The microscopic dynamics of quantum space as a group field theory

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Max Planck Institute for Gravitational Physics (Albert Einstein Institute)

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Non-commutativity and physics: spacetime quantum geometry

MAIN AIM(S) OF THIS TALK

- highlight some questions that may be addressed by a quantum theory of gravity, some ideas entering the GFT approach, and show room for a QFT formalism
- introduce the Group Field Theory approach to QG (focus on 3d case)
- discuss some recent results in GFT and point out what needs still to be done

QUANTUM GRAVITY

- Main lesson from current theory of gravity (GR): "Gravity is spacetime geometry", thus spacetime is itself a physical (and dynamical) system
- so, maybe Quantum Gravity is not so much a quantization of GR, but a microscopic quantum theory of spacetime structure (atomic theory of space)?
- background independence:
  - no spacetime geometry can be taken as fixed reference for processes
  - it should allow transitions between different backgrounds (e.g. topological BF theory not good enough): theory should be rich
  - still, above leaves room for presence of several "background structures"(see later)
- so, first QG questions:
  - what do space and time emerge from, at quantum level?
  - can we define a quantum theory *of* space & time, thus in absence of space and time?
  - if QFT framework, what are the fundamental quanta? ....quanta *of* space itself....
  - but can it be a QFT?

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FAILURE OF PERTURBATIVE QUANTIZATION AROUND FLAT SPACE

Quantum gravity is not a quantum field theory of gravitons on flat space:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \rightarrow S(h_{\mu\nu}) \rightarrow Z = \int \mathcal{D}h_{\mu\nu}\, e^{-S(h_{\mu\nu})}$$

such theory is perturbatively non-renormalizable (no more than effective field theory)

- missing ingredients?
  - new symmetries? (supergravity?)
  - unification? only gravity+matter can be quantized as above?
  - non-local fundamental structure? beyond point-like objects? (strings,...)
  - degrees of freedom? metric not correct variable?
  - GR itself only effective field theory (not to be quantized as such)?
- background independence!
  - cannot fix spacetime geometry as background
  - ok, are there other background structures (also in GR)?
- above does not rule out QFT as framework.......
  ....but QFT needs *some* background....

BACKGROUND STRUCTURES IN GENERAL RELATIVITY

What are the background structures in GR?

- continuum and local (field-theoretic) picture of space(time)
- dimensionality & signature
- local symmetry group (Lorentz)
- spatial topology
- spacetime topology
- space of geometries on given topology (Wheeler's superspace)

which of them is our quantum (field) theory of gravity to be based on?

which of them are turned into dynamical features of the world (thus, new d.o.f.)?

GFT keeps dimensionality, signature and local symmetry group, in discrete setting

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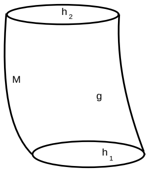
TRADITIONAL/CONSERVATIVE FRAMEWORKS (FORMAL)

The most conservative option is to retain all the background structures of GR, consider spacetime and geometry as fundamental, and "just quantize GR"

canonical approach (De Witt, Wheeler, Kuchar, Isham, ...)

- classical input: kinematics: $\rightarrow h_{ij}(x_\mu)$ on $S = \partial M$; dynamics: Hamiltonian constraint $\mathcal{H} = 0$
- quantum theory: kinematics $\rightarrow \Psi(h_{ij}(x)) \quad \hat{O}(h_{ij}(x))$

  dynamics $\rightarrow \widehat{\mathcal{H}}_{WdW}\, \Psi(h_{ij}) = 0$



covariant approach (Misner, Wheeler, Hawking, Hartle, Teitelboim, ...)

$$\langle h^F_{ij} | h^I_{ij} \rangle = G(h^F_{ij}, h^I_{ij}) = \int_{h^I}^{h^F} \mathcal{D}g_{\mu\nu}\, e^{-S_{EH}(g)} \quad , \quad \text{e.g.} \quad S_{EH}(g) = \int d^4x \sqrt{g}\, R(g)$$

having made sense of the above, "only problem" is semi-classical limit + quantum corrections to classical dynamics of geometry

making sense of it: discretize = divide S, M into chunks $\rightarrow \Delta$
$\Rightarrow h_{ij}, g_{\mu\nu} \rightarrow$ finite variables $\{L_e\}$, $S(g) \rightarrow S_\Delta(L_e)$ (discrete QG)

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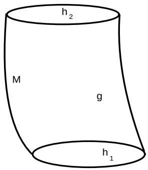

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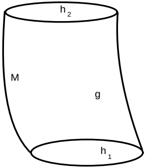

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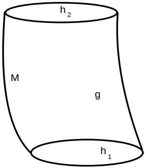

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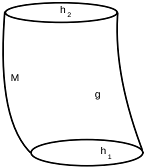

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LESS CONSERVATIVE (EVEN MORE FORMAL): DYNAMICAL TOPOLOGY?

2nd (3rd?) quantization of gravity? (Giddings, Strominger, Banks, Coleman, Hawking, Kuchar, Isham, McGuigan,...)
a) field on space of geometries (say, on S^3); b) all possible interactions (creation/annihilation) of universes (topology change)?

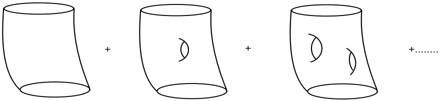
$\Psi(h_{ij}) \longrightarrow \phi(h_{ij})$ on (super-)space of geometries (Giulini, '09) on S^3

$$S(\phi) = -\frac{1}{2}\int \mathcal{D}h_{ij}\ \phi(h_{ij})\mathcal{H}_{WdW}\,\phi(h_{ij}) + \lambda\int \mathcal{D}h_{ij}\ \mathcal{V}(\phi),$$

with some (non-local) interaction term

idea of quantum theory:

Feynman diagrams M:



$$Z = \int \mathcal{D}\phi\ e^{-S(\phi)} = \sum_M \lambda^V Z_M = \sum_M \lambda^V \int \mathcal{D}g\ e^{i\,S(g;M)}$$

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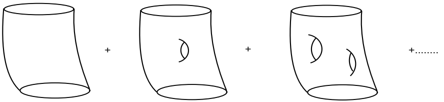

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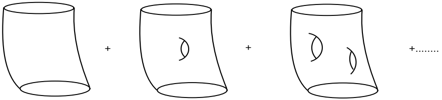

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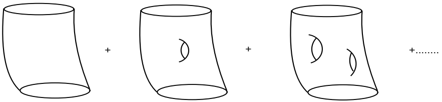

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GRAVITY/GEOMETRY: FUNDAMENTAL OR EMERGENT? GOING DISCRETE?

is the notion of gravity and/or geometry fundamental?

if not, what does it emerge from?

what are the pre-geometric data defining the 'substance"(kinematics) of QG?

and then,

what is the quantum dynamics of the quantum (pre-)geometric data?

which is an aspect of the issue:

is quantum gravity the quantization of General Relativity?
or should the GR dynamics be only emergent is some approximation?
in this case, how do we identify the quantum dynamics?

Using dynamical lattices (or any discrete structure) is highly non-trivial step:

- it means dropping *all background structures* of GR, together with continuum
- all have to be recovered in continuum approx.; non-trivial!!!

discrete, finite sets of data (classical or quantum), even if coming from discretizing a smooth geometry, can be understood as "pre-geometric/pre-spacetime"data, from which spacetime and geometry are *emergent*

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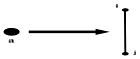
TOWARD GFT: MATRIX MODELS FOR 2D QUANTUM GRAVITY

- general idea: generalise combinatorics of Feynman diagrams from 1d to 2d, from graphs to discrete surfaces, from point particles to 1d objects



- ${M^i}_j \quad i,j = 1,...,N \quad N \times N$ hermitian matrix
- action:

$$S(M,g) \;=\; \frac{1}{2}trM^2 - \frac{g}{\sqrt{N}}trM^3 = \frac{1}{2}{M^i}_j K_{jkli} {M^k}_l - \frac{g}{\sqrt{N}} {M^i}_j {M^m}_n {M^k}_l V_{jmknli}$$

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- fundamental building blocks are 1d simplices with no additional data; microscopic dynamics: no GR, pure 2d combinatorics
- transition amplitudes defined in terms of Feynman diagrams

Toward GFT: Matrix models for 2d quantum gravity

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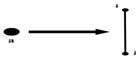

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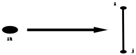

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$$K_{jkli} = \delta^j{}_k \, \delta^l{}_i \qquad V_{jmknli} = \delta^j{}_m \, \delta^n{}_k \, \delta^l{}_i \qquad \left(K^{-1}\right)_{jkli} = K_{jkli}$$

- fundamental building blocks are 1d simplices with no additional data; microscopic dynamics: no GR, pure 2d combinatorics
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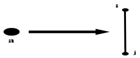
TOWARD GFT: MATRIX MODELS FOR 2D QUANTUM GRAVITY

- general idea: generalise combinatorics of Feynman diagrams from 1d to 2d, from graphs to discrete surfaces, from point particles to 1d objects



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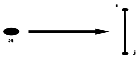

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MATRIX MODELS - FEYNMAN DIAGRAMS AND SIMPLICIAL COMPLEXES

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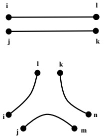

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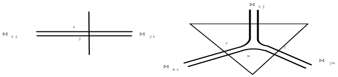
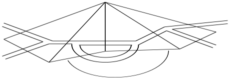


$\Gamma \simeq$ 2d simplicial complex Δ (triangulation)
$\simeq$ 2d discrete spacetime



Feynman amplitudes: join vertices with propagators and sum over common variables (indices) i

$$Z = \sum_{\Gamma} g^{V_\Gamma} N^{\chi(\Gamma)}$$

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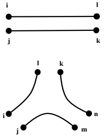

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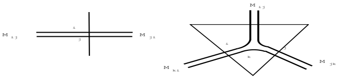

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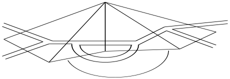

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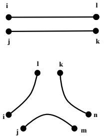

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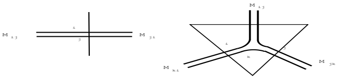

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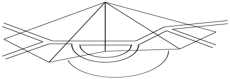

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Matrix models and simplicial 2d gravity

- continuum (Riemannian) 2d GR: $\int_S d^2x \sqrt{g} \, (-R(g) + \Lambda) = -4\pi \chi + \Lambda A_S$
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  (trivial) sum over histories of discrete GR on given 2d complex
  plus sum over all possible 2d complexes of all topologies
- discrete 2nd quantization of GR in 2d !!!!!
- question: control over sum over triangulations/topologies?
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  - task: continuum limit for trivial topology
  - expand $Z_0(g)$ in g: $Z_0(g) = \sum_V V^{\gamma-3}\left(\frac{g}{g_c}\right)^V \simeq_{V\to\infty} (g-g_c)^{2-\gamma} \quad (\gamma > 2)$
  - expectation value of area of surface:
    $\langle A\rangle = a\langle t_\triangle\rangle = \langle V_\Gamma\rangle = a\frac{\partial}{\partial g}\ln Z_0(g) \simeq \frac{a}{g-g_c}$, for large V
  - thus we can send area of triangle $a \to 0$ and $t = V \to \infty$ (continuum limit), while sending $g \to g_c$, to get finite continuum macroscopic area
  - this defines continuum limit (phase transition of discrete system!)
  - results match those of continuum 2d gravity path integral (GR as effective theory)
- can also define continuum limit with contributions from non-trivial topologies - double scaling limit
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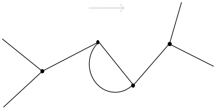
FROM POINT PARTICLES TO FIELDS, FROM MATRICES/TENSORS TO GFT

point particles $\longrightarrow$ fields

$$S(X) = \frac{1}{2}X^2 + \frac{\lambda}{3}X^3$$



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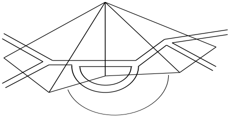

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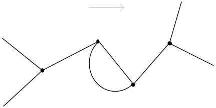

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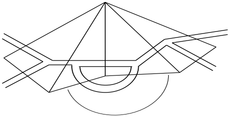

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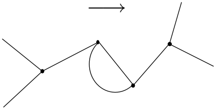

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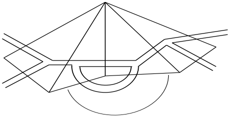

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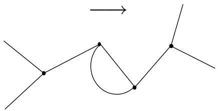

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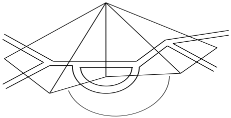
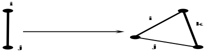


TENSOR MODELS

- generalize further in (combinatorial) dimension, from 2d to 3d (and higher) - from 1d objects (edges) to 2d objects (triangles) (and higher) - from 2d simplicial complexes as FD to 3d ones (and higher)



- $M^i{}_j \to T_{ijk} \quad i,j,k = 1,..,N \qquad N \times N \times N$ tensor
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  kinetic term $= K_{ijki'j'k'} = \delta_{ii'} \delta_{jj'} \delta_{kk'} = (K^{-1})_{ijki'j'k'} =$ propagator
  vertex term $= V_{ii'jj'kk'll'mm'nn'} = \delta_{ii'} \delta_{jj'} \delta_{kk'} \delta_{ll'} \delta_{mm'} \delta_{nn'}$
  with combinatorial pattern of edges in tetrahedron
- $Z = \int \mathcal{D}T\, e^{-S(T)} = \sum_\Gamma \lambda^{V_\Gamma} Z_\Gamma$
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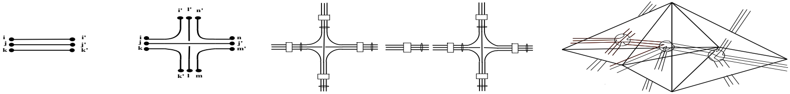

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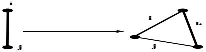

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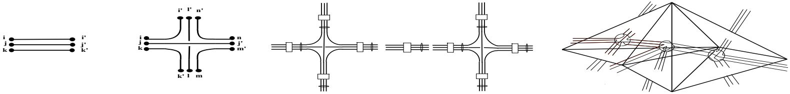

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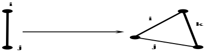
- generalize further in (combinatorial) dimension, from 2d to 3d (and higher) - from 1d objects (edges) to 2d objects (triangles) (and higher) - from 2d simplicial complexes as FD to 3d ones (and higher)



- $M^i{}_j \longrightarrow T_{ijk} \quad i,j,k = 1,..,N \qquad N \times N \times N$ tensor
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  kinetic term $= K_{ijki'j'k'} = \delta_{ii'}\delta_{jj'}\delta_{kk'} = (K^{-1})_{ijki'j'k'} =$ propagator
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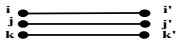

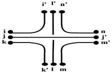

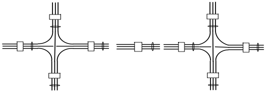

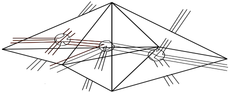

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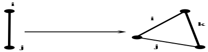

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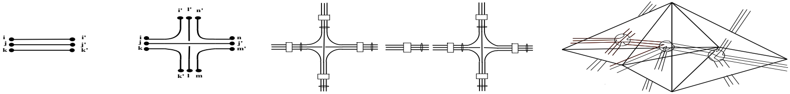

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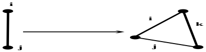

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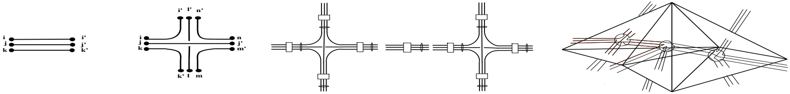

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- Z defined as sum over all 3d simplicial complexes (of manifolds and pseudo-manifolds)
  (pseudo-manifold = neighbourood of point not homeomorphic to a 3-Ball)
- why are they not good?
  - no topological expansion of amplitudes - no control over topology of diagrams
  - no way to separate manifolds from pseudo-manifolds
  - no direct/nice relation with 3d simplicial (classical or quantum) gravity - not enough structure/data in the amplitudes, and in boundary states
- in $d > 2$, gravity is -much- less trivial, both classically and quantum-mechanically
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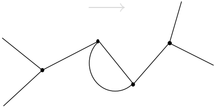
FROM POINT PARTICLES TO FIELDS, FROM MATRICES/TENSORS TO GFT

point particles $\longrightarrow$ fields

$$S(X) = \frac{1}{2}X^2 + \frac{\lambda}{3}X^3$$



$$S(\phi) = \frac{1}{2}\int dx\,\phi(x)^2 + \frac{\lambda}{3}\int dx\,\phi(x)^3$$

$\downarrow$

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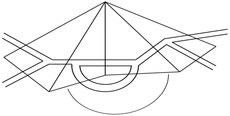
$$S(M) = \frac{1}{2}M_{ij}M_{ji} + \frac{\lambda}{3}M_{ij}M_{jk}M_{ki}$$



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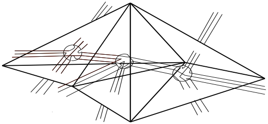

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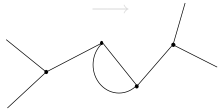

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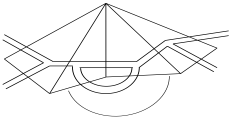

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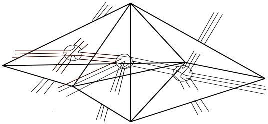

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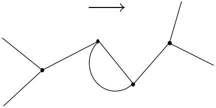

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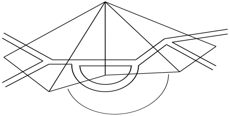

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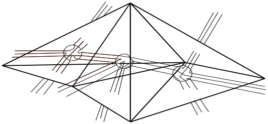

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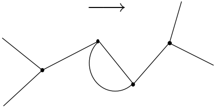

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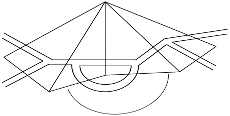

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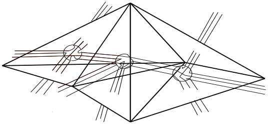

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The Group Field Theory formalism

general reviews:
Freidel, ’05, Oriti, ’06, ’07, ’10

work by:
Baratin, Ben Geloun, Bonzom, Boulatov, De Pietri, Fairbairn, Freidel, Girelli, Gurau, Livine, Louapre, Krajewski, Krasnov, Magnen, Noui, Ooguri, Oriti, Perez, Reisenberger, Rivasseau, Rovelli, Ryan, Smerlak, Tanasa, ...

3D QUANTUM GRAVITY AS A GFT : KINEMATICS OF 2D QUANTUM SPACE

"tensor models plus pre-geometric data" guided by LQG, simplicial QG, NCG

- Triangle in $\mathbb{R}^3$; (2nd quantized) kinematics encoded in field φ (space of triangle geometries)
- triangle geometries parametrized by three $\mathfrak{su}(2)$ Lie algebra elements x_i attached to edges = discrete triad variables (discretization of triad fields along edges)

$$\varphi \,:\, (x_1, x_2, x_3) \in \mathfrak{su}(2)^3 \longrightarrow \varphi(x_1, x_2, x_3) \in \mathbb{R}$$

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- from LQG (simplicial BF): phase space for edge = $\mathcal{T}^*\mathrm{SU}(2) \simeq \mathfrak{su}(2) \times \mathrm{SU}(2)$
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    $e_g(x) : \mathfrak{su}(2) \times \mathrm{SU}(2) \to \mathbb{C} : (x, g) \to e^{i\frac{1}{2}Tr(xg)}$ (fundamental representation)
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3D QUANTUM GRAVITY AS A GFT : KINEMATICS OF 2D QUANTUM SPACE

"tensor models plus pre-geometric data"guided by LQG, simplicial QG, NCG

- Triangle in $\mathbb{R}^3$; (2nd quantized) kinematics encoded in field φ (space of triangle geometries)
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- straightforward extension to functions of $\mathfrak{su}(2)^3$ (A. Baratin, DO, '10)

$$\varphi(x_1,x_2,x_3) = \int [dg]^3 \, \varphi(g_1,g_2,g_3) \, e_{g_1}(x_1) e_{g_2}(x_2) e_{g_3}(x_3)$$

  group elements = parallel transports of connection along links dual to the edges
- GFT field defined (initially) as irrep of $DSU(2)^{\times 3}$
- In order to define a geometric triangle, edge vectors have to 'close':

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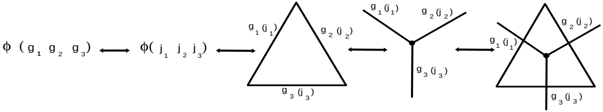
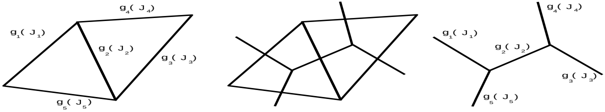
3D QUANTUM GRAVITY AS GFT: KINEMATICS OF 2D QUANTUM SPACE

- φ is building block of (quantum) 2d space



- fields can be convoluted (in group or Lie algebra picture) or traced (in representation picture) with respect to some common argument → gluing of multiple triangles along common edges → more complex simplicial structures, or, dually, more complicated graphs (many-GFT-particle states)



- generic observable/state/boundary configuration: $O(\varphi) = \sum_n O_n(\varphi^{*n})$
- in representation space, generic (polynomial) state is labeled by spin networks (also kinematical quantum states in Loop Quantum Gravity approach)

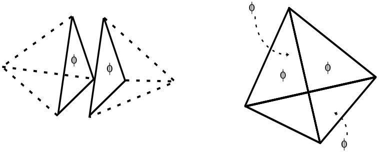
3D QG AS GFT: CLASSICAL DYNAMICS OF QUANTUM SPACE

- Define classical action for $\varphi_{123} = \varphi(x_1, x_2, x_3)$
  - interaction term: four geometric triangles glued pairwise along common edges to form tetrahedron
  - kinetic term: gluing of tetrahedra along common triangles, by edge identification
  - no gravity, no continuum, no GR input

$$S = \frac{1}{2}\int [dx]^3\, \varphi_{123} * \varphi_{123} - \frac{\lambda}{4!}\int [dx]^6\, \varphi_{123} * \varphi_{345} * \varphi_{526} * \varphi_{641}$$

where $\phi_i * \phi_i := (\phi * \phi_-)(x_i)$, with $\phi_-(x) = \phi(-x)$

- propagator and a vertex:



$$\int dh_t \prod_{i=1}^{3} (\delta_{-x_i} * e_{h_t})(y_i), \quad \int \prod_t dh_t \prod_{i=1}^{6} (\delta_{-x_i} * e_{h_{tt'}})(y_i)$$

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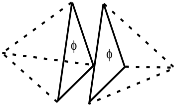

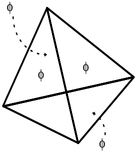

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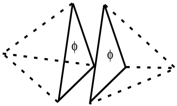

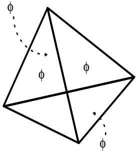

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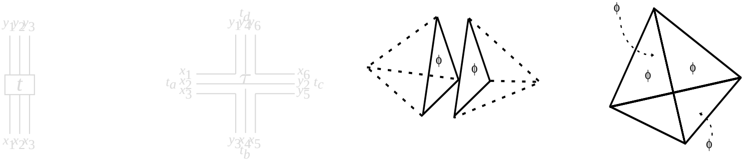

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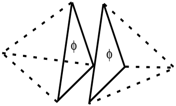

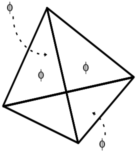

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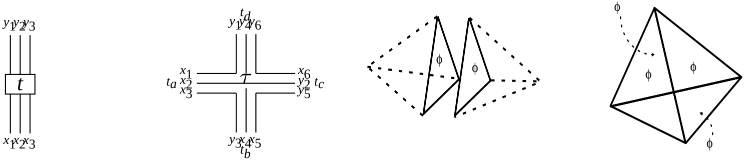

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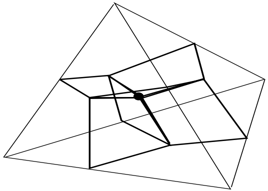
3D QG AS GFT: CLASSICAL DYNAMICS OF QUANTUM 2D SPACE



- geometrical meaning:
  - Group variables h_t and $h_{t\tau}$ are parallel transports through the triangle t, and from the center of the tetrahedron τ to triangle t
  - pair of variables (x_e, y_e) associated to the same edge $e =$ edges vectors seen from the frames associated to the two triangles t, t' sharing it
  - vertex functions: the two variables are identified, up to parallel transport $h_{tt'}$, and up to a sign for two opposite edge orientations
  - propagator encodes a similar gluing condition
- in group picture (Boulatov, '92):

$$\mathcal{K}(g_e, \tilde{g}_e) = \int dh_t \prod_{e=1}^{3} \delta(g_e h_t \tilde{g}_e^{-1}) \qquad \mathcal{V}(g_{tt'}) = \prod_{t\tau=1}^{4} \int dh_{t\tau} \prod_{t\neq t'} \delta(g_{tt'} h_{t\tau} h_{t'\tau}^{-1} \tilde{g}_{tt'}^{-1})$$

- geometric meaning: flatness of each wedge (portion of face inside tetrahedron): piecewise-flat context, trivial matching at boundary

3D QG AS GFT: CLASSICAL DYNAMICS OF QUANTUM 2D SPACE

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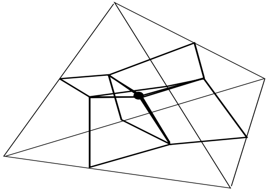

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3D QG AS GFT: CLASSICAL DYNAMICS OF QUANTUM 2D SPACE

- in representation space:

$$\begin{aligned} S(\varphi) &= \frac{1}{2}\sum_{\{j\},\{m\}} \varphi^{j_1j_2j_3}_{m_1m_2m_3}\,\varphi^{j_3j_2j_1}_{m_3m_2m_1} - \\ &- \frac{\lambda}{4!}\sum \varphi^{j_1j_2j_3}_{m_1m_2m_3}\,\varphi^{j_3j_4j_5}_{m_3m_4m_5}\,\varphi^{j_5j_2j_6}_{m_5m_2m_6}\,\varphi^{j_6j_4j_1}_{m_6m_4m_1}\left\{\begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array}\right\} \end{aligned}$$

- from which:

$$\begin{aligned} \mathcal{K} &= \mathcal{K}^{-1} = \delta_{j_1\tilde{j}_1}\delta_{m_1\tilde{m}_1}\delta_{j_2\tilde{j}_2}\delta_{m_2\tilde{m}_2}\delta_{j_3\tilde{j}_3}\delta_{m_3\tilde{m}_3} \\ \mathcal{V} &= \delta_{j_1\tilde{j}_1}\delta_{m_1\tilde{m}_1}\delta_{j_2\tilde{j}_2}\delta_{m_2\tilde{m}_2}\delta_{j_3\tilde{j}_3}\delta_{m_3\tilde{m}_3}\delta_{j_4\tilde{j}_4}\delta_{m_4\tilde{m}_4}\delta_{j_5\tilde{j}_5}\delta_{m_5\tilde{m}_5}\delta_{j_6\tilde{j}_6}\delta_{m_6\tilde{m}_6}\left\{\begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array}\right\} \end{aligned}$$

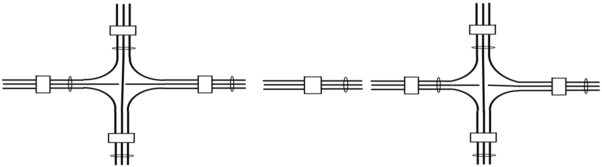
- geometry rather obscure - however, dynamics directly in terms of quantum numbers labelling quantum states of the theory

3D QG AS GFT: MICROSCOPIC QUANTUM DYNAMICS

- the quantum theory is defined by the partition function, in Feynman expansion:

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]} = \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym[\Gamma]} Z(\Gamma)$$

- building blocks of FD are:
  - lines of propagation, with 3 labelled strands (dual to triangles),
  - vertices of interaction (made of 4×3 labelled strands re-routed following the combinatorics of a tetrahedron)
- this produces: 2-cells, identified by strands of propagation passing through several vertices, and then closing (for closed FD), dual to edges; 'bubbles'= 3-cells bounded by the above 2-cells, dual to vertices of simplicial complex



- Feynman graphs Γ are fat graphs/cellular complexes topologically dual to 3d triangulated (pseudo-)manifolds of ALL topologies

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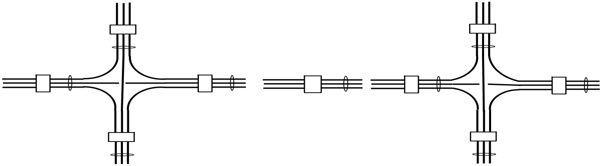

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3d QG as GFT: GFT Feynman amplitudes

Feynman amplitudes $Z(\Gamma)$ obtained by convoluting vertices with propagators

They can be expressed, equivalently, in Lie algebra, group or representation picture

Consider first the Lie algebra (non-commutative) representation

It shows explicit link with simplicial gravity path integrals
(solution to first problem of tensor models)

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FEYNMAN AMPLITUDES IN LIE ALGEBRA SPACE (A.BARATIN, DO, '10)

Propagator and vertex joined by $\star$-product (ordering of functions $\leftarrow$ orientation)
Each loop of strands bounds a face of Γ, dual to an edge of triangulation Δ
Under integration over $h_t \in \mathrm{SU}(2)$, $Z(\Gamma)$ factorizes into *face* amplitudes $A_f[h]$
Consider oriented loop bounding $f \in \Gamma$ and ordered sequence $\{\tau_j\}_{0\leq N}$ of vertices.
Each vertex τ_j: $(\delta_{x_j} \star e_{h_{jj+1}})(x_{j+1})$, with $h_{jj+1} = h_{\tau_j t_j} h_{t_j \tau_{j+1}}$

$$A_f[h] = \int \prod_{j=1}^{N} dx_j \vec{\bigstar}_{j=0}^{N+1} (\delta_{x_j} \star e_{h_{jj+1}})(x_{j+1}) \quad x_{N+1} = x_0$$

(identification, up to parallel transport, of metric variables x_e in different tetrahedra)
Integrate N variables $x_1, \cdots x_N$; plane waves compose to depend on total holonomy $H_0 := h_{01} \cdots h_{N0}$ around ∂f, then 'close the loop' by setting $x_{N+1} = x_0$
We obtain:

$$Z(\Gamma) = \int \prod_L dh_L \prod_f dx_f \, e^{i \sum_f \mathrm{Tr}\,(x_f H_f)}$$

This is simplicial path integral of 1st order 3d gravity (or 3d *BF* theory)

continuum theory: $S(e, \omega) = \int_{\mathcal{M}} tr\,(e \wedge F(\omega))$
for open FD, one gets 3d gravity with boundary terms (fixed boundary triad)

3D QG AS GFT: GFT FEYNMAN AMPLITUDES

In Lie algebra (discrete metric) x_e picture (A.Baratin, DO, '10):

$$Z(\Gamma) = \int_{\mathrm{SU}(2)} \prod_t dh_t \int_{\mathfrak{su}(2)} \prod_f dx_f \, e^{i \sum_f \mathrm{Tr}\, x_f H_f} \quad H_f = \prod_{L=tt' \in \partial f} h_L$$

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In group variables only, one obtains:

$$Z(\Gamma) = \int_{\mathrm{SU}(2)} \prod_L dh_L \prod_f \delta(H_f) \quad H_f = \prod_{L=tt' \in \partial f} h_L$$

volume of space of flat (discrete) connections (consistent with continuum picture)

In terms of group representations (quantum numbers of pre-geometry):

$$Z(\Gamma) = \left(\prod_f \sum_{j_f} \right) \prod_f (2j_f + 1) \prod_v \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

Ponzano-Regge spin foam (state sum) model

spin foam models are sum over histories of spin network states in Loop Quantum Gravity; correspondence GFT Feynman amplitudes - spin foam models is generic

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GFT: some recent results

- diffeomorphisms in GFT
- GFT perturbative renormalization
- From GFT to non-commutative QFT for matter

DIFFEOMORPHISM SYMMETRY IN 3D GRAVITY AND GFT

Continuum 3d BF theory:

- translation symmetry: $\delta^T_\phi e = d_\omega \phi \qquad \delta_\phi \omega = 0 \qquad \phi = \mathfrak{su}(2)$-valued scalar
- local rotation symmetry: $\delta^R_\Lambda e = [e, \Lambda] \qquad \delta^R_\Lambda \omega = d_\omega \Lambda \qquad \Lambda \in \mathfrak{su}(2)$
- diffeomorphism symmetry:
  $\delta^D_\xi e = d(\iota_\xi e) + \iota_\xi(de) \qquad \delta^D_\xi \omega = d(\iota_\xi \omega) + \iota_\xi(d\omega) \qquad \xi$ vector field
- on-shell (classically) diffeos obtained by combination of translation and rotation

In discrete gravity diffeos are generically broken (Dittrich-Bahr '09) but leave residual symmetry at least in 3d with $\Lambda = 0$ (Freidel-Louapre '02):

- discrete translation symmetry:
  $B_e \longrightarrow B_e + \phi_{v1} + [\Omega_{v1}(g_L), \phi_{v1}] - \phi_{v2} - [\Omega_{v2}(g_L), \phi_{v2}]$
- discrete rotation symmetry: $B_e(\sigma) \longrightarrow k_\sigma B_e(\sigma) k_\sigma^{-1} \qquad g_L \longrightarrow k_{\sigma 1} g_L k_{\sigma 2}^{-1}$

To identify diffeomorphism symmetry, need to work in (non-commutative) triad representation of GFT action - (necessary to) use "colored model"

DIFFEOMORPHISM SYMMETRY IN BOULATOV MODEL (A. BARATIN, F. GIRELLI, DO, '10)

label vertices in tetrahedron by $i = 1, 2, 3, 4$ - edges are labeled as $e = (ij)$ - color triangles of tetrahedron by their 3 vertices - define 4 fields: ϕ_{ijk} (coloring needed for field transformation)

$$S(\{\phi_{ijk}\}) = \sum_{(ijk)} \int [dx_{ij}](\phi_{ijk} * \phi_{ijk})(x_{ij}, x_{jk}, x_{ki}) +$$

$$+ \frac{\lambda}{4!} \int \phi_{123}(x_{12}, x_{23}, x_{31}) * \phi_{234}(x_{32}, x_{34}, x_{41}) * \phi_{124}(x_{21}, x_{24}, x_{14}) * \phi_{134}(x_{13}, x_{43}, x_{43})$$

- transformation of GFT field (for $\epsilon_v \in \mathfrak{su}(2)$) (translation of triangle vertices):

$$\left(T_{\{\epsilon_v\}} \triangleright \phi_{123}\right)(x_{12}, x_{23}, x_{31}) = `\phi(x_{12} - \epsilon_1 + \epsilon_2, x_{23} - \epsilon_2 + \epsilon_3, x_{31} - \epsilon_3 + \epsilon_1)'$$

$$\left(T_{\{\epsilon_v\}} \triangleright \phi_{123}\right)(g_{12}, g_{23}, g_{31}) = e^{i\mathrm{Tr}(\epsilon_1(g_{31}g_{12}^{-1}))}\, e^{i\mathrm{Tr}(\epsilon_2(g_{12}g_{23}^{-1}))}\, e^{i\mathrm{Tr}(\epsilon_3(g_{23}g_{31}^{-1}))}\phi(g_{12}, g_{23}, g_{31})$$

- can go to 'vertex variables": $\phi(x_{12}, x_{23}, x_{31}) \longrightarrow \phi(x_1, x_2, x_3)$
- action takes form (schematically):

$$S(\{\phi_{ijk}\}) = \sum_{(ijk)} \int [dx_i](\phi_{ijk} * \phi_{ijk})(x_i, x_j, x_k) +$$

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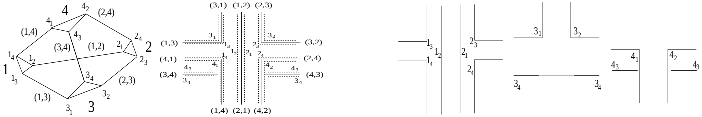
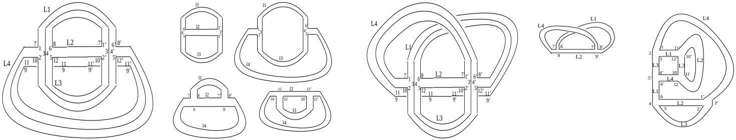


- see intertwiner of single copy of $D\mathrm{SU}(2)$ translation at each vertex of Δ
- can show that action $S(\{\phi_{ijk}\})$ is invariant (care with ordering, $*$-products,...)
- it indeed corresponds to the diffeomorphism symmetry at the level of the Feynman amplitudes (simplicial gravity path integrals)
- it can be related to simplicial Bianchi identity in each bubble (vertex of Δ) (evaluation of invariant diagram - need to take *braiding* into account)



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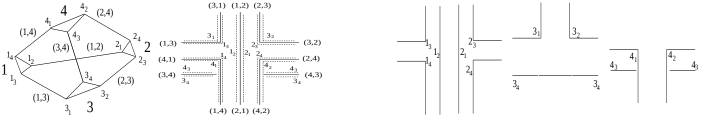

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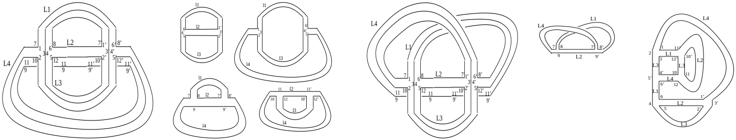

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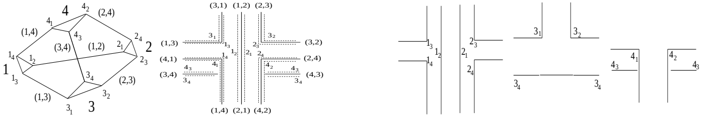

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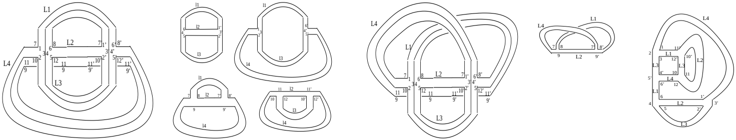

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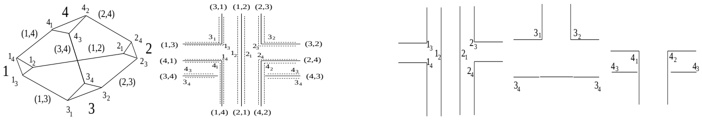

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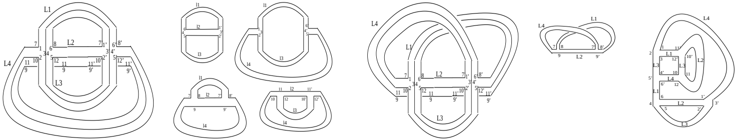

PERTURBATIVE GFT RENORMALIZATION - THE 3D CASE

(L. Freidel, R. Gurau, DO, '09), (J. Magnen et al., '09), (R. Gurau, '09), (J. Ben Geloun et al., '09, '10), (V. Bonzom, M. Smerlak, '10)

Question: can you control the perturbative GFT sum over Feynman diagrams (including sum over topologies)?

$$S = \frac{1}{2}\int (\phi(g_1, g_2, g_3))^2 + \frac{\lambda}{4!}\int \phi(g_1, g_2, g_3)\phi(g_3, g_4, g_5)\phi(g_5, g_6, g_1)\phi(g_6, g_4, g_2)$$

$$Z(\Gamma) = \prod_{L\in\Gamma}\int dg_L \prod_f \delta(\prod_{L\in\partial f} g_L)$$

FD are cellular complexes Γ dual to 3d triangulations

- divergences associated to bubbles (3-cells in FDs)
- highly involved combinatorics, all topologies and pseudo-manifolds $\longrightarrow$ difficult to isolate divergences, unclear which FDs need renormalization
- results:
  - algorithm identifying bubbles in FD, and their boundary triangulations (see earlier)
  - identification of 'Type 1'graphs, generalization of 2d planar graphs, allowing for contraction procedure, then proved to be -manifolds- of -trivial topology-
  - exact power counting of divergences for this class of FD
  - conjecture: these are the only relevant FD in generalized scaling limit
  - very general scaling bounds $Z_\Lambda(\Gamma) \leq K^n\Lambda^{6+3n/2}$, with n vertices

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Perturbative GFT renormalization - the 3d case

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EFFECTIVE NON-COMMUTATIVE MATTER FIELDS FROM GFT

- Insights from analogue gravity models in condensed matter
  (C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065:
  - effective metric and its dynamics from hydrodynamics of microscopic system $\Rightarrow$ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric $\Rightarrow$ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
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  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
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These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]),
e.g. in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

Task: derive such NC field theories from more fundamental GFT models
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EFFECTIVE NON-COMMUTATIVE MATTER FIELDS FROM GFT

- Insights from analogue gravity models in condensed matter
  (C. Barcelo, S. Liberati, M. Visser, gr-qc/0505065:
  - effective metric and its dynamics from hydrodynamics of microscopic system ⇒ (modified) GR from GFT hydrodynamics?
  - quasi-particles (perturbations around ground state) see only effective metric ⇒ effective QFT for matter in emergent metric from GFT? which effective matter QFT? incorporate QG effects? (DO, 0903.3970 [hep-th])
- General hypothesis
  - effective QG spacetime is non-commutative of Lie algebra type: $[x_\mu, x_\nu] = \mathcal{C}^\alpha_{\mu\nu} x_\alpha$
  - by duality, corresponding momentum space is a (curved) group manifold (Majid)
  - symmetry group of such NC spacetimes is Hopf algebra
  - deformed Casimir (dispersion relation), deformed addition of momenta (scattering thresholds)
  - suggests QFT in such NC spacetime, in momentum space, is a GFT

These NC spacetimes are arena for much current QG phenomenology (G. Amelino-Camelia, 0806.0339 [gr-qc]),
e.g. in the context of Deformed Special Relativity (DSR) (J. Kowalski-Glikman, hep-th/0405273)

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THE 3D (RIEMANNIAN) CASE - NC FIELD THEORY FROM GFT

We deal with: $SU(2) \leftrightarrow \mathfrak{su}(2) : [X_i, X_j] = \frac{i}{\kappa}\epsilon_{ijk}X_k$ (L. Freidel, E. Livine, '04, L. Freidel, S. Majid, '04)

NC Fourier transform introduced earlier : $C(SU(2)) \rightarrow C_{*,\kappa}(\mathbb{R}^3)$

Scalar field theory in momentum space is:

$$S[\psi] = \frac{1}{2}\int_{SU(2)} dg\psi(g)\mathcal{K}(g)\psi(g^{-1}) - \frac{\lambda}{3!}\int [dg]^3\,\psi(g_1)\psi(g_2)\psi(g_3)\delta(g_1g_2g_3),$$

Boulatov GFT model:

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Classical solution:

$$\varphi_0(g_1,g_2,g_3) \equiv \sqrt{\frac{3!}{\lambda}}\int dg\,\delta(g_1g)F(g_2g)\delta(g_3g).$$

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Interpretation: quantum flat space on some space topology (invariant under $DSU(2)$)

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NC field theory from GFT: 2d perturbations and effective FT

Look at "two-dimensional" perturbation $\psi(g_1, g_3) = \psi(g_1 g_3^{-1})$ of field φ around φ_0:
$\varphi(g_1, g_2, g_3) = \varphi_0(g_1, g_2, g_3) + \psi(g_1, g_3)$ Then effective action for ψ is

$$S_{eff}[\psi] \equiv S[\varphi_0 + \psi] - S[\varphi_0] = \frac{1}{2}\int \psi(g)\mathcal{K}(g)\psi(g^{-1}) -$$

$$-\frac{\mu}{3!}\int [dg]^3\, \psi(g_1)\psi(g_2)\psi(g_3)\delta(g_1 g_2 g_3) - \frac{\lambda}{4!}\int [dg]^4\, \psi(g_1)..\psi(g_4)\delta(g_1..g_4),$$

with

$$\mathcal{K}(g) = 1 - 2\left(\int F\right)^2 - \int dh F(h)F(hg), \qquad \frac{\mu}{3!} = \sqrt{\frac{\lambda}{3!}}\int F$$

The effective action S_{eff} depends on the solution φ_0 through F, and is invariant under $DSU(2)$ (quantum double of SU(2)), deformation of Poincaré group
F can be expanded in group characters: $F(g) = \sum_{j\in\mathbb{N}/2} F_j \chi_j(g)$

$$\mathcal{K}(g) = \sum_{j>0} F_j^2\left(1 - \frac{\chi_j(g)}{d_j}\right) - 2F_0^2 \equiv Q^2(g) - M^2.$$

$Q^2(g) \geq 0$, $Q(I) = 0$, generalized "Laplacian"
For $F(g) = a + \sqrt{1-a^2}\chi_1(g)$ we get $\mathcal{K}(g) = \frac{4}{3}(1-a^2)\vec{k}^2 - 2a^2$, $\vec{k} = \mathrm{Tr}(g\vec{\sigma})$

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GFT: SUMMARY

- GFTs are (combinatorially) non-local field theories on groups (Lie algebras), interpreted as 2nd quantized theories (generalization of matrix models)
  - of simplicial geometry and
  - of canonical LQG (QFT of spin networks)
- field ϕ represents "2nd quantized simplex"or "2nd quantized spin net vertex"
- arguments of field have interpretation of (quantum) pre-geometric data
- microscopic dynamics dictated by discrete (minimal) geometric considerations
- both geometry and topology are dynamical
- GFT realize duality of simplicial gravity path integrals and spin foam models
- GFT can be common framework for various QG approaches:
  - Loop Quantum Gravity and spin foam models:
    GFT states are Spin Networks, GFT perturbative expansion defines their dynamics
    GFT Feynman amplitudes are Spin Foam models (sum over histories of spin networks)
  - Quantum Regge Calculus: GFT Feynman amplitudes define simplicial QG path integrals, with unique (for given GFT) measure
  - Dynamical Triangulations: GFT describes QG (perturbatively) as sum over triangulations, weighted by simplicial path integral
- allow (almost) straightforward application of QFT tools
- being a "pre-geometric theory", recovering smooth geometry (and other background structures of GR) and GR dynamics is non-trivial task

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