \infty} - \int d^4x \text{Tr} \omega \int \frac{d^4p}{(2\pi)^4} \text{tr} \left\{ \left(\gamma_5 e^{-ipx} e^{-\frac{(i\mathcal{D}(\theta)(\mathbf{a}))^2}{\Lambda^2}} e^{ipx} \right) \right\} =$$

$$\mathcal{A}^{(\text{ordinary})}[\omega, \mathbf{a}] + \mathcal{A}^{(1)}[\omega, \mathbf{a}, \theta] + \mathcal{O}(\theta^2)$$

$$\mathcal{A}[\omega, \mathbf{a}]^{(\text{ordinary})} = -\frac{1}{32\pi^2} \int d^4x \text{Tr} \omega \epsilon^{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma},$$

$$\mathcal{A}^{(1)}[\omega, \mathbf{a}, \theta] = \int d^4x \text{Tr} \omega(x) [\mathcal{A}_1(x) + \mathcal{A}_2(x) + \mathcal{A}_3(x)]$$

$$\mathcal{A}_1 = - \sum_{l=0}^1 \lim_{\Lambda \rightarrow \infty} 2i \int \frac{d^4q}{(2\pi)^4} e^{-q^2} \frac{1}{2} \text{tr} \gamma_5 \mathcal{D}^{2l}(\Lambda q) \{ \mathcal{D}(\Lambda q), \mathcal{R}(\Lambda q) \} \mathcal{D}^{2(1-l)}(\Lambda q) \mathbf{I},$$

$$\mathcal{A}_2 = - \sum_{l=0}^2 \lim_{\Lambda \rightarrow \infty} 2i \int \frac{d^4q}{(2\pi)^4} e^{-q^2} \frac{1}{3! \Lambda^2} \text{tr} \gamma_5 \mathcal{D}^{2l}(\Lambda q) \{ \mathcal{D}(\Lambda q), \mathcal{R}(\Lambda q) \} \mathcal{D}^{2(2-l)}(\Lambda q) \mathbf{I},$$

$$\mathcal{A}_3 = - \sum_{l=0}^3 \lim_{\Lambda \rightarrow \infty} 2i \int \frac{d^4q}{(2\pi)^4} e^{-q^2} \frac{1}{4! \Lambda^4} \text{tr} \gamma_5 \mathcal{D}^{2l}(\Lambda q) \{ \mathcal{D}(\Lambda q), \mathcal{R}(\Lambda q) \} \mathcal{D}^{2(3-l)}(\Lambda q) \mathbf{I}.$$

The covariant form of the gauge anomaly. IV

- Some lengthy algebra and the fact that the a_μ 's commute –U(1) case– lead to

$$\mathcal{A}_1 = -\frac{i}{8\pi^2} \theta^{\alpha\beta} \epsilon^{\mu\nu\rho\sigma} \left(-\frac{1}{2} f_{\alpha\beta} f_{\mu\nu} f_{\rho\sigma} - f_{\nu\alpha} f_{\mu\beta} f_{\rho\sigma} \right) \\ + \frac{1}{16\pi^2} \theta^{\alpha\beta} \epsilon^{\mu\nu\rho\sigma} \left[f_{\mu\nu} (\partial_\rho f_{\sigma\alpha} D_\beta \mathbb{I} + \frac{1}{2} \partial_\rho f_{\alpha\beta} D_\sigma \mathbb{I}) + \partial_\mu f_{\nu\alpha} f_{\rho\sigma} D_\beta \mathbb{I} \right. \\ \left. + \frac{1}{2} \partial_\mu f_{\alpha\beta} f_{\rho\sigma} D_\nu \mathbb{I} \right],$$

$$\mathcal{A}_2 = -\frac{i}{2(4\pi)^2} \theta^{\alpha\beta} \epsilon^{\mu\nu\rho\sigma} f_{\alpha\beta} f_{\mu\nu} f_{\rho\sigma} + \\ -\frac{1}{16\pi^2} \theta^{\alpha\beta} \epsilon^{\mu\nu\rho\sigma} \left[f_{\mu\nu} \partial_\rho f_{\sigma\alpha} D_\beta \mathbb{I} + \partial_\mu f_{\nu\alpha} f_{\rho\sigma} D_\beta \mathbb{I} + \frac{1}{2} (f_{\mu\nu} \partial_\rho f_{\alpha\beta} D_\sigma \mathbb{I} \right. \\ \left. + \partial_\mu f_{\alpha\beta} f_{\rho\sigma} D_\nu \mathbb{I}) \right],$$

$$\mathcal{A}_3 = 0.$$

- Then,

$$\mathcal{A}^{(1)}[\omega, \mathbf{a}, \theta] = \int d^4x \operatorname{Tr} \omega(x) [\mathcal{A}_1(x) + \mathcal{A}_2(x) + \mathcal{A}_3(x)] = \\ \frac{i}{32\pi^2} \theta^{\alpha\beta} \epsilon^{\mu\nu\rho\sigma} \int d^4x \operatorname{Tr} \omega (f_{\alpha\beta} f_{\mu\nu} f_{\rho\sigma} + 4 f_{\nu\alpha} f_{\mu\beta} f_{\rho\sigma}) = 0$$

The covariant form of the gauge anomaly. IV

- In summary,

$$\mathcal{A}[\omega, \mathbf{a}, \theta]_{cov} = \mathcal{A}^{(ordinary)}[\omega, \mathbf{a}] + o(\theta^2)$$

- **NO FIRST-ORDER-IN- θ -CORRECTIONS TO THE ORDINARY GAUGE ANOMALY IN THE U(1) CASE: AGREEMENT WITH DIM. REG.**
- **NONABELIAN CASE SHOULD BE WORKED OUT!**

Renormalisability and the enveloping-algebra formalism. I

- The issue of the renormalisability of NC theories formulated within the enveloping-algebra formalism started off splendidly, for it was shown by Bichl, Grimstrup, Grosse, Popp, Schweda and Wulkenhaar –JHEP 0106(2001)013– that the photon 2pt function is renormalisable at any order in θ .
- Unfortunately, Wulkenhaar –JHEP 0203(2002)024– showed that this θ -expanded QED was not renormalisable mainly due to the infamous 4pt fermionic divergence:

$$\frac{c}{\epsilon} \theta^{\alpha\beta} \epsilon_{\mu\nu\rho\sigma} \int d^4x \bar{\psi} \gamma_5 \gamma^\rho \psi \bar{\psi} \gamma^\sigma \psi$$

- 4 years after Wulkenhaar's paper, there came along the encouraging results by Buric, Latas and Radovanovic –JHEP 02(2006)040– & Buric, Radovanovic and Trampetic –JHEP 03(2007) 030– that the gauge sector of SU(N) and the NC SM were one-loop renormalizable at first order in θ .

Renormalisability and the enveloping-algebra formalism: Matter sector

- However, due to the infamous 4pt fermionic divergence above, the construction of theories with a one-loop and first-order-in- θ matter sector remained an open issue.
- Then, it came along the paper by Buric, Latas, Radovanovic and Trampetic –PRD 77 (2008) 045031–, where they showed that the divergence of the 4pt fermionic function vanishes for a NC SU(2) chiral theory with a matter sector being an SU(2)-doublet of LH fermions.
- This result was later generalized –CPM & C. Tamarit, PRD 80 (2009) 065023– to any NC GUT inspired theory with only fermions as matter fields. \diamond NC GUT inspired theories: gauge theories whose noncommutative fermions are left-handed multiplets.
- Thus, one of the obstacles –what about the renormalisability of the other 1PI functions?– to achieve one-loop and first-order-in- θ renormalisability had been removed by selecting Grand Unification as a guiding principle.

One-loop & $\mathcal{O}(\theta)$ ren. GUT inspired models

- The absence of the infamous 4pt fermionic divergence opened up the possibility of building NC theories with massless fermionic chiral matter (and Nature seems to be fundamentally chiral) that are one-loop renormalisable at first in θ .
- Actually, Wulkenhaar had already pointed out in his non-renormalizability-of- θ -expanded-noncommutative-QED paper that, in the massless case, the theory is (off-shell) one-loop renormalisable, at first order in θ , if one forgets about the fermionic 4pt function.
- At long last, it was shown –CPM & C. Tamarit, JHEP 12(2009)042– that NC GUT inspired theories, with a matter sector made out of fermions and no scalars, were, on-shell, one-loop and first-order-in- θ renormalisable for any anomaly safe compact simple gauge group, if, and only if, all the flavour fermionic multiplets carry the same irrep of the gauge group. (SO(10), E_6 , not SU(5) –See C.TAMARIT, PRD81 (2010) 025006.

The on-shell renormalisability

- NC GUT inspired models:

$$S = \int d^4x - \frac{1}{2g^2} \text{Tr} F_{\mu\nu} \star F^{\mu\nu} + \bar{\Psi}_L i \not{D} \Psi_L, \quad (1)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]_\star, \quad D_\mu \psi_L = \partial_\mu \Psi_L - i\rho_\psi(A_\mu) \star \Psi_L,$$

ρ_ψ denotes an arbitrary unitary representation, which is a direct sum of irreducible representations, $\rho_\psi = \bigoplus_{r=1}^F \rho_\psi^r$.

The effective action divergent part

- Lengthy computations led to the following result:

Once ψ_L^r , g and θ have been renormalised as follows

$$\begin{aligned} \psi^r &= (Z_\psi^r)^{1/2} \psi_R^r, \quad g = \mu^{-\epsilon} Z_g g_R, \quad \theta^{\mu\nu} = Z_\theta \theta_R^{\mu\nu}, \\ Z_\psi^r &= 1 + \frac{g^2 C_2(r)}{16\pi^2 \epsilon}, \quad Z_g = 1 + \frac{g^2}{16\pi^2 \epsilon} \left[\frac{11}{6} C_2(G) - \frac{2}{3} \sum_r c_2(r) \right], \\ Z_\theta &= -Z_\psi^r - \frac{g^2}{48\pi^2 \epsilon} (13C_2(r) - 4C_2(G)), \end{aligned}$$

the UV divergences –one-loop and first order in θ – which remain in the background-field effective action are

$$S^{\text{ct}} = \int d^4x \frac{\delta S}{\delta a_\mu^a(x)} F_\mu^a[a, \psi] + \left(\sum_r \frac{\delta S}{\delta \psi^r(x)} G_r[a, \psi] + \text{c.c.} \right),$$

—**VANISHING ON-SHELL!**—, where

F and G functions

$$F_\mu = y_1 \theta^{\alpha\beta} \mathcal{D}_\mu f_{\alpha\beta} + y_2 \theta_\mu^\alpha D^\nu f_{\nu\alpha} + \sum_r y_3^r \theta_\mu^\alpha (\bar{\psi}_r \gamma_\alpha P_L T^a \psi^r) T^a + i \sum_r y_4^r \theta^{\alpha\beta} (\bar{\psi}_r \gamma_{\mu\alpha\beta} P_L T^a \psi^r) T^a + y_5 \tilde{\theta}_\mu^\beta D^\nu f_{\nu\beta},$$

$$G_{r,L} = k_1^r \theta^{\alpha\beta} f_{\alpha\beta} P_L \psi^r + k_2^r \theta^{\alpha\beta} \gamma_{\alpha\mu} P_L f_\beta^\mu \psi^r + k_3^r \theta^{\alpha\beta} \gamma_{\alpha\mu} P_L D_\beta D^\mu \psi^r + k_4^r \theta^{\alpha\beta} \gamma_{\alpha\beta} P_L D^2 \psi^r + k_5^r \tilde{\theta}^{\alpha\beta} \gamma_5 P_L f_{\alpha\beta} \psi^r; \quad y_i \in \mathbb{R}, k_i \in \mathbb{C},$$

with

$$y_1 = \text{Im} k_1^r, y_3^r = 2g^2 y_2,$$

$$y_4^r = -y_5 g^2 - \frac{g^4}{384\pi^2} (16C_2(r) - 13C_2(G)),$$

$$\text{Re} k_1^r = -\frac{1}{2} \text{Im} k_3^r - \frac{g^2}{384\pi^2 \epsilon} (13C_2(r) - 8C_2(G)),$$

$$\text{Im} k_5^r = -\frac{g^2}{384\pi^2 \epsilon} (11C_2(r) - 8C_2(G)),$$

$$\text{Im} k_4^r = \frac{g^2 C_2(r)}{384\pi^2 \epsilon}, \text{Re} k_2^r = -\frac{5g^2}{192\pi^2 \epsilon} (2C_2(r) - C_2(G)),$$

$$\text{Im} k_2^r = \text{Re} k_3^r = 2\text{Re} k_5^r = -2\text{Re} k_4^r.$$

Notice that y_1, y_2, y_5 and Z_θ must be flavour independent, and so must be

y_3, y_4 .

SUSY

WHAT ABOUT SUSY?

Details in CPM & C. Tamarit JHEP 0811 (2008) 087, JHEP 0911 (2009) 092.

- For $U(N)$ in the fundamental rep., $\mathcal{N} = 1$ SYM exists –at least in the WZ gauge– as a classical theory in the enveloping-algebra:

$$S_{NCSYM} = \frac{1}{2g^2} \text{Tr} \int d^4x \left[-\frac{1}{2} F^{\mu\nu} \star F_{\mu\nu} - 2i \Lambda^\alpha \star \sigma_{\alpha\dot{\alpha}}^\mu D_\mu \bar{\Lambda}^{\dot{\alpha}} + D \star D \right]$$

where

$$A_\mu = A_\mu[a, \lambda_\alpha, d, \theta], \Lambda_\alpha[a, \lambda_\alpha, d, \theta] \text{ and } D = D[a, \lambda_\alpha, d, \theta]$$

are SW maps.

S_{NCSYM} is invariant under $\mathcal{N} = 1$ SUSY:

- **linearly realized in terms of the NC fields,**
 (there is a local superfield formulation)
 and
- **nonlinearly realized in terms of the ordinary fields,**
 (no local superfield formulation, but nonlocal at least for $U(1)$)

Susy trans.

$$\begin{aligned}
 A_\mu[\varphi, \theta] &\rightarrow A_\mu^{(\epsilon)}[\varphi, \theta] = A_\mu[\varphi, \theta] + \delta_\epsilon A_\mu[\varphi, \theta] \\
 \Lambda_\alpha[\varphi, \theta] &\rightarrow \Lambda_\alpha^{(\epsilon)}[\varphi, \theta] = \Lambda_\alpha[\varphi, \theta] + \delta_\epsilon \Lambda_\alpha[\varphi, \theta] \\
 D[\varphi, \theta] &\rightarrow D^{(\epsilon)}[\varphi, \theta] = D[\varphi, \theta] + \delta_\epsilon D[\varphi, \theta]
 \end{aligned}$$

φ stands for the ordinary fields

$$\begin{aligned}
 A_\mu^{(\epsilon)}[\varphi, \theta] &= A_\mu[\varphi + \delta_\epsilon \varphi, \theta] \\
 \Lambda_\alpha^{(\epsilon)}[\varphi, \theta] &= \Lambda_\alpha[\varphi + \delta_\epsilon \varphi, \theta] \\
 D^{(\epsilon)}[\varphi, \theta] &= D[\varphi + \delta_\epsilon \varphi, \theta]
 \end{aligned}$$

where

$$\begin{aligned}
 \delta_\epsilon A^\mu &= i\epsilon^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \bar{\Lambda}^{\dot{\alpha}} + i\bar{\epsilon}^{\dot{\alpha}} \bar{\sigma}_{\dot{\alpha}\alpha}^\mu \Lambda^\alpha, \\
 \delta_\epsilon \Lambda_\alpha &= (\sigma^{\mu\nu})_{\alpha\beta} \epsilon_\beta F_{\mu\nu} + i\epsilon_\alpha D, \\
 \delta_\epsilon D &= -\epsilon^\alpha \sigma_{\alpha\dot{\alpha}}^\mu D_\mu \bar{\Lambda}^{\dot{\alpha}} + \bar{\epsilon}^{\dot{\alpha}} \bar{\sigma}_{\dot{\alpha}\alpha}^\mu D_\mu \Lambda^\alpha.
 \end{aligned}$$

and, up to first order in θ , $\longrightarrow \longrightarrow \longrightarrow$

nonlinear susy trans.

$$\delta_\epsilon \mathbf{a}_\mu = \frac{1}{4} \epsilon \sigma_\mu \bar{\lambda} - \frac{1}{4} \bar{\epsilon} \bar{\sigma}_\mu \lambda + \frac{1}{16} \theta^{\nu\rho} \left[\{ \mathbf{a}_\nu, 2D_\rho(\epsilon \sigma_\mu \bar{\lambda} - \bar{\epsilon} \bar{\sigma}_\mu \lambda) - i[\mathbf{a}_\rho, \epsilon \sigma_\mu \bar{\lambda} - \bar{\epsilon} \bar{\sigma}_\mu \lambda] \} \right. \\ \left. - \{ \epsilon \sigma_\nu \bar{\lambda} - \bar{\epsilon} \bar{\sigma}_\nu \lambda, \partial_\rho \mathbf{a}_\mu + f_{\rho\mu} \} - \{ \mathbf{a}_\nu, \partial_\rho(\epsilon \sigma_\mu \bar{\lambda} - \bar{\epsilon} \bar{\sigma}_\mu \lambda) + D_\rho(\epsilon \sigma_\mu \bar{\lambda} - \bar{\epsilon} \bar{\sigma}_\mu \lambda) \} \right. \\ \left. - D_\mu(\epsilon \sigma_\rho \bar{\lambda} - \bar{\epsilon} \bar{\sigma}_\rho \lambda) \right] + \theta^2,$$

$$\delta_\epsilon \lambda_\alpha = -\epsilon_\alpha \mathbf{d} + 2i \epsilon_\gamma (\sigma^{\mu\nu})^\gamma_\alpha f_{\mu\nu} + \frac{1}{4} \theta^{\nu\rho} \left[-\frac{1}{4} \{ \epsilon \sigma_\nu \bar{\lambda} - \bar{\epsilon} \bar{\sigma}_\nu \lambda, 2D_\rho \lambda_\alpha - i[\mathbf{a}_\rho, \lambda_\alpha] \} \right. \\ \left. - \{ \mathbf{a}_\nu, 4iD_\rho(\epsilon_\gamma (\sigma^{\mu\rho})^\gamma_\alpha f_{\mu\lambda}) + 2[\mathbf{a}_\rho, \epsilon_\gamma (\sigma^{\mu\lambda})^\gamma_\alpha f_{\mu\lambda}] + \frac{i}{4} [\epsilon \sigma_\rho \bar{\lambda} - \bar{\epsilon} \bar{\sigma}_\rho \lambda, \lambda_\alpha] \} \right] \\ + \theta^2$$

$$\delta_\epsilon \mathbf{d} = i\bar{\epsilon} \bar{\sigma}^\mu D_\mu \lambda + i\epsilon \sigma^\mu D_\mu \bar{\lambda} + \frac{1}{4} \theta^{\nu\rho} \left[2i \{ f_{\mu\nu}, \bar{\epsilon} \bar{\sigma}^\mu D_\rho \lambda + \epsilon \sigma^\mu D_\rho \bar{\lambda} \} \right. \\ \left. + i \{ \mathbf{a}_\nu, (\partial_\rho + D_\rho)(\bar{\epsilon} \bar{\sigma}^\mu D_\mu \lambda + \epsilon \sigma^\mu D_\mu \bar{\lambda}) \} - \frac{1}{4} \{ \epsilon \sigma_\nu \bar{\lambda} - \bar{\epsilon} \bar{\sigma}_\nu \lambda, 2D_\rho \mathbf{d} - i[\mathbf{a}_\rho, \mathbf{d}] \} \right. \\ \left. - \{ \mathbf{a}_\nu, 2D_\rho(i\bar{\epsilon} \bar{\sigma}^\mu D_\mu \lambda + i\epsilon \sigma^\mu D_\mu \bar{\lambda}) - i[\mathbf{a}_\rho, i\bar{\epsilon} \bar{\sigma}^\mu D_\mu \lambda + i\epsilon \sigma^\mu D_\mu \bar{\lambda}] \} \right. \\ \left. + \frac{i}{4} [\epsilon \sigma_\rho \bar{\lambda} - \bar{\epsilon} \bar{\sigma}_\rho \lambda, \mathbf{d}] \right] + \theta^2.$$

Comments

COMMENTS ON THE NONLINEAR SUSY TRANSFORMATIONS

- They are truly SUSY transformations,

$$\{\delta_{\epsilon_2}, \delta_{\epsilon_1}\}(\text{fields}) = i(\epsilon_2 \sigma^\mu \bar{\epsilon}_1 - \epsilon_1 \sigma^\mu \bar{\epsilon}_2) \partial_\mu (\text{fields}) + \text{gauge transformations},$$

due to the fact that the NC fields carry a linear realisation of $\mathcal{N} = 1$ SUSY. This holds at any order in θ –see CPM and C.Tamarit, JHEP 2008.

- $\delta_\epsilon a_\mu$, $\delta_\epsilon \lambda_\alpha$ and $\delta_\epsilon d$ belong to the Lie algebra of the ordinary gauge group, only for $U(N)$ in the fundamental rep. and its siblings, i.e.,
- for an arbitrary Lie algebra they take values on the enveloping-algebra: they are not ordinary field variations which are also ordinary fields.

Susy for NC GUTS? I

- For simple gauge groups it still makes sense to consider

$$S = \frac{1}{2g^2} \text{Tr} \int d^4x \left[-\frac{1}{2} F^{\mu\nu} \star F_{\mu\nu} - 2i \Lambda^\alpha \star \sigma_{\alpha\dot{\alpha}}^\mu D_\mu \bar{\Lambda}^{\dot{\alpha}} + D \star D \right]$$

where

$$A_\mu = A_\mu[a, \lambda_\alpha, d, \theta], \Lambda_\alpha[a, \lambda_\alpha, d, \theta] \text{ and } D = D[a, \lambda_\alpha, d, \theta]$$

are SW maps.

- It looks like as if it were a SUSY invariant NC action, but, the **CATCH** is that the invariance is under

$$A_\mu[\varphi, \theta] \rightarrow A_\mu^{(\epsilon)}[\varphi, \theta] = A_\mu[\varphi, \theta] + \delta_\epsilon A_\mu[\varphi, \theta]$$

$$\Lambda_\alpha[\varphi, \theta] \rightarrow \Lambda_\alpha^{(\epsilon)}[\varphi, \theta] = \Lambda_\alpha[\varphi, \theta] + \delta_\epsilon \Lambda_\alpha[\varphi, \theta]$$

$$D[\varphi, \theta] \rightarrow D^{(\epsilon)}[\varphi, \theta] = D[\varphi, \theta] + \delta_\epsilon D[\varphi, \theta]$$

φ stands for the ordinary fields

and $A_\mu^{(\epsilon)}[\varphi, \theta], \Lambda_\alpha[\varphi, \theta], D^{(\epsilon)}[\varphi, \theta]$ ARE NOT SW MAPS! $\longrightarrow \longrightarrow \longrightarrow$

$\longrightarrow \longrightarrow \longrightarrow$

Susy for NCGUTS? II

- Those transformations are therefore defined from the space of NC "physical" fields –those defined by the SW map— into the space of general fields taking values on the enveloping algebra.
- The so remaining question is whether this invariance has any physical consequences.
- In this regard, it is worth noticing that –unlike in $U(N)$ case–the "SUSY" NC $SU(N)$ theory thus obtained is one-loop and first-order-in- θ (off-shell) renormalisable. This would be a like chance unless, at first order in θ , there is a symmetry at work that relates the gluon and gluino dynamics –see CPM and C. Tamarit JHEP 0911 (2009) 092

Open problems

PRESSING PROBLEMS

- For $SO(10)$, E_6 , inclusion of a phenomenologically relevant Higgs and study of the one-loop renormalisability at first order in θ .
- Construction and analysis of the properties of NC $SO(10)$, E_6 "SUSY".
- Gauge anomalies, Fujikawa's method and index theorems.

Recall that the index theorem in $2n+2$ dimensions gives the gauge anomaly in $2n$ dimensions, that the index of the Dirac operator does not change under small deformations of it and that in our formalism we are considering small deformations of the ordinary Dirac operator \implies No θ -dependent anomalous terms.

- Will this NC GUTS eventually find accommodation within F-theory?