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NC gauge models

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Content

Introduction

UV/IR mixing in scalar NCQFT
Renormalizable models

NC gauge theory

Oscillator model
joint work with Blaschke, Grosse, Kronberger, Schweda
$1/p^2$ model
joint work with Blaschke, Heindl, Rofner, Sedmik, Schweda

Outlook

Introduction

We consider 4D canonically deformed Euclidean space:

$$[x^i \stackrel{\star}{,} x^j] = \mathrm{i}\Theta^{ij} \,,$$

$\Theta^{ij} = -\Theta^{ji} = \text{const}$, with Weyl-Moyal $\star$-product

$$f \star g\,(x) = \mathrm{e}^{\frac{\mathrm{i}}{2}\Theta^{ij}\partial_i^x \partial_j^y} f(x)\,g(y)\Big|_{y \to x} \,.$$

Introduction

NC scalar ϕ^4

$$S = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \, \partial_\mu \phi + \frac{m^2}{2} \phi^2 + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right)$$

Feynman rules:

- propagator $G(p) = \frac{1}{p^2 + m^2}$
- vertex function $\Gamma(p_1, \ldots, p_4) = \lambda \delta^{(4)}(p_1 + p_2 + p_3 + p_4) \mathrm{e}^{-\mathrm{i} \sum_{i<j} p_i \Theta p_j}$

Introduction

2-point tadpole

$$\Pi(\Lambda, p) \propto \int d^4k \frac{2 + \cos k\tilde{p}}{k^2 + m^2} = \Pi^{UV}(\Lambda) + \Pi^{IR}(\Lambda, p)$$

with the IR-divergent non-planar part

$$\Pi^{IR} \sim \frac{1}{\tilde{p}^2}$$

$\tilde{p}_\mu = \Theta_{\mu\nu} p_\nu$;
UV/IR mixing destroys renormalizability.

Introduction

2 different strategies to cure UV/IR mixing:

1 - Adding an oscillator potential (Grosse, Wulkenhaar 03, 05):

$$S = \int d^D x \left(\frac{1}{2} \phi \star [\tilde{x}_\nu, [\tilde{x}^\nu, \phi]_\star]_\star + \frac{\Omega^2}{2} \phi \star \{\tilde{x}^\nu, \{\tilde{x}_\nu, \phi\}_\star\}_\star + \frac{\mu^2}{2} \phi \star \phi + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right)(x) \, ,$$

where $\tilde{x}_\nu = \theta^{-1}_{\nu\alpha} x^\alpha$ and $i\partial_\mu f = [\tilde{x}_\mu, f]_\star$

Introduction

Remarks:

- Langmann-Szabo duality
- no UV/IR mixing due to oscillator term; propagator given by the Mehler kernel - IR damping implemented

$$K_M(p,q) = \frac{\omega^3}{8\pi^2} \int_0^\infty \frac{d\alpha}{\sinh^2 \alpha} \mathrm{e}^{-\frac{\omega}{4}(p-q)^2 \coth \frac{\alpha}{2} - \frac{\omega}{4}(p+q)^2 \tanh \frac{\alpha}{2}}$$

- theory perturbatively renormalisable
- Oscillator term can be interpreted as coupling of the scalar field to the curvature of a NC background (Buric, MW 08)

Introduction

2 - Adding a non-local term (Gurau, Magnen, Rivasseau, Tanasa 09): in momentum space:

$$S_{nl} = \int d^4p \, \frac{a}{2} \phi(p) \frac{1}{\tilde{p}^2} \phi(-p)$$

Also implements IR damping ($G(p) \to 0$, for $k \to 0$):

$$G(p) = \frac{1}{p^2 + m^2 + \frac{a^2}{p^2}}$$

$\Rightarrow$ perturbatively renormalizable to all orders.

NC gauge theory

Aim is to generalize both approaches to NC $U(1)$ gauge theory.

- ad 1 - oscillator approach
  action proposed by (Blaschke, Grosse and Schweda 07)

$$S = \int d^4x \left(\frac{1}{4} F_{\mu\nu} \star F^{\mu\nu} + s(\bar{c} \star \partial_\mu A_\mu) - \frac{1}{2} B^2 + \frac{\Omega^2}{8} s(\tilde{c}_\mu \star \mathcal{C}_\mu) \right)$$

where $\mathcal{C}_\mu = \{\{\tilde{x}_\mu \stackrel{\star}{,} A_\nu\} \stackrel{\star}{,} A_\nu\} + [\{\tilde{x}_\mu \stackrel{\star}{,} \bar{c}\} \stackrel{\star}{,} c] + [\bar{c} \stackrel{\star}{,} \{\tilde{x}_\mu \stackrel{\star}{,} c\}]$,

$$\begin{aligned} sA_\mu &= D_\mu c, \quad s\bar{c} = B, \qquad sc = igc \star c, \\ sB &= 0, \qquad s\tilde{c}_\mu = \tilde{x}_\mu \end{aligned}$$

NC gauge theory

propagator of the gauge field is given by Mehler kernel

1-loop calculations (Blaschke, Grosse, Kronberger, Schweda, MW 09)

– power counting
– correction to 1-, 2-, 3- and 4-point functions computed
– tadpole terms UV-divergent $\rightarrow$ linear counter terms needed

NC gauge theory

Induced gauge action (de Goursac, Wallet, Wulkenhaar 07; Grosse, MW 07)

$$
\begin{aligned}
S &= \int d^4x \left\{ \frac{3}{\theta}(1-\rho^2)(\tilde{\mu}^2-\rho^2)(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2) \right. \\
&\qquad \left. + \frac{3}{2}(1-\rho^2)^2 \big((\tilde{X}_\mu \star \tilde{X}_\mu)^{\star 2} - (\tilde{x}^2)^2 \big) - \frac{\rho^4}{4} F_{\mu\nu} F_{\mu\nu} \right\},
\end{aligned}
$$

where $F_{\mu\nu} = -i[\tilde{x}_\mu, A_\nu]_\star + i[\tilde{x}_\nu, A_\mu]_\star - i[A_\mu, A_\nu]_\star$

$$
\rho = \frac{1-\Omega^2}{1+\Omega^2},\ \tilde{\mu}^2 = \frac{m^2\theta}{1+\Omega^2}
$$

NC gauge theory

- $\Omega \to 0$ ($\rho \to 1$): usual NCYM
- $\Omega \to 1$ ($\rho \to 0$): obtain interesting matrix models
- non-trivial vacuum
- computation of the propagator and Feynman rules

NC gauge theory

action proposed by Buric, Grosse, Madore 10

$$
\begin{aligned}
S &= \int d^2x \Big((1-\alpha^2)F_{12}^{*2} - 2(1-\alpha^2)\mu\, F_{12} \star \phi + (5-\alpha^2)\mu^2\phi^2 \\
&+4i\alpha F_{12} \star \phi^{\star 2} + (D_i\phi)^2 - \alpha^2\{p_i + A_i \stackrel{\star}{,} \phi\}^2 \Big)
\end{aligned}
$$

NC gauge theory

- ad 2 - $1/p^2$ model

  Gauge invariant generalizations

  additional term

$$F_{\mu\nu}\frac{1}{\tilde{D}^2 D^2}F_{\mu\nu}$$

  proposed by Blaschke, Gieres, Kronberger, Schweda, MW 08;
  when expanded in the gauge field, vertices with arbitrary number of photon legs occur $\Rightarrow$ localisation

NC gauge theory

two different ways to implement the localization

Blaschke, Rofner, Schweda, Sedmik 08:

$$\int d^4x F_{\mu\nu} \frac{a^2}{\tilde{D}^2 D^2} F_{\mu\nu} \to \int d^4x \left(a B_{\mu\nu} F_{\mu\nu} - B_{\mu\nu} \star \tilde{D}^2 D^2 B_{\mu\nu} \right)$$

additional degrees of freedom introduced

Vilar, Ventura, Tedesco, Lemes 09:
used BRST doublet structure in order to avoid introduction of additional degrees of freedom

not renormalizable

NC gauge theory

IR Damping can also be implemented in the "soft breaking" part

Gribov-Zwanziger approach to QCD (Zwanziger 89, 93), where a IR modification of the propagator is suggested to cure the Gribov ambiguities

NC gauge theory

$$
\begin{aligned}
S &= S_{inv} + S_{gf} + S_{aux} + S_{break} + S_{ext} \, , \\
S_{inv} &= \int d^4x \, \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \, , \\
S_{gf} &= \int d^4x \, s(\bar{c} \partial_\mu A_\mu) \, , \\
S_{aux} &= \int d^4x \, s \, (\bar{\psi}_{\mu\nu} B_{\mu\nu}) \quad , \\
S_{break} &= \int d^4x \, s \left((\bar{Q}_{\mu\nu\alpha\beta} B_{\mu\nu} + Q_{\mu\nu\alpha\beta} \bar{B}_{\mu\nu}) \frac{1}{\tilde{\Box}} (f_{\alpha\beta} + \sigma \frac{\theta_{\alpha\beta}}{2} \tilde{f}) \right) \quad , \\
S_{ext} &= \int d^4x \, (\Omega^A_\mu s A_\mu + \Omega^c s c) \, ,
\end{aligned}
$$

where $f_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, $\Theta_{\alpha\beta} = \epsilon \, \theta_{\alpha\beta}$ and $\tilde{f} = \theta_{\alpha\beta} f_{\alpha\beta}$.

NC gauge theory

1-loop results:

vertex functions are the usual ones, as in "naive" NC theories;
propagators are complicated:

$$G^A_{\mu\nu}(k) = \left(k^2 + \frac{\gamma^4}{\tilde{k}^2}\right)^{-1} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} - \frac{\bar{\sigma}^4}{(k^2 + (\bar{\sigma}^4 + \gamma^4)\frac{1}{\tilde{k}^2})} \frac{\tilde{k}_\mu \tilde{k}_\nu}{(\tilde{k}^2)^2}\right)$$

but for loop-calculation can be approximated:

$$G^A_{\mu\nu} \sim \frac{1}{k^2}(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}), \; k^2 \to \infty$$

NC gauge theory

- 1-loop corrections reduce to the ones known from "naive" NCQED (e.g. Hayakawa 99; Matusis, Susskind, Toumbas 00)
- modifications at higher loops expected
- divergences can be absorbed in the tree level action

Outlook

- 1-loop calculations for the induced gauge model
- Renormalization (dis-)proof of the latest $1/p^2$-model applying multi-scale analysis