Field theory on curved NC spacetimes

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Spacetime Quantum Geometry

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Motivation

- QFT on curved spacetimes is important for physics

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  $\longrightarrow$ test their physical implications by using QFTCS

Scalar field theory on a class of curved NC spacetimes

- *Simple example of a twist:* [Moyal product/twist]

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  - normalization: $(\epsilon \otimes_{\mathbb{C}} \mathrm{id})\mathcal{F} = (\mathrm{id} \otimes_{\mathbb{C}} \epsilon)\mathcal{F} = 1$
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  - algebra of functions $(C^\infty(\mathcal{M})[[\lambda]], \star)$, where $h \star k := \bar{f}^\alpha(h) \cdot \bar{f}_\alpha(k)$

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  - pairing $\langle v,\omega\rangle_\star := \langle\bar{f}^\alpha(v),\bar{f}_\alpha(\omega)\rangle$ among vflds and 1forms

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$$S_\Phi = \int L_\Phi = -\frac{1}{2}\int \Big(\langle\langle \mathrm{d}\Phi, g_\star^{-1_\star}\rangle_\star, \mathrm{d}\Phi\rangle_\star + M^2\,\Phi\star\Phi\Big)\star \mathrm{vol}_\star$$

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NB: $P_\star$ is formally self adjoint w.r.t. SP $(\varphi,\psi)_\star = \int \varphi^*\star\psi\star\mathrm{vol}_\star$, i.e.

$$(\varphi, P_\star[\psi])_\star = (P_\star[\varphi],\psi)_\star$$

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1.) $\mathcal{F}^{-1} = \exp\left(\frac{i\lambda}{2}(\partial_t \otimes_{\mathbb{C}} \partial_\varphi - \partial_\varphi \otimes_{\mathbb{C}} \partial_t)\right) \quad \Rightarrow \quad [e^{i\varphi} \overset{\star}{,} t] = \lambda\, e^{i\varphi}$

$$-\left(\partial_t^2 + 3H\partial_t + M^2\right)\frac{1 + e^{i3\lambda H \partial_\varphi}}{2}\,\Phi + e^{-2Ht}\triangle\frac{e^{-i\lambda H\partial_\varphi} + e^{i4\lambda H \partial_\varphi}}{2}\,\Phi = 0$$

NB: depends only on λH $\Rightarrow$ no deformation for $H \to 0$!
very rough estimate: $\lambda H \approx t_{\mathrm{pl}} H_{\mathrm{today}} \approx 10^{-60}$

2.) $\mathcal{F}^{-1} = \exp\left(\frac{i\lambda}{2}(x^i\partial_i \otimes_{\mathbb{C}} \partial_\varphi - \partial_\varphi \otimes_{\mathbb{C}} x^i\partial_i)\right) \Rightarrow \quad e^{i\varphi} \star r = e^{\lambda}\, r \star e^{i\varphi}$

$$-\left(\partial_t^2 + 3\frac{\dot a}{a}\partial_t + M^2\right)\frac{1 + e^{i3\lambda\partial_\varphi}}{2}\,\Phi + \frac{1}{a^2}\triangle\frac{e^{-i\lambda\partial_\varphi} + e^{i4\lambda\partial_\varphi}}{2}\,\Phi = 0$$

- spherical wave $\Phi = \phi(t)\, j_l(k\, r)\, Y_{lm}(\theta, \varphi)$ and for simplicity $a(t) \equiv 1$

$$\partial_t^2\phi(t) + \left(M^2 + k^2\,\frac{e^{\lambda m} + e^{-4\lambda m}}{1 + e^{-3\lambda m}}\right)\phi(t) = 0\,, \quad E(k, m)^2 \geqq M^2 + k^2$$

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- UV improvement of Φ^4-theory in $d = 4$:

 $\sim \sqrt{\Lambda_{\text{UV}}} + \text{finite}$,  $= \text{finite}$

Let's go back to curved NC spacetimes and talk about the quantization of scalar fields.

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- based on strong results for the commutative case we find:

  there exist unique Green's operators $\Delta_{\star\pm} := \sum \lambda^n \Delta_{(n)\pm}$ satisfying

(i) $P_\star \circ \Delta_{\star\pm} = \mathrm{id}_{C^\infty_0(\mathcal{M})[[\lambda]]}$,

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(iii) $\mathrm{supp}(\Delta_{(n)\pm}[\varphi]) \subseteq J_\pm(\mathrm{supp}(\varphi))$, for all $n \in \mathbb{N}^0$ and $\varphi \in C^\infty_0(\mathcal{M})$,

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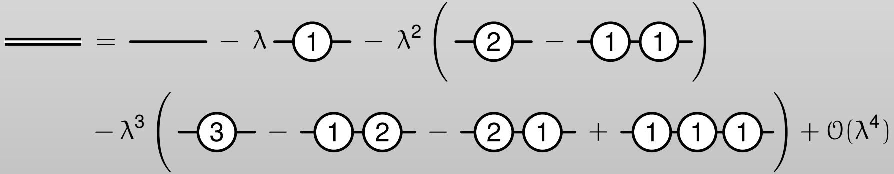
NB: also possible for deformed normally hyperbolic operators $P_\star$

Explicit formula for $\Delta_{\star\pm}$ in terms of $\Delta_\pm := \Delta_{(0)\pm}$:

$$
\begin{aligned}
\Delta_{\star\pm} = \Delta_\pm& \\
&- \lambda\, \Delta_\pm \circ P_{(1)} \circ \Delta_\pm \\
&- \lambda^2 \left(\Delta_\pm \circ P_{(2)} \circ \Delta_\pm - \Delta_\pm \circ P_{(1)} \circ \Delta_\pm \circ P_{(1)} \circ \Delta_\pm\right) \\
&+ \mathcal{O}(\lambda^3) \quad \text{[higher orders follow the same structure]}
\end{aligned}
$$

Graphically:



→ perturbative approach to deformed Green's operators

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Proposition (TO, AS)

$(V_\star, \omega_\star)$ *with* $V_\star := H/\mathit{Ker}(\Delta_\star)$ *and*

$$\omega_\star([\varphi],[\psi]) := (\varphi, \Delta_\star[\psi])_\star = \int \varphi^* \star \Delta_\star[\psi] \star \mathrm{vol}_\star$$

is a symplectic vector space.

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NB: twisting requires CCR-compatible $\mathcal{F}$ [Poisson geometry: Aschieri, Lizzi, Vitale]

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  - Can one include convergent deformations? $\rightarrow$ hopefully C^*-properties