Bayrischzell workshop

Noncommutativity and Physics: General Relativity and Quantum Geometry

Neutrino self energy and dispersion relation in noncommutative spacetimes

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based on

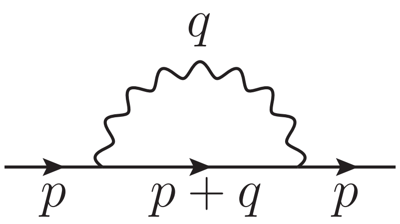
R. Horvat, A. I., P. Schupp, J. Trampetić and J. You,

JHEP 1204 (2012) 108

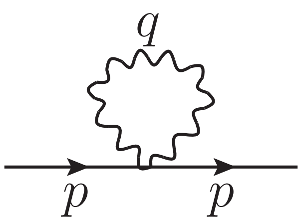
CONTENTS

Problem

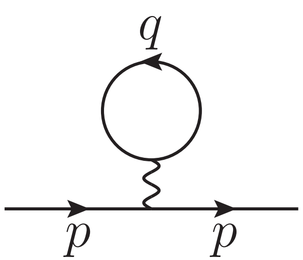
Evaluation of the neutrino self energy in the θ exact way in $U_\star(1)$ noncomutative field theories with vertices obtained using Seiberg-Witten maps.



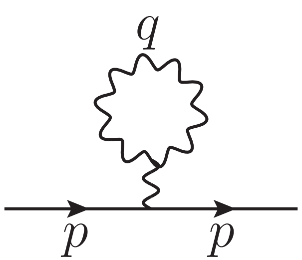
Σ_1



Σ_2



Σ_3



Σ_4

Models and Feynman rules

Model 1

Fields: a_μ, ψ
Seiberg-Witten (SW) maps: A_μ, Ψ ($U(1)_\star$)
Action, covariant derivative, field strength:

$$
\begin{aligned}
S &= \int -\frac{1}{4} F^{\mu\nu} \star F_{\mu\nu} + i\bar{\Psi} \star \not{D} \Psi \\
D_\mu \Psi &= \partial_\mu \Psi - i[A_\mu \stackrel{\star}{,} \Psi] \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu \stackrel{\star}{,} A_\nu]
\end{aligned}
$$

A_μ–ψ coupling [P.Schupp, J.Trampetić, J.Wess, G.Raffelt, NPB651(2003)45, P.Minkowski, P.Schupp, J.Trampetić, EPJC36(2004),405]:

$$
[A_\mu \stackrel{\star}{,} \Psi] \equiv A_\mu \star \Psi - \Psi \star A_\mu + \mathcal{O}(a^3)
$$

SW solutions (expansion of S) [P. Schupp, J. You JHEP08(2008)107]:

$$A_\mu = a_\mu - \frac{1}{2}\theta^{\nu\rho} a_\nu \star_2 (\partial_\rho a_\mu + f_{\rho\mu}) + \mathcal{O}(a^3) = \hat{A}_\mu(a)$$

$$\begin{aligned}\Psi &= \psi - \theta^{\mu\nu} a_\mu \star_2 \partial_\nu \psi + \frac{1}{2}\theta^{\mu\nu}\theta^{\rho\sigma}\Big\{(a_\rho \star_2 (\partial_\sigma a_\mu + f_{\sigma\mu})) \star_2 \partial_\nu \psi + 2 a_\mu \star_2 (\partial_\nu (a_\rho \star_2 \partial_\sigma \psi)) \\ &- a_\mu \star_2 (\partial_\rho a_\nu \star_2 \partial_\sigma \psi) - [a_\rho \partial_\mu \psi (\partial_\nu a_\sigma + f_{\nu\sigma}) - \partial_\rho \partial_\mu \psi a_\nu a_\sigma]_{\star_3} + \mathcal{O}(a^3)\psi = \hat{\Psi}(\psi)\end{aligned}$$

Generalized star products

$$f(x) \star_2 g(x) = \frac{\sin \frac{\partial_1 \theta \partial_2}{2}}{\frac{\partial_1 \theta \partial_2}{2}} f(x_1) g(x_2)\Big|_{x_1 = x_2 = x}$$

$$[f(x) g(x) h(x)]_{\star_3} = \left[\frac{\sin \frac{\partial_2 \theta \partial_3}{2} \sin \frac{\partial_1 \theta (\partial_2 + \partial_3)}{2}}{\frac{(\partial_1 + \partial_2)\theta \partial_3}{2} \frac{\partial_1 \theta (\partial_2 + \partial_3)}{2}} + \{1 \leftrightarrow 2\}\right] f(x_1) g(x_2) h(x_3)\Big|_{x_i = x}$$

Interaction parts of the action

$$S_g = i \partial_\mu a_\nu \star [a^\mu \stackrel{\star}{,} a^\nu] + \frac{1}{2}\partial_\mu (\theta^{\rho\sigma} a_\rho \star_2 (\partial_\sigma a_\nu + f_{\sigma\nu})) \star f^{\mu\nu} + \mathcal{O}(a^4)$$

$$
\begin{aligned}
S_f \quad = & \int \bar{\psi}\gamma^\mu[a_\mu \stackrel{\star}{;} \psi] + i(\theta^{ij}\partial_i\bar{\psi} \star_2 a_j)\partial\!\!\!/\psi - i\bar{\psi} \star \partial\!\!\!/(\theta^{ij}a_i \star_2 \partial_j\psi) \\
+ \quad & (\theta^{ij}\partial_i\bar{\psi} \star_2 a_j)\gamma^\mu[a_\mu \stackrel{\star}{;} \psi] - \bar{\psi}\gamma^\mu[a_\mu \stackrel{\star}{;} \theta^{ij}a_i \star_2 \partial_j\psi] - \frac{1}{2}\bar{\psi}\gamma^\mu[\theta^{ij}a_i \star_2 (\partial_j a_\mu + f_{j\mu}) \stackrel{\star}{;} \psi] \\
- \quad & i(\theta^{ij}\partial_i\bar{\psi} \star_2 a_j)\partial\!\!\!/(\theta^{kl}a_k \star_2 \partial_l\psi) + \frac{i}{2}\theta^{ij}\theta^{kl}\Big((a \star_2 (\partial_l a_i + f_{li})) \star_2 \partial_j\bar{\psi} \\
+ \quad & 2a_i \star_2 (\partial_j(a_k \star_2 \partial_l\bar{\psi})) - a_i \star_2 (\partial_k a_j \star_2 \partial_l\bar{\psi}) + [a_i\partial_k\bar{\psi}(\partial_j a_l + f_{jl}) - \partial_k\partial_i\bar{\psi}a_j a_l]_{\star_3}\Big)\partial\!\!\!/\psi \\
+ \quad & \frac{i}{2}\theta^{ij}\theta^{kl}\bar{\psi}\partial\!\!\!/\Big((a_k \star_2 (\partial_\ell a_i + f_{li})) \star_2 \partial_j\psi + 2a_i \star_2 (\partial_j(a_k \star_2 \partial_l\psi)) - a_i \star_2 (\partial_k a_j \star_2 \partial_l\psi) \\
+ \quad & [a_i\partial_k\psi(\partial_j a_l + f_{jl} - \partial_k\partial_l)]_{\star_3}\Big) + \bar{\psi}\mathcal{O}(a^3)\psi
\end{aligned}
$$

Model 2

Action for a neutral massless free fermion field S_f promoted by SW maps ($\hat{\Psi}(\bar{\psi})$, $\hat{\Phi}(\partial_\mu \psi)$) to noncommutative action $S_{f,2}$

$$S_f \;=\; \int \bar{\psi}\gamma^\mu \partial_\mu \psi \;\rightarrow\; S_{f_{M2}} = \int \hat{\Psi}(\bar{\psi})\gamma^\mu \hat{\Phi}(\partial_\mu \psi)$$

assuming that the SW maps and corresponding map $\hat{\Lambda}$ for the gauge parameter λ satisfy relations

$$\begin{aligned} \delta_\lambda(\hat{\Psi}(\bar{\psi})) &= i[\hat{\Lambda}(\lambda) \stackrel{\star}{,} \hat{\Psi}(\bar{\psi})] \\ \delta_\lambda(\hat{\Phi}(\partial_\mu \psi)) &= i[\hat{\Lambda}(\lambda) \stackrel{\star}{,} \hat{\Phi}(\partial_\mu \psi)] \end{aligned}$$

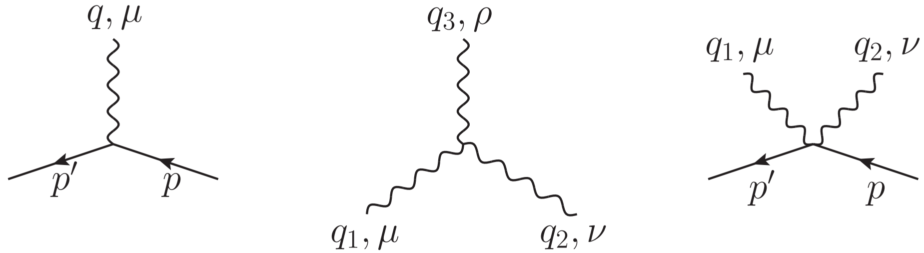
assuring the gauge invariance of the NC action S_{f_2}. $\hat{\Psi}$ taken same as in model 1 and $\hat{\Phi}$ same as $\hat{\Psi}$ ($\delta\psi = 0$, $\delta(\partial_\mu \psi) = 0$),

$$\begin{aligned} \hat{\Phi}(\partial_\mu \psi) &= \hat{\Psi}(\partial_\mu \psi) \;=\; \partial_\mu \psi - \theta^{ij} a_i \star_2 \partial_j(\partial_\mu \psi) \\ &+ \frac{1}{2}\theta^{ij}\theta^{kl}\Big((a_k \star_2 (\partial_l a_i + f_{li})) \star_2 \partial_j(\partial_\mu \psi) + 2a_i \star_2 (\partial_j(a_k \star_2 \partial_l(\partial_\mu \psi))) \\ &- a_i \star_2 (\partial_k a_j \star_2 \partial_l(\partial_\mu \psi))[a_k \partial_i(\partial_\mu \psi)(\partial_j a_l + f_{jl}) - \partial_k \partial_i(\partial_\mu \psi) a_j a_l] + \mathcal{O}(a^3)\psi \end{aligned}$$

Action

$$
\begin{aligned}
S_{f_{M2}} \quad = \quad & \int \Big(i\bar{\psi}\partial\!\!\!/\psi - i(\theta^{ij}\partial_j\bar{\psi} \star_2 a_i)\partial\!\!\!/\psi - i\bar{\psi}(\theta^{ij}a_i \star_2 \partial\!\!\!/\partial_j\psi)\Big) \\
+ \quad & i(\theta^{ij}\partial_j\bar{\psi} \star_2 a_i)\partial\!\!\!/(\theta^{kl}a_k \star_2 \partial_l(\partial_\mu\psi)) \\
+ \quad & \frac{i}{2}\theta^{ij}\theta^{kl}\Big((a_k \star_2 (\partial_l a_i + f_{li})) \star_2 \partial_j\bar{\psi} + 2a_i \star_2 (\partial_j(a_k \star_2 \partial_l\bar{\psi})) \\
- \quad & a_i \star_2 (\partial_k a_j \star_2 \partial_l\bar{\psi}) - [a_k\partial_i\bar{\psi}(\partial_j a_l + f_{jl}) - \partial_k\partial_i(\bar{\psi})a_j a_l]_{\star_3}\Big)\partial\!\!\!/\psi \\
+ \quad & i(\theta^{ij}\partial_j\bar{\psi}\gamma^\mu\Big((a_k \star_2 (\partial_l a_i + f_{li})) \star_2 \partial_j(\partial_\mu\psi) + 2a_i \star_2 (\partial_j(a_k \star_2 \partial_l(\partial_\mu\psi))) \\
- \quad & a_i \star_2 (\partial_k a_j \star_2 \partial_l(\partial_\mu\psi)) - [a_k\partial_i(\partial_\mu\psi)(\partial_j a_l + f_{jl}) - \partial_k\partial_i(\partial_\mu\psi)a_j a_l]_{\star_3}\Big) + h.c.
\end{aligned}
$$

Vertices



V_1 $\quad$ V_2 $\quad$ V_3

$q = p - p'$ $\quad$ q_1, q_2, q_3 *incoming* $\quad$ q_1, q_2 *incoming*

$$V_1^\mu = -F(q,p)[\gamma^\mu q\theta p - \not{q}\tilde{p}^\mu + \not{p}\tilde{q}^\mu], \qquad F(q,p) = \frac{\sin\frac{1}{2}q\theta p}{\frac{1}{2}q\theta p}$$

$$V_{1,M_2}^\mu = -F(p,q)\not{p}\tilde{q}^\mu, \qquad \tilde{p}^\mu = \theta^{\mu\nu}p_\nu$$

$$\begin{aligned}
V_2^{\mu\nu\rho} &= -2\Big\{\sin\frac{q_1\theta q_2}{2}[(q_1-q_2)^\rho g^{\mu\nu} + (q_2-q_3)^\mu g^{\nu\rho} + (q_3-q_1)^\nu g^{\rho\mu}] \\
&+ F(q_1,q_2)\Big[\theta^{\mu\nu}(q_2q_3q_1^\rho - q_1q_3q_2^\rho) + \theta^{\mu\rho}(q_2q_3q_1^\nu - q_1q_2q_3^\nu) + \theta^{\nu\rho}(q_1q_3q_2^\mu - q_1q_2q_3^\mu) \\
&- g^{\mu\nu}(q_2^2\tilde{q}_1^\rho + q_1^2\tilde{q}_2^\rho) - g^{\mu\rho}(q_1^2\tilde{q}_3^\nu + q_3^2\tilde{q}_1^\nu) - g^{\nu\rho}(q_3^2\tilde{q}_2^\mu + q_2^2\tilde{q}_3^\mu) \\
&+ q_3^\rho(\tilde{q}_2^\mu q_3^\nu + \tilde{q}_1^\nu q_3^\mu) + q_2^\nu(\tilde{q}_1^\rho q_2^\mu + \tilde{q}_3^\mu q_2^\rho) + q_1^\mu(\tilde{q}_2^\rho q_1^\nu + \tilde{q}_3^\nu q_1^\rho)\Big]\Big\}
\end{aligned}$$

$$
\begin{aligned}
V_3^{\mu\nu}(q_1,q_2,p,p') \;=\; & 2i\Bigg\{2\frac{\sin\frac{q_1\theta p}{2}\sin\frac{q_2\theta p'}{2}}{q_1\theta p}\tilde{p}^{\mu}\gamma^{\nu} - 2\frac{\sin\frac{q_1\theta p}{2}\sin\frac{q_2\theta p'}{2}}{q_2\theta p'}\tilde{p}'^{\nu}\gamma^{\mu} \\
- \; & \frac{\sin\frac{p\theta p'}{2}\sin\frac{q_1\theta q_2}{2}}{q_1\theta q_2}(2\gamma^{\nu}\tilde{q}_2^{\mu} - \not{q}_2\theta^{\mu\nu}) - 2\frac{\sin\frac{q_1\theta p}{2}\sin\frac{q_2\theta p'}{2}}{q_1\theta p q_2\theta p'}(\not{q}_2+\not{p}')\tilde{p}^{\mu}\tilde{p}'^{\nu} \\
+ \; & \not{p}'\Bigg[\frac{\sin\frac{p\theta p'}{2}\sin\frac{q_1\theta q_2}{2}}{q_1\theta q_2 p\theta p'}(q_2\theta p\theta^{\mu\nu} - 2\tilde{q}_2^{\mu}\tilde{p}^{\nu}) \\
- \; & \frac{\sin\frac{q_1\theta p'}{2}\sin\frac{q_2\theta p}{2}}{q_1\theta p' q_2\theta p}2(\tilde{q}_2-\tilde{p})^{\mu}\tilde{p}^{\nu} + \frac{\sin\frac{q_1\theta p'}{2}\sin\frac{q_2\theta p}{2}}{q_1\theta p'}\theta^{\mu\nu} \\
+ \; & \left(\frac{\sin\frac{q_2\theta p}{2}\sin\frac{q_1\theta p'}{2}}{q_2\theta p' q_1\theta p'} + \frac{\sin\frac{q_1\theta q_2}{2}\sin\frac{p\theta p'}{2}}{q_2\theta p' p\theta p'}\right)(2\tilde{p}^{\nu}\tilde{q}_2^{\mu} + \theta^{\mu\nu}p\theta q_2 - \tilde{p}^{\mu}\tilde{p}^{\nu})\Bigg] \\
+ \; & \not{p}\Bigg[\frac{\sin\frac{p'\theta p}{2}\sin\frac{q_1\theta q_2}{2}}{q_1\theta q_2 p'\theta p}(2\tilde{q}_2^{\mu}\tilde{p}'^{\nu} - q_2\theta p'\theta^{\mu\nu}) \\
+ \; & \frac{\sin\frac{q_1\theta p}{2}\sin\frac{q_2\theta p'}{2}}{q_1\theta p q_2\theta p'}2(\tilde{q}_2+\tilde{p}')^{\mu}\tilde{p}'^{\nu} - \frac{\sin\frac{q_1\theta p}{2}\sin\frac{q_2\theta p'}{2}}{q_1\theta p}\theta^{\mu\nu} \\
- \; & \left(\frac{\sin\frac{q_2\theta p'}{2}\sin\frac{q_1\theta p}{2}}{q_2\theta p q_1\theta p} + \frac{\sin\frac{q_2\theta q_1}{2}\sin\frac{p'\theta p}{2}}{q_2\theta p p'\theta p}\right)(2\tilde{p}'^{\nu}\tilde{q}_2^{\mu} + \theta^{\mu\nu}p'\theta q_2 + \tilde{p}'^{\mu}\tilde{p}'^{\nu})\Bigg]\Bigg\} + \left\{\begin{matrix} q_1 \leftrightarrow q_2 \\ \mu \leftrightarrow \nu \end{matrix}\right\}
\end{aligned}
$$

$$
\begin{aligned}
V^{\mu\nu}_{3,M_2}(q_1,q_2,p,p') \;=\;& 2i\not{p}'\Bigg\{ -2\frac{\sin\frac{q_1\theta p}{2}\frac{q_2\theta p'}{2}}{q_1\theta p q_2\theta p'}\tilde{p}^{\mu}\tilde{p}'^{\nu} \\
+\;& \Bigg[\frac{\sin\frac{p\theta p'}{2}\frac{q_1\theta q_2}{2}}{q_1\theta q_2 p\theta p'}(q_2\theta q_1\theta^{\mu\nu} - 2\tilde{q}_2^{\mu}\tilde{p}^{\nu} + 2\tilde{q}_2^{\mu}\tilde{p}'^{\nu}) \\
-\;& \frac{\sin\frac{q_1\theta p'}{2}\sin\frac{q_2\theta p}{2}}{q_1\theta p' q_2\theta p}2(\tilde{q}_2-\tilde{p})^{\mu}\tilde{p}^{\nu} + \Bigg(\frac{\sin\frac{q_1\theta p'}{2}\sin\frac{q_2\theta p}{2}}{q_1\theta p'} - \frac{\sin\frac{q_1\theta p}{2}\sin\frac{q_2\theta p'}{2}}{q_1\theta p}\Bigg)\theta^{\mu\nu} \\
+\;& \Bigg(\frac{\sin\frac{q_2\theta p}{2}\sin\frac{q_1\theta p'}{2}}{q_2\theta p' q_1\theta p'} + \frac{\sin\frac{q_1\theta q_2}{2}\sin\frac{p\theta p'}{2}}{q_2\theta p' p\theta p'}\Bigg)(2\tilde{p}^{\nu}\tilde{q}_2^{\mu} + \theta^{\mu\nu}p\theta q_2 - \tilde{p}^{\mu}\tilde{p}^{\nu}) \\
-\;& \Bigg(\frac{\sin\frac{q_2\theta p'}{2}\sin\frac{q_1\theta p}{2}}{q_2\theta p q_1\theta p} + \frac{\sin\frac{q_2\theta q_1}{2}\sin\frac{p'\theta p}{2}}{q_2\theta p p'\theta p}\Bigg)(2\tilde{p}'^{\nu}\tilde{q}_2^{\mu} + \theta^{\mu\nu}p'\theta q_2 - \tilde{p}'^{\mu}\tilde{p}'^{\nu}) \\
+\;& \frac{\sin\frac{q_1\theta p}{2}\sin\frac{q_2\theta p'}{2}}{q_1\theta p q_2\theta p'}2(\tilde{q}_2-\tilde{p}')^{\mu}\tilde{p}'^{\nu}\Bigg\} + \begin{Bmatrix} q_1 \leftrightarrow q_2 \\ \mu \leftrightarrow \nu \end{Bmatrix}
\end{aligned}
$$

Charge conjugation

Choice: $\theta^{\mu\nu} \stackrel{C}{\to} -\theta^{\mu\nu}$ [P. Aschieri, B, Jurčo, P. Schupp, J. Wess NPB 651 (2003) 45]

Consequences:

$$
\begin{aligned}
A_\mu &\stackrel{C}{\to} -A_\mu \\
L^1_{f,M1} &= \bar\psi\gamma^\mu[a_\mu \stackrel{\star}{;} \psi] + i\theta^{ij}\partial_i\bar\psi \star_2 a_j \partial\!\!\!/ \psi - i\theta^{ij}\bar\psi a_i \star_2 \partial_j\psi \;\equiv\; j^\mu_\psi a_\mu \\
&\stackrel{C}{\to} -(\bar\psi\gamma^\mu[a^C_\mu \stackrel{\star}{;} \psi] + i\theta^{ij}\partial_i\bar\psi \star_2 a^C_j \partial\!\!\!/ \psi - i\theta^{ij}\bar\psi a^C_i \star_2 \partial_j\psi) \equiv (-j^\mu_\psi)a^C_\mu \\
L^3_{g,M1} &= a^\mu\partial^\nu\Big(-\theta^{\rho\sigma}\partial_\rho a_\nu \star_2 \partial_\sigma a_\mu + \frac{1}{2}\partial_\nu\theta^{\rho\sigma}a_\rho \star_2 (\partial_\sigma a_\mu + f_{\sigma\mu})\Big) \;\equiv\; a_\mu j^\mu_a \\
&\stackrel{C}{\to} (-j^\mu_a)a^C_\mu
\end{aligned}
$$

Therefore in model 1

$$
\langle 0|j^\mu_\psi|0\rangle = 0, \qquad \langle 0|j^\mu_a|0\rangle = 0
$$

and $\Sigma_3 = 0$, $\Sigma_4 = 0$. Similar proof is valid in the model 2.

Methods: parametrizations

1. Schwinger parametrization

$$\frac{1}{a^n} = \frac{1}{\Gamma(n)} \int_0^\infty e^{a\alpha} \alpha^{n-1} d\alpha$$

$$\frac{1}{a_1^{n_1} a_2^{n_2}} = \frac{1}{\Gamma(n_1)\Gamma(n_2)} \int_0^\infty \int_0^\infty e^{-a_1\alpha_1 - a_2\alpha_2} \alpha_1^{n_1-1} \alpha_2^{n_2-1} d\alpha_1 d\alpha_2$$

α_1 and α_2 are dimensionfull parameters.

2. Feynman parametrization

Used to combine the propagator denominators having the same maximal power of loop momentum (a_1 and a_2). Obtained from Schwinger parametrization putting $\alpha_1 = x\alpha$ and $\alpha_2 = (1-x)\alpha$ (x is dimensionless and α is dimensionfull parameter), and integrating over α

$$\frac{1}{a_1^{n_1} a_2^{n_2}} = \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} \int_0^1 \frac{x^{n_1-1}(1-x)^{n_2-1} dx}{[a_1 x + a_2(1-x)]^{n_1+n_2}}$$

3. "HQET" parametrization

Used to simplify a product of propagator denominators linear (a_1) and quadratic in loop momenta. Obtained from Schwinger parametrization putting $\alpha_1 = y\alpha$ and $\alpha_2 = \alpha$ (now both y and α are dimensionfull parameters) and integrating over α

$$\frac{1}{a_1^{n_1} a_2^{n_2}} = \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} \int_0^\infty \frac{y^{n_1-1} dy}{(a_1 y + a_2)^{n_1+n_2}}$$

Amplitudes and results

Model 1

$\Sigma_3 = 0$, $\Sigma_4 = 0$: charge conjugation symmetry

$\Sigma_2 = 0$: shown by explicit calculation

Σ_1:

$$
\begin{aligned}
-i\Sigma_1 &= -e^2 \int \frac{\mu^{4-D} d^D q}{(2\pi)^D} \underbrace{\frac{(2 - e^{iq\theta p} - e^{-iq\theta p})}{(q\theta p)^2}}_{F(p,q)^2} \frac{1}{q^2} \frac{1}{(q+p)^2} \\
&\times \Big[(q\theta p)^2 [(4-D) 2 (\not{p} + \not{q})] + (q\theta p) [\tilde{\not{q}} (2p^2 + 2pq) - \tilde{\not{p}} (2q^2 + 2pq)] \\
&\quad + [\not{p} (\tilde{q}^2 (p^2 + 2qp) - q^2 (\tilde{p}^2 + 2\tilde{p}\tilde{q})) + \not{q} (\tilde{p}^2 (q^2 + 2pq) - p^2 (\tilde{q}^2 + 2\tilde{p}\tilde{q}))] \Big] \\
&= \{ -((4-D)[2I_1 - I_2 - I_3] + [2I_4 - I_5 - I_6] + [2I_7 - I_8 - I_9]) \} \\
&\equiv -((4-D) I_{123} + I_{456} + I_{789})
\end{aligned}
$$

$$\varepsilon I_{123} = \frac{ie^2}{(4\pi)^2}\not{p}\varepsilon A_1 = \frac{ie^2}{(4\pi)^2}2\not{p}, \qquad \varepsilon = 4 - D$$

$$I_{456} = \frac{ie^2}{(4\pi)^2}\frac{\tilde{\not{p}}p^2}{\tilde{p}^2}(2A_1)$$

$$I_{789} = \frac{ie^2}{(4\pi)^2}\Big\{\Big[\not{p}\Big(\frac{\varepsilon}{2} - \frac{p^2 Tr\theta^2}{2\tilde{p}^2} - \frac{p^2\tilde{\tilde{p}}^2}{\tilde{p}^4}\Big) + \tilde{\not{p}}\frac{p^2}{\tilde{p}^2}\Big](-2A_1)$$

$$+\Big[\not{p}\Big(-\frac{1}{2} - \frac{p^2 Tr\theta^2}{2\tilde{p}^2} - \frac{p^2\tilde{\tilde{p}}^2}{2\tilde{p}^4}\Big) + \tilde{\not{p}}\frac{p^2}{\tilde{p}^2}\Big](-2A_2)\Big\}$$

$$A_1 = 2(4\pi)^{2-\frac{D}{2}}\Big[\Big(\frac{\mu^2}{-p^2}\Big)^{2-\frac{D}{2}}\Gamma\Big(2-\frac{D}{2}\Big)B\Big(\frac{D}{2}-1,\frac{D}{2}\Big)$$

$$- \quad 2^{\frac{D}{2}-1}(-1)^{D-2}\int_0^1 dx(1-x)\Big(\frac{p^2}{\tilde{p}^2\mu^4}x(1-x)\Big)^{\frac{D}{4}-1}K_{\frac{D}{2}-2}((p^2\tilde{p}^2x(1-x))^{\frac{1}{2}})$$

$$A_2 = -\frac{1}{8}(4\pi)^{2-\frac{D}{2}}(\mu^2\tilde{p}^2)^{2-\frac{D}{2}}\frac{\pi}{\sin\frac{\pi D}{2}}\int_0^1 dx(1-x)$$

$$\times \quad \Big[4(p^2\tilde{p}^2x(1-x))^{\frac{D}{2}-1}\Gamma\Big(\frac{1}{2}\Big)\,{}_1\tilde{F}_2\Big(\frac{1}{2};\frac{3}{2},\frac{D}{2};\frac{p^2\tilde{p}^2x(1-x)}{4}\Big)$$

$$- \quad 2^D\Gamma\Big(\frac{3-D}{2}\Big)\,{}_1\tilde{F}_2\Big(\frac{3-D}{2};2-\frac{D}{2},\frac{5-D}{2};\frac{p^2\tilde{p}^2x(1-x)}{4}\Big)$$

Form factors, dimensionless structures, divergences, θ-series, counterterm

Form factors, dimensionless structures, divergences

$$
\begin{aligned}
\Sigma_1 &= \not{p} A + \tilde{\not{p}} \frac{p^2}{\tilde{p}^2} B \\
&= \frac{e^2}{(4\pi)^2} \Big(\not{p} (A_1 (s_1 + 2 s_2) + A_2 (1 + s_1 + s_2)) + \tilde{\not{p}} \frac{p^2}{\tilde{p}^2} (-2 A_2) \Big) \\
s_1 &= \frac{p^2 tr\theta\theta}{\tilde{p}^2}, \qquad s_1 = \frac{p^2 \tilde{\tilde{p}}^2}{\tilde{p}^4}
\end{aligned}
$$

$$
\begin{aligned}
A_1 &= \frac{2}{\varepsilon} + 2 + \ln(-4\pi) + \ln \frac{\mu^2 \tilde{p}^2}{16} - \psi_0 \Big(\frac{3}{2}\Big) \\
&+ \frac{1}{2} \sum_{n=1}^{\infty} \frac{\sqrt{\pi}}{\Gamma(n + \frac{3}{2})} \frac{1}{n!} \Big(\frac{p^2 \tilde{p}^2}{16}\Big)^n \Big(\ln \frac{p^2 \tilde{p}^2}{16} - \psi_0(n+1) + \psi_0 \Big(n + \frac{3}{2}\Big)\Big) \\
A_2 &= 2 - \frac{1}{2} \sum_{n=0}^{\infty} \frac{\pi^{\frac{1}{2}} \Gamma(\frac{1}{2} + n)}{\Gamma(\frac{3}{2} + n) \Gamma(\frac{5}{2} + n)} \frac{1}{n!} \Big(\frac{p^2 \tilde{p}^2}{16}\Big)^{n+1} \Big(\frac{p^2 \tilde{p}^2}{16} \\
&+ \psi_0 \Big(n + \frac{1}{2}\Big) - \psi_0(1+n) - \psi_0 \Big(n + \frac{3}{2}\Big) - \psi_0 \Big(n + \frac{5}{2}\Big)\Big)
\end{aligned}
$$

θ expansion : convergence

$$
\begin{aligned}
A_1 \simeq\ & \frac{2}{\varepsilon} + \ln(-\pi e^{\gamma_E}\mu^2\tilde{p}^2) \\
& - \frac{11}{72}p^2\tilde{p}^2\Big(1 + \frac{137}{8800}(p^2\tilde{p}^2) + \frac{33}{313600}(p^2\tilde{p}^2)^2 + \frac{7129}{17882726400}(p^2\tilde{p}^2)^3 + \dots\Big) \\
& + \gamma_E\left(1 + \ln\left(\frac{p^2\tilde{p}^2}{4}\right)^{\frac{1}{2\gamma_E}}\right)\frac{p^2\tilde{p}^2}{12}\Big(1 + \frac{p^2\tilde{p}^2}{80} + \frac{(p^2\tilde{p}^2)^2}{13440} + \frac{(p^2\tilde{p}^2)^3}{3870720} + \dots\Big) \\
A_2 =\ & 2 + \frac{7}{18}p^2\tilde{p}^2\Big(1 + \frac{71}{8400}p^2\tilde{p}^2 + \frac{1103}{21952000}(p^2\tilde{p}^2)^2 + \frac{3587}{19914854400}(p^2\tilde{p}^2)^3 + \dots \\
& - 2\gamma_E\left(1 + \ln\left(\frac{p^2\tilde{p}^2}{4}\right)^{\frac{1}{2\gamma_E}}\right)\frac{p^2\tilde{p}^2}{12}\Big(1 + \frac{p^2\tilde{p}^2}{120} + \frac{(p^2\tilde{p}^2)^2}{22400} + \frac{(p^2\tilde{p}^2)^3}{6773760} + \dots\Big)
\end{aligned}
$$

UV divergence and counterterm, UV/IR mixing term, $p^2 = 0$

$$
\begin{aligned}
\Sigma_{UV} &= \frac{2e^2}{(4\pi)^2\varepsilon}\not{p}\left[p^2\Big(\frac{Tr\theta\theta}{(\theta p)^2} + 2\frac{(\theta\theta p)^2}{(\theta p)^4}\Big)\right] \\
\Sigma_{C} &= \delta_2\bar{\psi}i\not{\partial}\left[\partial^2\Big(\frac{Tr\theta\theta}{(\theta\partial)^2} + 2\frac{(\theta\theta\partial)^2}{(\theta\partial)^4}\Big)\right]\psi \\
\Sigma_{UV/IR} &= \frac{2e^2}{(4\pi)^2}\ln\left(\frac{\mu^2(\theta p)^2}{16}\right)^{\frac{1}{2}}\not{p}\left[p^2\Big(\frac{Tr\theta\theta}{(\theta p)^2} + 2\frac{(\theta\theta p)^2}{(\theta p)^4}\Big)\right]
\end{aligned}
$$

Renormalization procedure: p^2 terms in Σ_{UV} and $\Sigma_{UV/IR}$: $p^2 = 0$ is valid

Restriction of θ and dispersion relations in model 1

Restriction of θ

- θ^{0i} breaks unitarity $\longrightarrow$ choose $\theta^{0i} = 0$
- restrict θ^{ij} to obtain $s_1 + 2s_2 = 0$: elimination of div. A_1 term (UV, IR/UV)

$$\theta = \frac{1}{\Lambda_{NC}^2} C = \frac{1}{\Lambda_{NC}^2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -c & b \\ 0 & c & 0 & -a \\ 0 & -b & a & 0 \end{pmatrix} \qquad a^2 + b^2 + c^2 = 1 \quad (norm. : \Lambda_{NC})$$

$$\Rightarrow \quad s_1 + 2s_2 = \frac{p^2 TrC^2}{(Cp)^2} + 2\frac{p^2 (CCp)^2}{(Cp)^4} = 0, \quad (CCp)^2 = (Cp)^2, \quad TrC^2 = -2$$

$$\Downarrow$$

$$\Sigma_1 = \hat{A}_2 \Big[\Big(1 + \frac{p^2}{p_r^2} \Big) \not{p} + 2\tilde{\not{\tilde{p}}} \frac{p^2}{p_r^2} \Big], \qquad \hat{A}_2 = \frac{e^2 A_2}{(4\pi)^2}, \qquad p_r^2 = -(Cp)^2$$

- propagator (one loop) : (spontaneous) breaking of Lorentz invariance

$$\frac{i}{\not{p} - \Sigma_1} \equiv \frac{i}{\Sigma} = \frac{i\Sigma}{\Sigma^2}$$

- Σ^2 depends on $z \equiv \frac{p^2}{p_r^2}$ and $y \equiv -p^2\tilde{p}^2 = \frac{p^2 p_r^2}{\Lambda_{NC}^4}$, (through A_2); $\tilde{p}^2 \equiv -\frac{p_r^2}{\Lambda_{NC}^4}$

$$\Sigma^2 \;=\; p^2\Big[\hat{A}_2^2(y)(z^2+2z+5) - \hat{A}_2^2(y)(6+2z)+1\Big] \;\equiv\; p^2\Sigma'$$

Dispersion relations

- defined by zeros of the denominator of the propagator

$$\begin{aligned}
\Sigma^2 \;&=\; 0 \\
&\overset{1}{\Rightarrow}\; p^2 \;=\; 0 \\
&\overset{2}{\Rightarrow}\; \hat{A}_2^2 z^2 - 2(\hat{A}_2 - \hat{A}_2^2)z + (1 - 6\hat{A}_2 + 5\hat{A}_2^2) \;=\; 0 \\
&\overset{2}{\Rightarrow}\; z \;=\; \frac{1}{\hat{A}_2}\Big[\Big(1-\hat{A}_2\Big) \pm 2\Big(\hat{A}_2 - \hat{A}_2^2\Big)^{\frac{1}{2}}\Big], \qquad p^2 \;=\; z p_r^2
\end{aligned}$$

- second dispersion relation is direction dependent : birefrigence
- two interesting regimes $y \ll 1$ and $y \gg 1$ with Λ_{NC} independent solutions

$y \ll 1$

- Zeroth term in y expansion of $\hat{A}_2$ used

$$\begin{aligned} \hat{A}_2 &\simeq \frac{e^2}{8\pi^2} \\ \Rightarrow \quad z &\simeq \Big(\frac{8\pi^2}{e^2} - 1\Big) \pm 2\Big(\frac{8\pi^2}{e^2} - 1\Big)^{\frac{1}{2}} \simeq 859 \pm 59 \\ \Rightarrow \quad p^2 \equiv z p_r^2 &\simeq (859 \pm 59) p_r^2 \end{aligned}$$

- Two positive real solutions independent on Λ_{NC}: unfamiliar in NC theories.
- Quantum effect : maximal neutrino velocity :

$$\frac{v}{c} = \frac{\partial E}{\partial p} \simeq \sqrt{1 + (859 \pm 59)\sin^2\theta} \qquad \sin\theta = \hat{p}\hat{p}_r \qquad \hat{p}_r \perp (a, b, c)$$

$y \gg 1$

- asymptotic form of A_2 analyzed

$$
\begin{aligned}
A_2 \;&=\; \frac{\pi}{2\sin\frac{\pi\varepsilon}{2}}(4\pi)^{\frac{\varepsilon}{2}}\left(\frac{-\mu^2 p_r^2}{\Lambda_{NC}^4}\right)^{\frac{\varepsilon}{2}}\int_0^1 dx(1-x) \\
&\times\; \Big[(-yx(1-x))^{1+\frac{\varepsilon}{2}}\Gamma\Big(\frac{1}{2}\Big)\,{}_1\tilde{F}_2\Big(\frac{1}{2};\frac{3}{2},2-\frac{\varepsilon}{2};\frac{-yx(1-x)}{4}\Big) \\
&-\; 2^{2-\varepsilon}\Gamma\Big(-\frac{1}{2}+\frac{\varepsilon}{2}\Big)\,{}_1\tilde{F}_2\Big(-\frac{1}{2}+\frac{\varepsilon}{2};\frac{\varepsilon}{2},\frac{1}{2}+\frac{\varepsilon}{2};\frac{-yx(1-x)}{4}\Big) \\
&=\; \left(\frac{-\mu^2 p_r^2}{\Lambda_{NC}^4}\right)^{\frac{\varepsilon}{2}}(4\pi)^{\frac{\varepsilon}{2}}\,2^{-1-\varepsilon}\frac{\pi^{\frac{3}{2}}}{\sin\frac{\pi\varepsilon}{2}}\Big[\Big(\frac{-y}{16}\Big)^{1-\frac{\varepsilon}{2}}\Gamma\Big(\frac{1}{2}\Big)\,{}_1\tilde{F}_2\Big(\frac{1}{2};\frac{3}{2},\frac{5}{2}-\frac{\varepsilon}{2};-\frac{y}{16}\Big) \\
&-\; \Gamma\Big(-\frac{1}{2}+\frac{\varepsilon}{2}\Big)\,{}_2\tilde{F}_3\Big(-\frac{1}{2}+\frac{\varepsilon}{2},1;\frac{\varepsilon}{2},\frac{1}{2}+\frac{\varepsilon}{2},\frac{3}{2};-\frac{y}{16}\Big)\Big]
\end{aligned}
$$

- asymptotic form for the hypergeometric functions in the $y \to \infty$ used leading to:

$$
A_2 \;\simeq\; i\pi^2\frac{\sqrt{y}}{8} \propto \sqrt{y} \;\Rightarrow\; z \;\simeq\; -1 \pm 2i \Rightarrow\; p_0^2 \;\simeq\; p_3^2 \pm 2ip_r^2
$$

- no real solutions $\Rightarrow$ no additional stable particles

Model 2

$\Sigma_3 = 0$, $\Sigma_4 = 0$: charge conjugation symmetry

$\Sigma_2 = 0$: shown by explicit calculation

Σ_1 :

$$\Sigma_1 \quad = \quad -\frac{\not p}{(4\pi)^2}\Big[\frac{8}{3}\frac{1}{\tilde{p}^2}\Big(\frac{Tr\theta^2}{\tilde{p}^2} + 4\frac{\tilde{\tilde{p}}^2}{\tilde{p}^4}\Big)\Big]$$

- one form factor only $\propto \not p$
- no UV divergence
- strong IR/UV mixing
- no additional disp. relation – the expresion in square bracket independent of p^0

Conclusion

1. Self energy of neutral fermion evaluated in two $U(1)_*$ gauge invariant models obtained by two different SW maps. Potential contributions coming from four diagrams Σ_1, Σ_2, Σ_3 and Σ_4,

2. Only the Σ_1 gives contribution to self energy. $\Sigma_3 = 0$ and $\Sigma_4 = 0$ by charge conjugation invariance in both models, and $\Sigma_2 = 0$ shown explicitly in both models

3. Σ_1 in model 1:
   - Contains two spinor structures ($\not{p}$, $\tilde{\tilde{\not{p}}}$) and therefore two form factors
   - One of these form factors contains UV divergence and logarithmic IR/UV mixing proportional to p^2, while the other one is finite
   - UV divergence requests for nonlocal counterterm
   - The renormalization procedure preserves $p^2 = 0$ dispersion relation

- for NC matrix θ containing space components only the form factor containing UV divergence and IR/UV mixing vanishes. The dispersion relation can explicitly be found and it contains besides the $p^2 = 0$ solution another solution leading to a superluminal neutrinos if $p^2\tilde{p}^2 \ll 1$. That leads to a birefrigence of neutrino.

4. Σ_1 in model 2:
   - contains one spinor structure only ($\not{p}$) and one form factor
   - there is no UV divergence but the form factor has strong IR/UV mixing $(1/\tilde{p}^2)$
   - for θ containing space components only there is no additional dispersion relation – the $p^2 = 0$ dispersion relation is fullfilled

5. Comment: the results we obtained are quantum effects which strongly depend on the choice of the SW map

Thank you