The experimental status of General Relativity

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The Bremen drop tower

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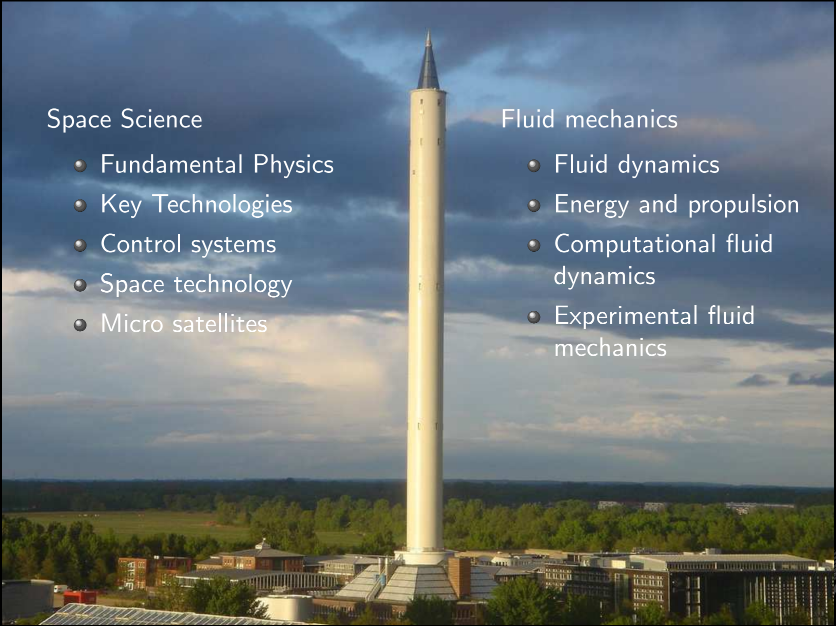

Space Science

- Fundamental Physics
- Key Technologies
- Control systems
- Space technology
- Micro satellites

Fluid mechanics

- Fluid dynamics
- Energy and propulsion
- Computational fluid dynamics
- Experimental fluid mechanics

Main theme of this talk

Gravity can only be explored through the motion of test particles

Test particles

- Orbits and clocks
- Massive particles and light

What is gravity depends on the structure of the equation of motion

- Existence of inertial systems
- Order of differential equation
- Dependence on particle parameters

Outline

1. Testing Newton’s laws
    - Newton’s first law: Inertial systems
    - Newton’s second law: The law of inertia
    - Newton’s third law: Law of reciprocal action

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    - Universality of Free Fall
    - Universality of Gravitational Redshift
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    - Black Holes
    - Binary systems
    - Gravitational waves
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Non–existence of inertial systems: Finsler geometry

Motivation

- generic generalization of GR
- model for violation of Lorentz invariance
- counter example for Schiff's conecture
- has been discussed within Quantum Gravity (Jacobson, Liberati & Mattingly)
- Very Special Relativity (Cohen & Glashow)

Equation of motion

General structure of equation of motion in Finsler geometry

$$0 = \frac{d^2 x^\mu}{ds^2} + \Gamma(x, \dot{x}) \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds}$$

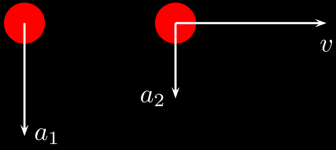
connection depends on velocity

Finsler geometry

Main characteristics of geodesic motion

- Geodesic equation fulfills Universality of Free Fall
- $\Gamma(x, \dot{x})$ cannot be transformed to zero $\forall \dot{x}$
  $\Rightarrow$ gravity cannot be transformed away locally
  $\Leftrightarrow$ Einstein's elevator does not hold
  $\Leftrightarrow$ no inertial system (there are always forces present)

- Condition to be able to transform away gravity is stronger then pure UFF.
- Acceleration toward the Earth depends on horizontal velocity.
- Speculation:
  - violation of UGR
  - temperature dependent $G = G(T)$



Parametrizing deviations from Riemann/Minkowski

Model

Deviation from Riemann/Minkowksi

$$ds^{2r} = \left(g_{\mu_1\mu_2} \cdots g_{\mu_{2r-1}\mu_{2r}} + \phi_{\mu_1\ldots\mu_{2r}}\right) dx^{\mu_1} \cdots dx^{\mu_{2r}}$$

Additional assumption: $\phi_{\mu_1\ldots\mu_{2r}}$ possesses spatial indices only (from light propagation)

Consequences

- Modified tangent space ⇒ violation of Lorentz invariance
- Velocity–dependent gravity

if we have Finslerian gravity, then we also have Finslerian kinematics

Finslerian structure in manifold	⇔	Finserian structure in tangent space
Finslerian gravity	⇔	Finslerian Special Relativity

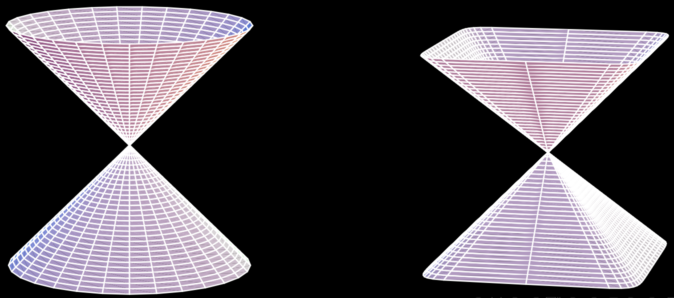
Finsler violation of Lorentz invariance

Light propagation

- $$0 = \left(g_{\mu_1\mu_2} \cdots g_{\mu_{2r-1}\mu_{2r}} + \phi_{\mu_1\ldots\mu_{2r}}\right) l^{\mu_1} \ldots l^{\mu_{2r}}$$
- anisotropy of light propagation
- Michelson–Morley yields $\phi \leq 10^{-17}$ (C.L., Lorek & Dittus, GRG 2009)

Minkowskian light cone

Finslerian light cone



Finsler violation of Lorentz invariance

Finslerian gravity

Finsler gravity

For spherically symmetric Finsler geometry:

- 3rd Kepler law for circular orbits

$$\frac{r^3}{T^2} = \left(1 - \frac{A(r)}{r^4}\right) \frac{GM}{4\pi^2}$$

- radial free fall

$$\frac{d^2 r}{dt^2} = -\left(1 - B(r)\right) \frac{GM}{r^2}$$

- Free fall "sees" another gravitational "constant" than planetary bound orbits
- Free fall experiment and planetary and satellite and planetary observations

$$A(r), B(r) \leq 10^{-9} \qquad \text{in a certain range}$$

- may apply to Pioneer anomaly or galactic rotation curves

(C.L. & Perlick 2012)

Quantum mechanics in Finsler space

Finslerian Hamilton operator

$$H = -\frac{1}{2m}\left(\Delta^2 + \phi^{ijkl}\partial_i\partial_j\partial_k\partial_l\right)^{\frac{1}{2}} \approx -\frac{1}{2m}\Delta\left(1 + \frac{1}{2}\frac{\phi^{ijkl}\partial_i\partial_j\partial_k\partial_l}{\Delta^2}\right)$$

(Göklü, Herrmann, Müntinga, C.L. 2010)

Effects

- Hughes–Drever: $H_{\text{tot}} = H + \boldsymbol{\sigma}\cdot\boldsymbol{B}$ may yield estimate $\phi \leq 10^{-30}$
- Atomic interferometry, atom–photon interaction

$$\delta\phi \sim H(p+k) - H(p) = \frac{k^2}{2m} + \frac{1}{m}\left(\delta^{il} + \frac{\phi^{ijkl}p_j p_k}{p^2}\right)p_i k_l$$

modified Doppler term: gives different Doppler term while rotating the whole apparatus

Problem: Finslerian version of Einstein field equations

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Order of equation of motion?

Usual framework

$$L = L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}) \qquad \Rightarrow \qquad \frac{d}{dt}(m\dot{\boldsymbol{x}}) = \boldsymbol{F}(t, \boldsymbol{x}, \dot{\boldsymbol{x}})$$

Most important equation in physics!

More general equations?

- $\boldsymbol{p} = m\dot{\boldsymbol{x}}$ is a constitutive law. Can be more general (as is many cases)

  $$\boldsymbol{p} = \boldsymbol{f}(\dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}, \dddot{\boldsymbol{x}}, \ldots)$$

  Then equations of motion of higher order
- Influence of external fluctuations (e.g. space–time fluctuations, gravitational wave background): generalized Langevin equation with extra force term

  $$\int_0^t C(t-t')\dot{x}(t')dt'$$

Order of equation of motion?

Generalized framework

$$L = L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) \qquad \Rightarrow \qquad \epsilon \frac{d^4}{dt^4} \ddot{\boldsymbol{x}} = \boldsymbol{F}(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}, \dddot{\boldsymbol{x}})$$

Gauge principle is used to introduce interactions

Our specific model

$$L(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) = L_0(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) \quad \underbrace{-q_0 A_a \dot{x}^a}_{\text{1st order gauge fields}} \quad + \quad \underbrace{q_1 A_{ab} \dot{x}^a \dot{x}^b}_{\text{2nd order gauge fields}}$$

with (Pais–Uhlenbeck oscillator)

$$L_0(t, \boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}) = -\frac{\epsilon}{2} \ddot{\boldsymbol{x}}^2 + \frac{m}{2} \dot{\boldsymbol{x}}^2 \qquad\qquad \text{dim}\epsilon = \mathrm{kg\,s^2}$$

2nd order gauge field = space–time metric

C.L. & Rademaker 2009

Equation of motion

simplest case: constant electric field

$$\epsilon \ddddot{\boldsymbol{x}} + m\ddot{\boldsymbol{x}} = q\boldsymbol{E}_0$$

solution in 1D with initial conditions $x(0) = 0$, $\dot{x}(0) = 0$, $\ddot{x}(0) = 0$, and $\dddot{x}(0) = 0$

$$x(t) = \frac{q}{m}E_0\left(\frac{1}{2}t^2 + \frac{\epsilon}{m}\left(\cos\left(\omega t\right) - 1\right)\right) \qquad \text{small deviation}$$

$$\dot{x}(t) = \frac{q}{m}E_0\left(t - \sqrt{\frac{\epsilon}{m}}\sin\left(\omega t\right)\right) \qquad \text{small deviation}$$

$$\ddot{x}(t) = \frac{q}{m}E_0\left(1 - \cos\left(\omega t\right)\right) \qquad \mathcal{O}(1) \text{ deviation}$$

$$\dddot{x}(t) = \frac{q}{m}E_0\sqrt{\frac{m}{\epsilon}}\sin\left(\omega t\right) \qquad \omega = \sqrt{\frac{m}{\epsilon}} \qquad \text{large deviation}$$

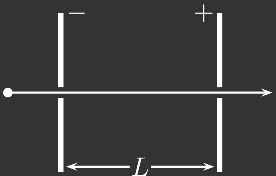
- $x(t)$ shows *zitterbewegung*
- Limit $\epsilon \to 0$ does not exist

Search for ϵ

Accelerated flight

Flight through accelerator

$$\frac{\langle \dot{x}(L)\rangle - \dot{x}_0}{\dot{x}_0} = \frac{\epsilon}{4m}\frac{\dot{x}_0^2}{L^2}$$



Ion interferometric measurement of acceleration

phase shift

$$\Delta\phi = A(\omega)\boldsymbol{k}\cdot\ddot{\boldsymbol{x}}(\omega)\,T^2$$

with transfer function $A(\omega)$

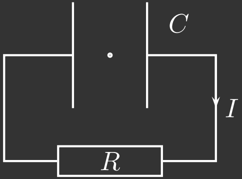
$$A(\omega) = \frac{sin^2(\omega t)}{\omega}$$

Search for ϵ

Electronic devices

Zitterbewegung of a charged particle induces voltage noise

$$\frac{1}{2}C\langle U^2\rangle_t = m\langle\dot{x}^2\rangle = \frac{1}{2}\epsilon\left(\frac{q}{m}E_0\right)^2$$



Consistent estimate $\epsilon \leq 10^{-50}\,\mathrm{kg}\,\mathrm{s}^2$ (not good compared with Quantum Gravity scale)

Linearity of law of inertia

Why is the relation between acceleration and force linear? (motivation: MOND)

It is a definition

$m\boldsymbol{a} = \boldsymbol{F}$: exploration of forces through observation of orbits

Meaningful question 1: Test linearity

Taking elements of the field equation into account:

- If $\boldsymbol{F} = -\boldsymbol{\nabla} U$ with $U = M/r$, then one can ask

$$M \;\to\; \alpha M \qquad \overset{?}{\Longrightarrow} \qquad \boldsymbol{a} \;\to\; \alpha \boldsymbol{a}$$

- Test of field equation/dynamics in the weak field/small acceleration domain
- Same for gravity and electromagnetism?
- Experiments
  - Abramovici & Vager, PRD 1986 $F = q\Delta\phi/L$, down to $10^{-9}\ \mathrm{m/s^2}$
  - Gundlach et al, PRL 2007 down to $10^{-14}\ \mathrm{m/s^2}$

Linearity of law of inertia

These questions are motivated by MOND

Meaningful question 2: Free fall experiment

- MOND – dark matter: requires a certain frame of reference (galactic frame)
- MOND–situation possible on Earth once a year for 0.1 s within 1 l volume (Ignatiev, PRL 2007)
- Until now there is no (laboratory) test of MOND — MOND needs space

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Active and passive mass

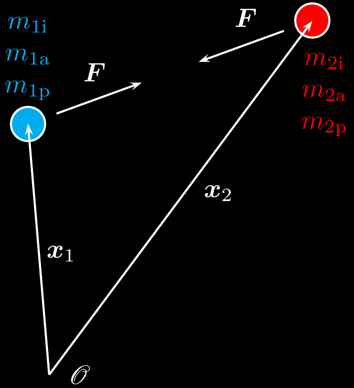
Gravitationally bound two–body system (Bondi, RMP 1957)

$$\begin{aligned} m_{1\mathrm{i}}\ddot{\boldsymbol{x}}_1 &= m_{1\mathrm{p}} m_{2\mathrm{a}} \frac{\boldsymbol{x}_2 - \boldsymbol{x}_1}{|\boldsymbol{x}_2 - \boldsymbol{x}_1|^3} \\ m_{2\mathrm{i}}\ddot{\boldsymbol{x}}_2 &= m_{2\mathrm{p}} m_{1\mathrm{a}} \frac{\boldsymbol{x}_1 - \boldsymbol{x}_2}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|^3} \end{aligned}$$

center–of–mass and relative coordinate

$$\begin{aligned} \boldsymbol{X} &:= \frac{m_{1\mathrm{i}}}{M_\mathrm{i}}\boldsymbol{x}_1 + \frac{m_{2\mathrm{i}}}{M_\mathrm{i}}\boldsymbol{x}_2 \\ \boldsymbol{x} &:= \boldsymbol{x}_2 - \boldsymbol{x}_1 \end{aligned}$$

$M_\mathrm{i} = m_{1\mathrm{i}} + m_{2\mathrm{i}}$ = total inertial mass. Then



Active and passive mass

Decoupled dynamics of relative coordinate

$$\begin{aligned}\ddot{\boldsymbol{X}} &= \frac{m_{1\mathrm{p}} m_{2\mathrm{p}}}{M_\mathrm{i}} C_{21} \frac{\boldsymbol{x}}{|\boldsymbol{x}|^3} \qquad \text{with} \qquad C_{21} = \frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}} - \frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}} \\ \ddot{\boldsymbol{x}} &= -\frac{m_{1\mathrm{p}} m_{2\mathrm{p}}}{m_{1\mathrm{i}} m_{2\mathrm{i}}} \left(m_{1\mathrm{i}} \frac{m_{1\mathrm{a}}}{m_{1\mathrm{p}}} + m_{2\mathrm{i}} \frac{m_{2\mathrm{a}}}{m_{2\mathrm{p}}} \right) \frac{\boldsymbol{x}}{|\boldsymbol{x}|^3}\end{aligned}$$

- $C_{21} = 0$: ratio of the active and passive masses are equal for both particles
- $C_{21} \neq 0$: $\Rightarrow$ self–acceleration of center of mass

Interpretation

$\ddot{\boldsymbol{X}} \neq 0 \quad \Leftrightarrow \quad C_{12} \neq 0 \quad \Leftrightarrow$

- Violation of law of reciprocal action or of *actio* $=$ *reactio* for gravity
- The gravitational field created by masses of same weight depends on its composition. Has the same status as the Weak Equivalence Principle.

Requires experimental tests …

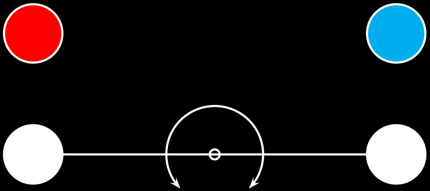
Experiment testing $m_{\mathrm{ga}} = m_{\mathrm{gp}}$

Measurement of relative acceleration

Step 1: Take two masses with $m_{\mathrm{pg1}} = m_{\mathrm{pg2}}$ (equal weight)

Step 2: Test active equality of these two masses with torsion balance

Experimental setup: Torsion balance with equal passive masses reacting on m_{ag1} and m_{ag2}



No effect has been seen: $C_{12} \leq 5 \cdot 10^{-5}$ (Kreuzer, PR 1868)

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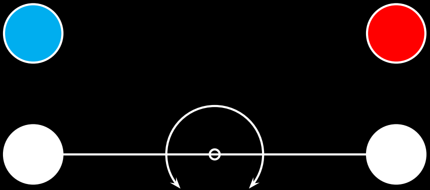

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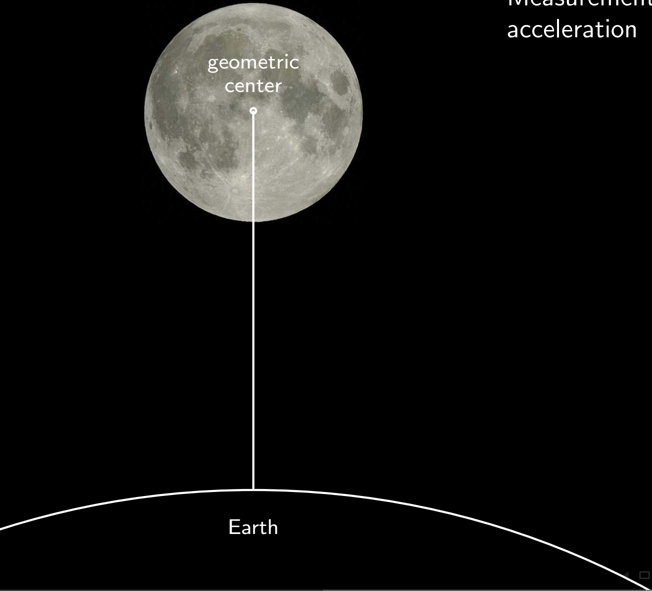

Measurement of center–of–mass acceleration

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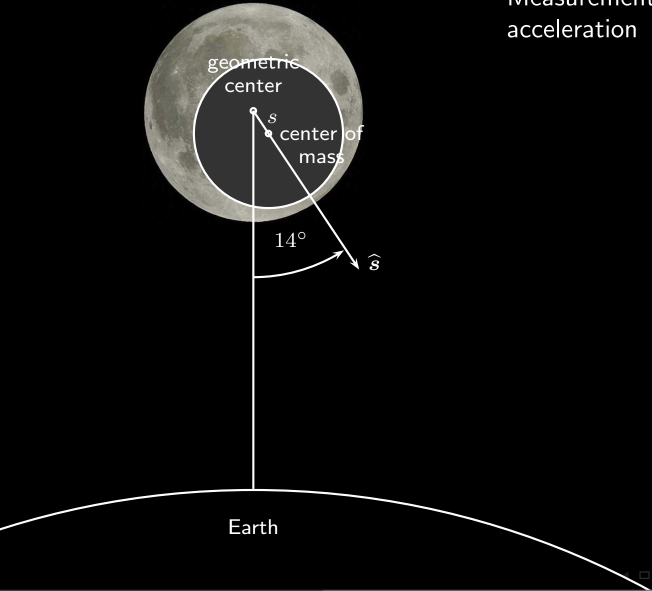

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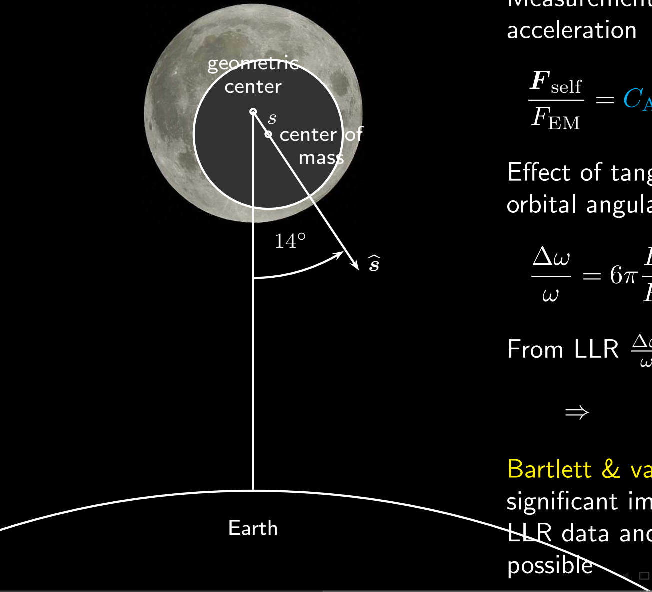

Measurement of center–of–mass acceleration

$$\frac{\boldsymbol{F}_{\text{self}}}{F_{\text{EM}}} = C_{\text{Al}-\text{Fe}} \frac{M_{\text{M}}}{M_{\oplus}} \frac{r_{\text{EM}}^2}{r_{\text{M}}^2} \frac{s}{r_{\text{M}}} \frac{\rho}{\Delta\rho} \widehat{\boldsymbol{s}}$$

Effect of tangential part: increase of orbital angular velocity

$$\frac{\Delta\omega}{\omega} = 6\pi \frac{F_{\text{self}}}{F_{\text{EM}}} \sin 14° \text{ per month}$$

From LLR $\frac{\Delta\omega}{\omega} \leq 10^{-12}$ per month

$$\Rightarrow \qquad C_{\text{Al}-\text{Fe}} \leq 7 \cdot 10^{-13}$$

Bartlett & van Buren, PRL 1986

significant improvement with new LLR data and moon orbiter data possible

Active and passive charges: Dynamics

C.L., Macias, Müller, PRA 2007

Dynamics of two electrically bound particles ($\boldsymbol{E}$ = external electric field)

$$\begin{aligned} m_{1\mathrm{i}}\ddot{\boldsymbol{x}}_1 &= q_{1\mathrm{p}}q_{2\mathrm{a}}\frac{\boldsymbol{x}_2-\boldsymbol{x}_1}{|\boldsymbol{x}_2-\boldsymbol{x}_1|^3}+q_{1\mathrm{p}}\boldsymbol{E}(\boldsymbol{x}_1)\,, \\ m_{2\mathrm{i}}\ddot{\boldsymbol{x}}_2 &= q_{2\mathrm{p}}q_{1\mathrm{a}}\frac{\boldsymbol{x}_1-\boldsymbol{x}_2}{|\boldsymbol{x}_1-\boldsymbol{x}_2|^3}+q_{2\mathrm{p}}\boldsymbol{E}(\boldsymbol{x}_2)\,, \end{aligned}$$

- Similar phenomena
- New feature: Active and passive neutrality
- Very good neutrality measurements $\Rightarrow C_{12} \leq 10^{-21}$
- Other approach through fine structure constant for H and He^+
- Also: active and passive magnetic moment

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The present situation

All predictions of General Relativity are experimentally well tested and confirmed

Foundations

The Einstein Equivalence Principle

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Implication

Gravity is a metrical theory

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Implication

Gravity is a metrical theory

$\Rightarrow$

Predictions for metrical theory

- Solar system effects
  - Perihelion shift
  - Gravitational redshift
  - Deflection of light
  - Gravitational time delay
  - Lense–Thirring effect
  - Schiff effect
- Strong gravitational fields
  - Binary systems
  - Black holes
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General Relativity

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Consequences

BH, binary systems, lensing, ...

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The Einstein Equivalence Principle

- Universality of Free Fall
- Universality of Gravitational Redshift
- Local Lorentz Invariance

⇓

Implication

Gravity is a metrical theory

⇒

Predictions for metrical theory

- Solar system effects
  - Perihelion shift
  - Gravitational redshift
  - Deflection of light
  - Gravitational time delay
  - Lense-Thirring effect
  - Schiff effect
- Strong gravitational fields
  - Binary systems
  - Black holes
- Gravitational waves

⇓

General Relativity

⇐

Consequences

BH, binary systems, lensing, ...

The present situation

All predictions of General Relativity are experimentally well tested and confirmed

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The Einstein Equivalence Principle

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General Relativity

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Consequences

BH, binary systems, lensing, ...

Outline

1. Testing Newton's laws
   - Newton's first law: Inertial systems
   - Newton's second law: The law of inertia
   - Newton's third law: Law of reciprocal action
2. Experimental tests of foundations of GR
   - Universality of Free Fall
   - Universality of Gravitational Redshift
   - Tests of local Lorentz invariance
3. The consequences of the experiments
4. Solar system tests: The search for the gravitational field equations
5. The full theory
6. Testing the full theory
   - Black Holes
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   - Gravitational waves
   - Test of Newton potential
7. Problems in Gravity theories?

Description of tests of the Universality of Free Fall

Haugan formalism (Haugan, AP 1979)

- Ansatz: Hamiltonian

$$E = mc^2 + \frac{1}{2} m \left(\delta_{ij} + \frac{\delta m_{\mathrm{i}ij}}{m} \right) v^i v^j + m \left(\delta_{ij} + \frac{\delta m_{\mathrm{g}ij}}{m} \right) U^{ij}(\boldsymbol{x})$$

- Canonical equations

$$a^i = \delta^{ij} \partial_j U(\boldsymbol{x}) + \frac{\delta m_{\mathrm{i}}^{ij}}{m} \partial_j U(\boldsymbol{x}) + \delta^{ij} \frac{\delta m_{\mathrm{g}kl}}{m} \partial_j U^{kl}(\boldsymbol{x}),$$

- For diagonal mass tensors $\delta m_{\mathrm{i}ij} = \delta m_{\mathrm{i}} \delta_{ij}$, $\delta m_{\mathrm{g}ij} = \delta m_{\mathrm{g}} \delta_{ij}$:

$$a^i = \delta^{ij} \frac{m_{\mathrm{g}}}{m_{\mathrm{i}}} \partial_j U$$

- Comparison of acceleration of two different particles: Eötvös coefficient

$$\eta = \frac{a_2 - a_1}{\frac{1}{2}(a_2 + a_1)} = \frac{(m_{\mathrm{g}}/m_{\mathrm{i}})_2 - (m_{\mathrm{g}}/m_{\mathrm{i}})_1}{\frac{1}{2}\left((m_{\mathrm{g}}/m_{\mathrm{i}})_2 + (m_{\mathrm{g}}/m_{\mathrm{i}})_1\right)}$$

UFF and charge

Standard theory

- In standard theory from ordinary coupling (deWitt & Brehme, AP 1968) $a^\mu = \alpha \lambda_C R^\mu{}_\nu v^\nu \sim 10^{-35}\ \mathrm{m/s}^2$

Anomalous coupling

- Anomalous coupling (Dittus, C.L., Selig, GRG 2004)

$$H = \frac{\boldsymbol{p}^2}{2m} + mU(\boldsymbol{x}) + \kappa e U(\boldsymbol{x}) = \frac{\boldsymbol{p}^2}{2m} + m\left(1 + \kappa \frac{e}{m}\right) U(\boldsymbol{x}) \, .$$

- Charge dependent anomalous gravitational mass
- Can be generalized to charge dependent anomalous inertial mass (e.g. Rohrlich 2000)

$\Rightarrow$ Charge dependent Eötvös factor

- It is possible to choose κ's such that for neutral composite matter UFF is fulfilled while for isolated charges UFF is violated

UFF and spin

Standard theory

- In standard theory from ordinary coupling: $a^\mu = \lambda_C R^\mu{}_{\nu\rho\sigma} v^\nu S^{\rho\sigma} \Rightarrow$ violation of UFF at the order $10^{-20}\ \mathrm{m/s^2}$, beyond experiment

Anomalous coupling

- Speculations: violation P, C, and T symmetry in gravitational fields (Leitner & Okubo 1964, Moody & Wilczek 1974) suggest

$$V(r) = U(r)\left[1 + A_1(\boldsymbol{\sigma}_1 \pm \boldsymbol{\sigma}_2)\cdot\widehat{\boldsymbol{r}} + A_2(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)\cdot\widehat{\boldsymbol{r}}\right] ,$$

- One body (e.g., the Earth) is unpolarized →

$$V(r) = U(r)\left(1 + A\boldsymbol{\sigma}\cdot\widehat{\boldsymbol{r}}\right) .$$

Hyperfine splittings of H ground state: $A_p \leq 10^{-11}$, $A_e \leq 10^{-7}$

- Hari Dass 1976, 1977, includes velocity of the particles

$$V(r) = U_0(r)\left[1 + A_1\boldsymbol{\sigma}\cdot\widehat{\boldsymbol{r}} + A_2\boldsymbol{\sigma}\cdot\frac{\boldsymbol{v}}{c} + A_3\widehat{\boldsymbol{r}}\cdot\left(\boldsymbol{\sigma}\times\frac{\boldsymbol{v}}{c}\right)\right]$$

Tests of UFF

1 Tests with bulk matter

Method	Grav field	Accuracy	Experiment
Torsion pendulum	Sun	$\eta \leq 2 \cdot 10^{-13}$	Adelberger 2006

2 Tests with quantum matter

Method	Grav field	Accuracy	Experiment
Atom interferometery	Earth	$\eta \leq 10^{-9}$	Chu, Peters 1999

3 Gravitational self energy

Method	Grav field	Accuracy	Experiment
Torsion pendulum and LLR	Sun	$\eta \leq 1.3 \cdot 10^{-3}$	Baessler et al 1999

4 Charged particles

Method	Grav field	Accuracy	Experiment
Free fall of electron	Earth	$\eta \leq 10^{-1}$	Witteborn & Fairbank 1967

Tests of UFF

3 Gravitational self energy

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4 Charged particles

Method	Grav field	Accuracy	Experiment
Free fall of electron	Earth	$\eta \leq 10^{-1}$	Witteborn & Fairbank 1967

5 Particles with spin

Method	Grav field	Accuracy	Experiment
Weighting polarized bodies	Earth	$\eta \leq 10^{-8}$	Hsie et al 1989

6 Anti–particles

Method	Grav field	Accuracy	Experiment
Free fall of anti–Hydrogen	Earth	$\eta \leq 10^{-3} - 10^{-5}$	(estimate)

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Tests of the Universality of Gravitational Redshift

Description

- Gravitational redshift

$$\nu(x_1) = \left(1 - (1 + \alpha_{\text{clock}}) \frac{U(x_1) - U(x_0)}{c^2}\right) \nu(x_0)$$

may depend on used clock

- Comparison of two colocated clocks

$$\frac{\nu_1(x_1)}{\nu_2(x_1)} \approx \left(1 - (\alpha_{\text{clock2}} - \alpha_{\text{clock1}}) \frac{U(x_1) - U(x_0)}{c^2}\right) \frac{\nu_1(x_0)}{\nu_2(x_0)} .$$

- Null test
- Tested quantity: $\alpha_{\text{clock2}} - \alpha_{\text{clock1}}$
- Need of large differences in the gravitational potential

Tests of the Universality of Gravitational Redshift

Tests

Comparison	Accuracy	Experiment
Cs – Resonator	$2 \cdot 10^{-2}$	Turneaure & Stein 1987
Mg – Cs (fine structure)	$7 \cdot 10^{-4}$	Godone et al 1995
Resonator – I_2 (electronic)	$4 \cdot 10^{-2}$	Braxmaier et al 2002
Cs – H-Maser (hf)	$2.5 \cdot 10^{-5}$	Bauch et al 2002
Cs – Hg	$5 \cdot 10^{-6}$	Fortier et al 2007

Clocks

- Atomic clocks
  - Based on principal transitions
  - Based on fine structure
  - Based on hyperfine transitions
- Molecular clocks
  - Based on rotational transitions
  - Based in vibrational transitions
- Light clocks
- Gravitational clocks
  - Planetary motion
  - Binary systems
- Rotation
  - Earth
  - Pulsars
- Decay of particles

All based on different physical principles, laws.
Clocks of different nature exhibit a different dependence on fundamental constants

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Postulates

Postulates of SR

Postulate 1: The velocity of light is constant.

Postulate 2: The relativity principle.

The meaning of the postulates

- c does not depend on
  - the velocity of the source ⇒ **uniqueness** of c
  - the velocity of the observer
  - the direction of propagation
  - its frequency and polarization
- The meaning of c
  - For all particles the velocity of light is the limiting velocity
  $$c = c_+ = c_- = c_\nu = v_p^{\max} = v_e^{\max} = v_{\mathrm{grav}}$$
  - c is universal and can, thus, be interpreted as geometry
- **All** physics is the same in **all** inertial systems
  - Experimental results do not depend on the orientation of the laboratory
  - Experimental results do not depend on the velocity of the laboratory

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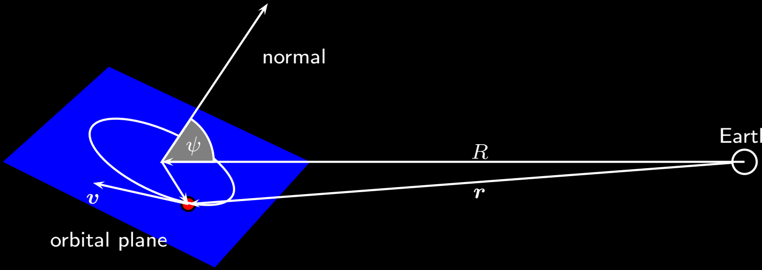
$$c = c_{\text{[illegible]}} = c_{\text{[illegible]}} = c_\nu = v_p^{\max} = v_e^{\max} = v_{\text{grav}}$$

  - c is universal and can, thus, be interpreted as geometry
- **All** physics is the same in **all** inertial systems
  - Experimental results do not depend on the orientation of the laboratory
  - Experimental results do not depend on the velocity of the laboratory

quite complex structure

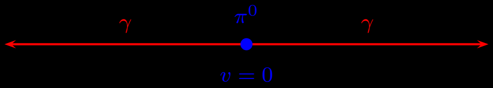
Independence of c from the velocity of the source



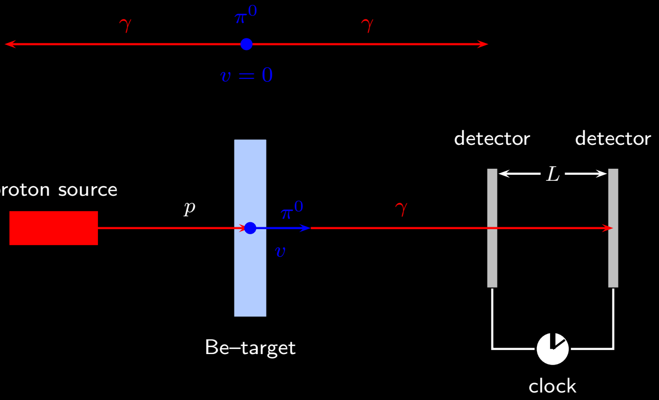
Description and result (Brecher, PRL 1977)

- Model $c' = c + \kappa v$
- Time of flight consideration: May happen
  - Reversal of chronological order (one light ray may overtake the other)
  - Multiple images
  - ...
- Result $|\kappa| \leq 10^{-11}$

Independence of c from the velocity of the source



Independence of c from the velocity of the source



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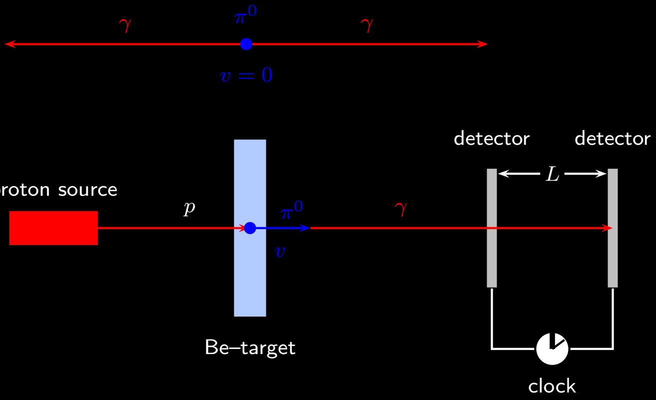
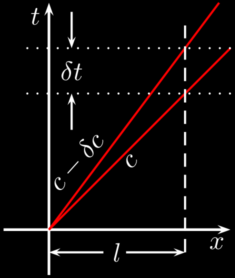


Description and result (Alväger et al 1964)

- Model $c' = c + \kappa v$
- Result $|\kappa| \leq 10^{-6}$ at $v = 0.99975\,c$

Tests of the universality of c



Polarization / dispersion of light

Method: Comparison of arrival times of light with different polarization / different frequency from distant galaxies

Result

Carrol, Field and Jackiw 1992, Kauffmann and Haugan 1995, Kostelecky and Mathews 2002

$$\left|\frac{c_+ - c_-}{c_+}\right| \leq 10^{-32}$$

Schafer 1999, Biller 1999

$$\left|\frac{c_\nu - c_{\nu'}}{c_\nu}\right| \leq 6 \cdot 10^{-21}$$

Tests of the universality of c

Velocity of neutrinos
Method: Comparison of arrival times of photons and neutrinos from supernova SN1987a

Result

Longo 1987, Stodolsky 1988

$$\left|\frac{c - v_\nu}{c}\right| \leq 10^{-8}$$

Limiting velocity of protons

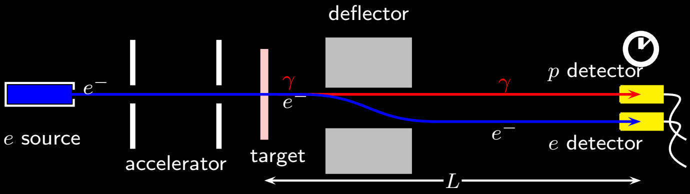
Result

Coleman and Glashow 1998

$$\left|\frac{v_p^{\max} - c}{c}\right| \leq 10^{-21}$$

Tests of the universality of c

Limiting velocity of electrons



Result

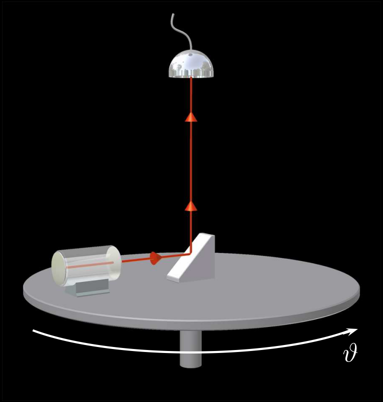
Giurogossian et al, PRD 1965

$$\left|\frac{v_e^{\max} - c}{c}\right| \leq 10^{-6}$$

Other tests by Brown et al, PRD 1963, Alspector et al, PRL 1976, Kalbfleisch et al, PRL 1976 with result of same order of accuracy

Test of isotropy of c

- Interference experiment – Michelson–Morley ....
- Measuring frequency of light in rotating resonator



Model independent description

- Light wave
  $\varphi = Ae^{i(k_+ \cdot x - \omega t)} + Be^{i(k_- \cdot x - \omega t)}$
- Dispersion relation $\omega = k_\pm c_\pm$
- Boundary conditions
- Effective: 2–way velocity of light

$$\frac{2}{c} = \frac{1}{c_+} + \frac{1}{c_-}$$

- Observable frequency

$$\nu = \frac{m}{2L} c$$

Test of isotropy of c

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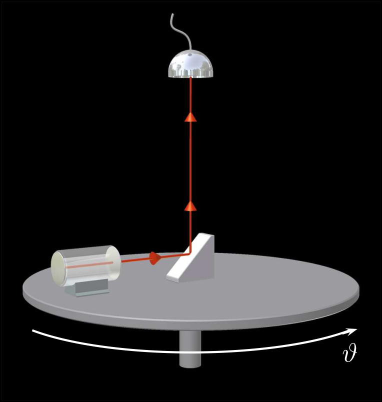

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- Dispersion relation $\omega = k_\pm c_\pm$
- Boundary conditions
- Effective: 2–way velocity of light

$$\frac{2}{c} = \frac{1}{c_+} + \frac{1}{c_-}$$

- Observable frequency

$$\nu(\vartheta) = \frac{m}{2L} c(\vartheta)$$

Test of isotropy of c

- Interference experiment – Michelson–Morley ....
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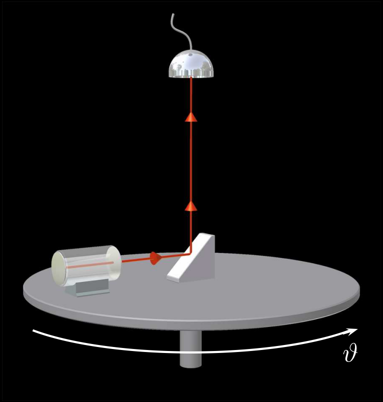


Result

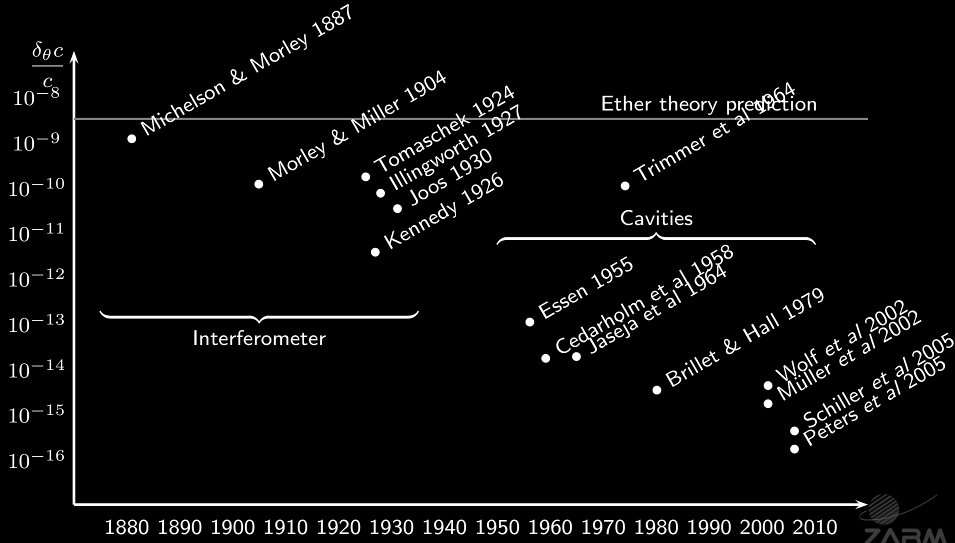
- Herrmann, Peters et al, PRD 2010
- Cryogenic resonators, one rotating, one fixed
- Result

$$\left|\frac{\Delta_\vartheta c}{c}\right| \leq 10^{-17}$$

Interpretation (Müller et al 2004)

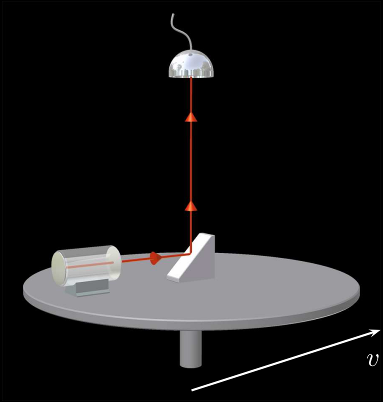
- Velocity of light
- Property of solid
- Electron properties (in L)

Summary: Test of isotropy of c



Test of independence of c from laboratory velocity

- Interference experiment – Kennedy–Thorndike ....
- Measuring frequency of light in moving resonator



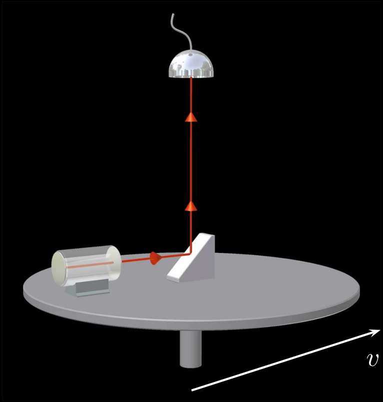
Model independent description

- Same description as above
- Observable frequency

$$\nu = \frac{m}{2L} c$$

Test of independence of c from laboratory velocity

- Interference experiment – Kennedy–Thorndike ....
- Measuring frequency of light in moving resonator



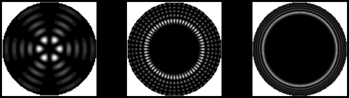
Model independent description

- Same description as above
- Observable frequency

$$\nu(v) = \frac{m}{2L} c(v)$$

Test of independence of c from laboratory velocity

Cavity–clock comparison



p = 5 m = 3 | p = 5 m = 30 | p = 5 m = 100

Whispering gallery modes

Method and result

- Wolf et al, PRL 2004
- Whispering gallery modes
- Comparison with H–maser
- Result

$$\left|\frac{\Delta_v c}{c}\right| \leq (4.5 \pm 4.5) \cdot 10^{-16}$$

- $\delta v \leftrightarrow$ rotation of Earth

Hughes–Drever experiments

The model

Modified Schrödinger equation

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\left(\delta^{ij} + \alpha^{ij}\right)\partial_i\partial_j\psi$$

Leads to a splitting of the Zeeman singlett

Hughes–Drever experiments

The model

Modified Schrödinger equation

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\left(\delta^{ij} + \alpha^{ij}\right)\partial_i\partial_j\psi$$

Leads to a splitting of the Zeeman singlett

Experiments

experiment	method	estimate
Hughes et al 1960	NMR with ^{7}Li	$\lvert\alpha^{ij}\rvert \leq 10^{-20}$
Drever 1961	NMR with ^{7}Li	$\lvert\alpha^{ij}\rvert \leq 2\times 10^{-23}$
Prestage et al. 1985	NMR with ^{9}Be$^+$	$\lvert\alpha^{ij}\rvert \leq$
Lamoreaux et al. 1986, 1989, 1990	NMR with ^{201}Hg	$\lvert\alpha^{ij}\rvert \leq 2\times 10^{-28}$
Chupp et al 1989	NMR with ^{21}Ne	$\lvert\alpha^{ij}\rvert \leq 5\times 10^{-30}$

Methods for testing time–dilation

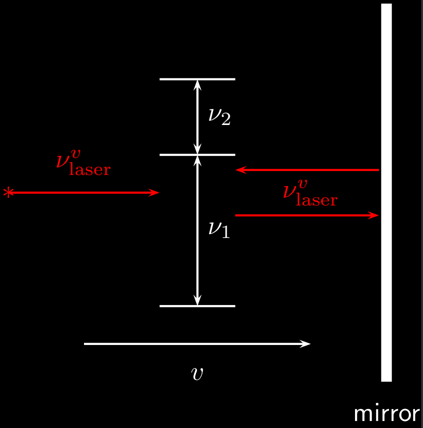
General: Comparison of identical cloks in different motion

- Transport of macrocopic clocks
- Photon absorption / emission
- Two–photon absorption
- Rotor experiments
- Saturation spectroscopy
- Particle decay

Test of time–dilation: Transport of clocks

The Experiment by Hafele and Keating 1968

Test of time–dilation: 2 Photon absorption



Description

- Resonance condition for $v = 0$

$$2v^{v=0}_{\text{laser}} = \nu_1 + \nu_2$$

- Resonance condition for $v \neq 0$

$$\begin{aligned} \nu_1 + \nu_2 &= \nu_+ + \nu_- \\ \nu_\pm &= \nu^v_{\text{laser}} (1 \pm v) \sqrt{1 - v^2} \end{aligned}$$

- Consequence

$$\nu^{v=0}_{\text{laser}} = \nu^v_{\text{laser}} \sqrt{1 - v^2}$$

with

$$v = \frac{\nu_+ - \nu_-}{\nu_+ + \nu_-}$$

- No need of synchronization

Further tests

- Sagnac effect
- Experiments testing $E = mc^2$
- Test of dispersion (mass of photon, QG induced anomalous dispersion)
- ...

Outline

Metric theory

Implication

Gravity = Riemannian geometry = space–time metric $g_{\mu\nu}$

- proper time $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
- paths $D_v v = 0 \quad \leftrightarrow \quad \frac{d^2 x^\mu}{ds^2} + \left\{ {\mu \atop \rho\sigma} \right\} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds} = 0$
- Dirac, Maxwell, ...

Predictions

All metric theories imply

- gravitational redshift
- light deflection
- perihelion shift
- gravitational time delay
- Lense–Thirring effect
- Schiff effect
- geodetic precession

Einstein's theory is singled out by certain values for these effects

How to find Einstein's theory? → PPN

PPN formalism

Physical situation

Spherically symmetric metric ⇒

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{tt} dt^2 - g_{rr} dr^2 - r^2 d\vartheta^2 - r^2 \sin^2 \vartheta d\varphi^2$$

g_{tt}, g_{rr} $\leftrightarrow$ Gravitational field equations: not known

Parametrization for

- asymptotically flat
- weak fields: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} \ll 1$

$$\begin{aligned} g_{00} &= -1 + 2\alpha \frac{U}{c^2} - 2\beta \frac{U^2}{c^4}\,, \qquad U = \text{ Newton potential} \\ g_{0i} &= 0 \\ g_{ij} &= (1 + 2\gamma) \frac{U}{c^2} \end{aligned}$$

PPN formalism

Physical situation

Axially symmetric metric $\Rightarrow$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = g_{tt} dt^2 - g_{rr} dr^2 - r^2 d\vartheta^2 - r^2 \sin^2\vartheta d\varphi^2 - g_{t3} dt d\varphi$$

g_{tt}, g_{rr}, g_{ti} $\leftrightarrow$ Gravitational field equations: not known

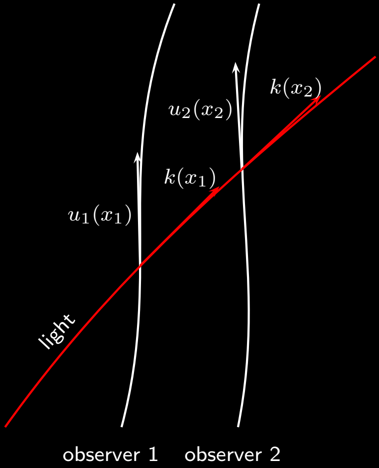
Parametrization for

- asymptotically flat
- weak fields: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu} \ll 1$

$$\begin{aligned} g_{00} &= -1 + 2\alpha\frac{U}{c^2} - 2\beta\frac{U^2}{c^4}\,, \qquad U = \text{ Newton potential} \\ g_{0i} &= 4\mu\frac{(\boldsymbol{J}\times\boldsymbol{r})_i}{c^3 r^3} \\ g_{ij} &= (1+2\gamma)\frac{U}{c^2} \end{aligned}$$

Outline

Gravitational redshift



gravitational redshift for stationary observer

$$\Rightarrow \qquad \frac{\nu_2}{\nu_1} \approx 1-(U(r_2)-U(r_1))$$

Pound & Rebka, PRL 1960: confirmation $\sim 1\%$

Vessot, Levine et al, GRG 1978, PRL 1980: GP-A, confirmation $\sim 10^{-4}$

Post–Newton

Equations of motion

Relativistic approximation of geodesic equation, PPN equation of motion

$$\frac{d^2x^i}{dt^2} = \delta^{ij}\left(1 - 2(\beta + \gamma)U + \gamma\delta_{lk}\frac{dx^l}{dt}\frac{dx^k}{dt}\right)\partial_j U - 2\partial_j U\,(1+\gamma)\,\frac{dx^j}{dt}\frac{dx^i}{dt}$$

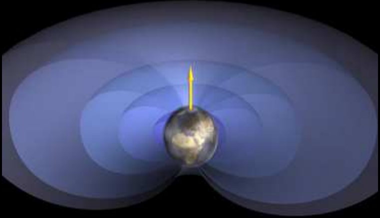
Consequences

- Perihelion shift

$$\left|\frac{2(1+\gamma)-\beta}{3} - 1\right| \leq 10^{-4}$$

- Light defection (VLBI, Eubanks et al 2001) $|\gamma - 1| \leq 10^{-4}$
- Gravitational time delay (Cassini, Bertotti et al 2005) $|\gamma - 1| \leq 2 \cdot 10^{-5}$

The gravitomagnetic field



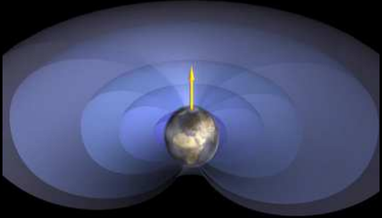
The gravitomagnetic field

- Space–time component of metric

$$g_{0i} = -\frac{G}{2} \frac{(\boldsymbol{r} \times \boldsymbol{J})^i}{r^3}$$

- Genuine post–Newtonian gravitational field
- Analogue of magnetic field

The gravitomagnetic field



Notions

- Angular momentum

$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$$

- Runge–Lenz vector

$$\boldsymbol{A} = \boldsymbol{L} \times \dot{\boldsymbol{r}} + Gm\frac{\boldsymbol{r}}{r}$$

The gravitomagnetic field

- Space–time component of metric

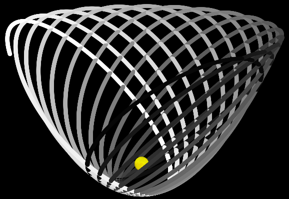
$$g_{0i} = -\frac{G}{2}\frac{(\boldsymbol{r} \times \boldsymbol{J})^i}{r^3}$$

- Genuine post–Newtonian gravitational field
- Analogue of magnetic field

Main consequences

- $\frac{d}{dt}\boldsymbol{L} \neq 0$ (Newton: $= 0$)
- $\frac{d}{dt}\boldsymbol{A} \neq 0$ (Newton: $= 0$)

Lense–Thirring effect



A particular orbit showing the Lense–Thirring effect

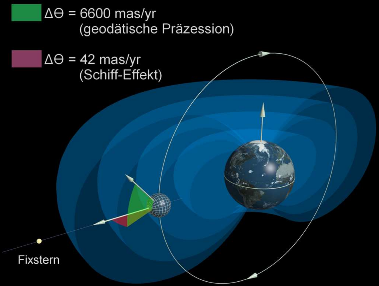
Observations

- Observed quantities

$$\begin{aligned} \dot{\Omega} &= \frac{2GJ}{c^2 a^3 (1-e^2)^{3/2}} \\ \dot{\omega} &= -\frac{6GJ\cos i}{c^2 a^3 (1-e^2)^{3/2}} \end{aligned}$$

- Measurement with LAGEOS satellites, together with data from CHAMP and GRACE
- Result: confirmation with approx 10 % error (Ciufolini 2005)
- LARES launch 2010, improved mission scenario

Schiff effect



GP-B

Description

- Dynamics of direction of spin $D_v S = 0$
- Compared with direction given by distant stars
- Effective dynamics

$$\dot{\boldsymbol{S}} = \boldsymbol{\Omega} \times \boldsymbol{S}$$

with

$$\boldsymbol{\Omega} = \boldsymbol{\nabla} \times \boldsymbol{g}$$

- Ongoing data analysis
- Originally aimed accuracy: 0.1 mas/a $\sim$ 0.5%

Result

Result

Within the range of experimental accuracy, all tests are compatible with

$$\beta = 1\,, \qquad \gamma = 1\,, \qquad \dots$$

All measured values of PPN parameters are compatible with Einsteins General Relativity.

Unfortunately, no complete operational foundation of Einstein's equation is known
Deviations from Einstein different from PPN: Finslerian gravity (Perlick & C.L. 2012)

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1. Testing Newton's laws
   - Newton's first law: Inertial systems
   - Newton's second law: The law of inertia
   - Newton's third law: Law of reciprocal action
2. Experimental tests of foundations of GR
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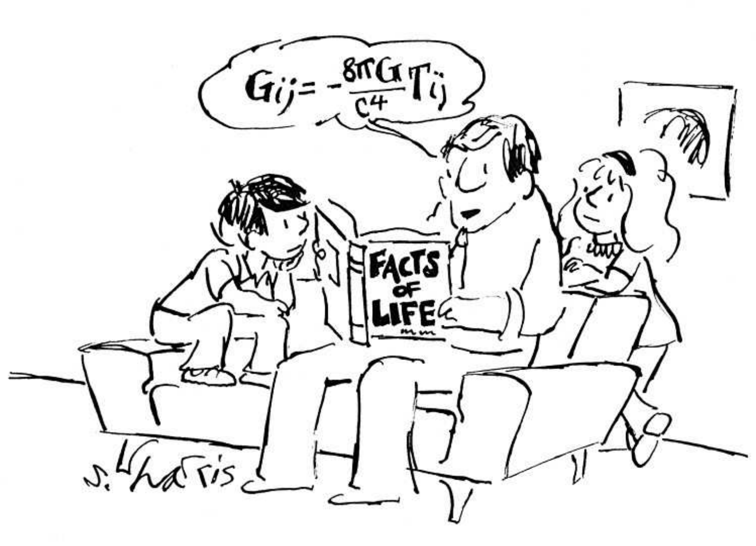
General Relativity



Possible tests beyond PPN

- test particle motion in strong field regime of given solution
- gravitationally bound systems (binary systems)
  includes gravitational waves

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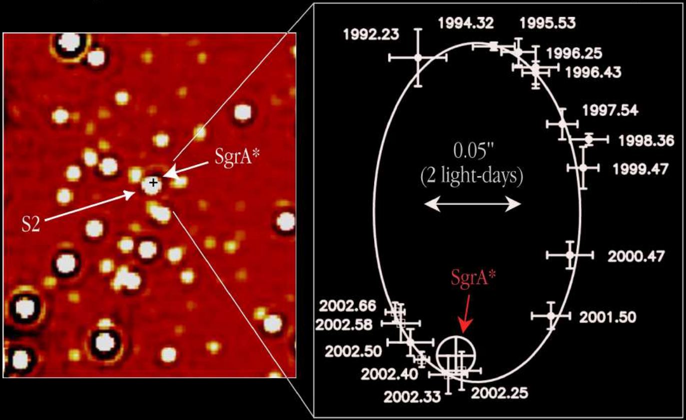
Black Hole at the Center of the Milky Way

Most important prediction: Black Holes ⟶ Search for Black Holes



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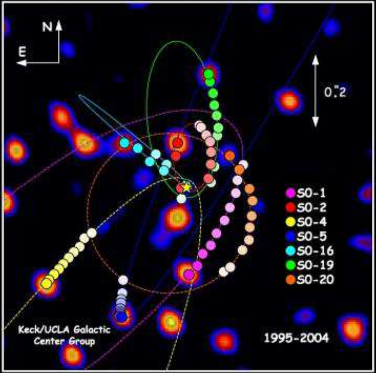
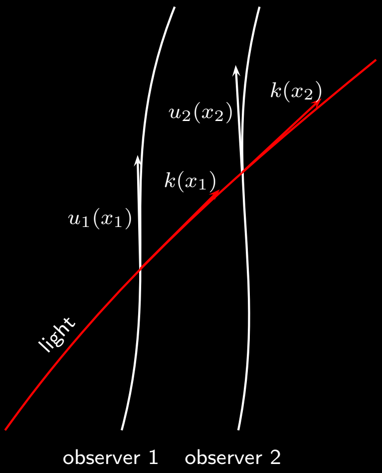


Black Hole: mass $3.7 \cdot 10^6 \; M_\odot$ (Yusuf–Zasdeh *et al.* *Astrophys. J.* **644**, 198 (2006))
angular velocity $\sim 1/17\,\mathrm{min}$
R. Genzel (1995 – 2006)

Gravitational redshift



stationary gravitational field

- $\Rightarrow$ $k(\xi) = const$, ξ Killing vector
- $\Rightarrow$ gravitational redshift for stationary observer $u \sim \xi$

$$\frac{\nu_2}{\nu_1} = \frac{k(u_2)}{k(u_1)} = \sqrt{\frac{g_{tt}(r_2)}{g_{tt}(r_1)}} = \sqrt{\frac{1 - \frac{2M}{r_2}}{1 - \frac{2M}{r_1}}}$$

may approach ∞

Orbits in Schwarzschild space–time

Equations of motion

Geodesic equation

$$D_v v = 0 \qquad \leftrightarrow \qquad \frac{d^2 x^\mu}{ds^2} + \left\{ {\mu \atop \rho\sigma} \right\} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds} = 0$$

Solution for orbits (Hagihara 1931)

$$r(\varphi) = \frac{2m}{\frac{1}{3} + \wp\left(\frac{\varphi}{2}; g_2, g_3\right)}\,, \qquad r(\varphi) = \frac{2m}{\frac{1}{3} + \wp\left(\frac{\varphi}{2} + i\omega_2; g_2, g_3\right)}$$

ω_1 and ω_2 periods of $\wp$

Applications in:

- motion of stars around black holes
- EMRI
- accretion discs

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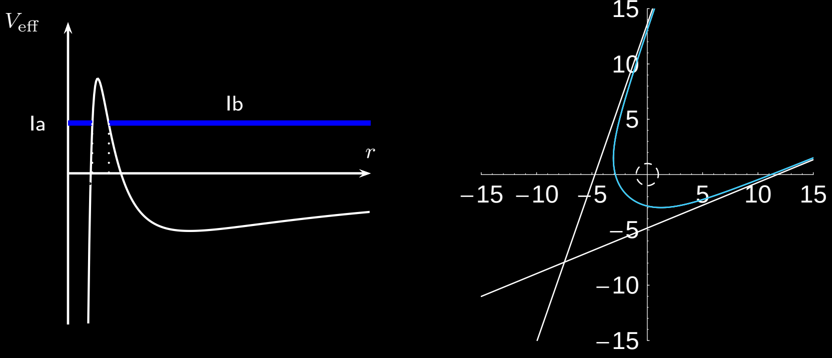
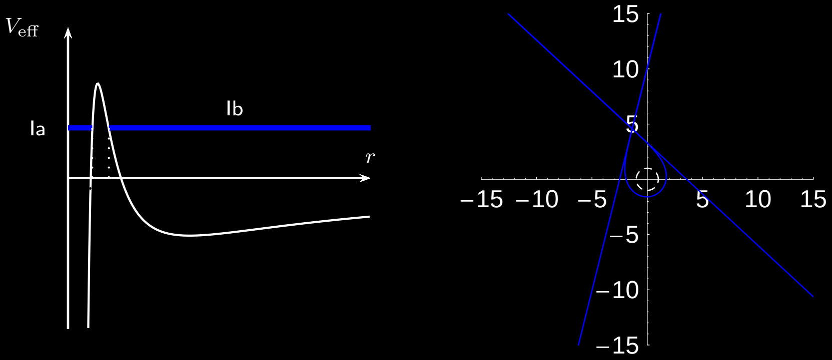


pseudo–hyperbolic — quasi hyperbolic

Orbits in Schwarzschild space–time



pseudo–hyperbolic — quasi hyperbolic

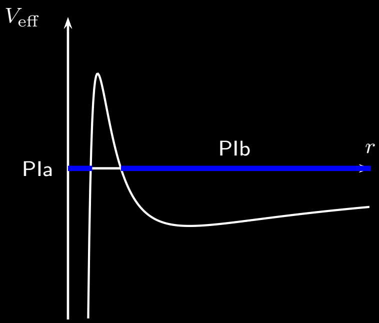
Particle deflection

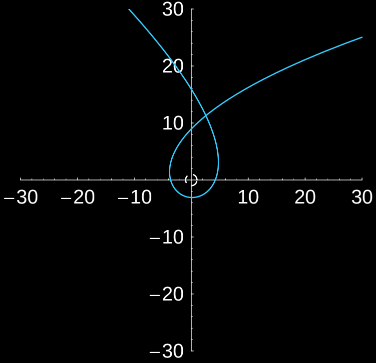
Deflection angle

$$\varphi_{1,2} = \frac{2}{\sqrt{e_1 - e_3}} F(\alpha, k)\,, \qquad \alpha = \arcsin\sqrt{-\frac{e_3 + \frac{1}{3}}{e_2 - e_3}}\,, \quad k = \sqrt{\frac{e_2 - e_3}{e_1 - e_3}}$$

- Can be represented as function of $r_{\min}$ (impact parameter) and E.
- This can be used to perform a well–defined approximation for small $r_{\mathrm{S}}/r_{\min}$.

Orbits in Schwarzschild space–time





pseudo parabolic — quasi parabolic

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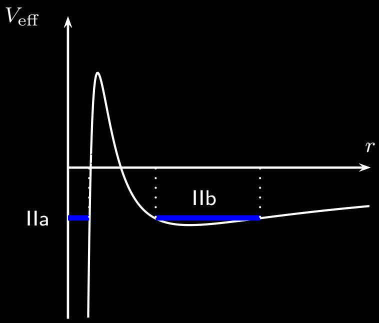
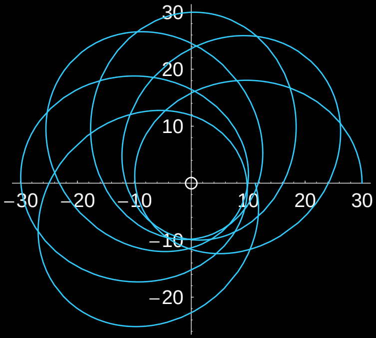
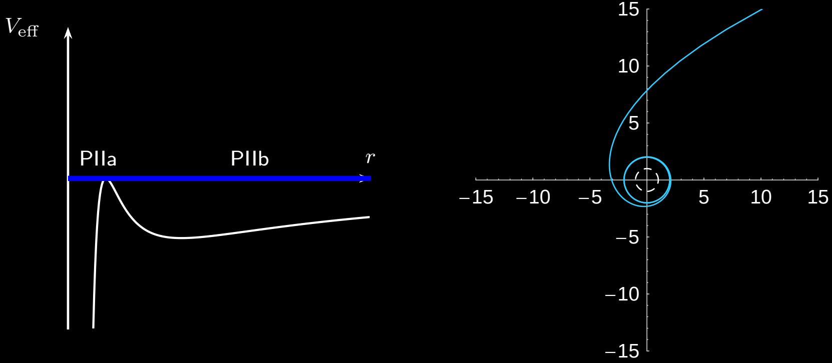




pseudo–elliptic — quasi elliptic: perihelion shift: $\delta\varphi = \omega_1 - 2\pi$

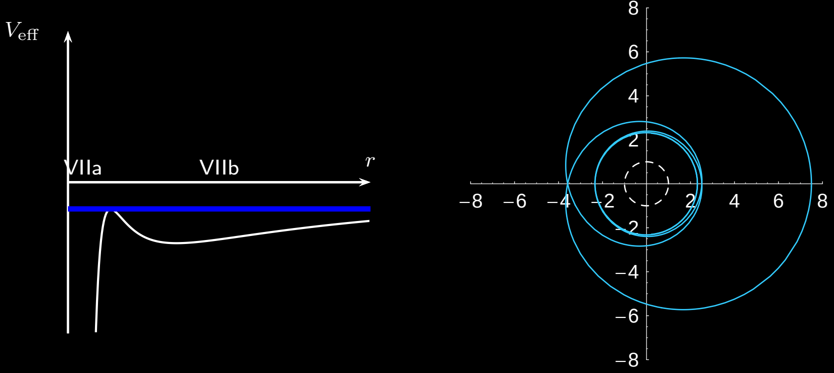
Application: Quasar QJ287: $\delta\varphi \approx 39^\circ$ per revolution (Valtonen et al 2008)

Orbits in Schwarzschild space–time



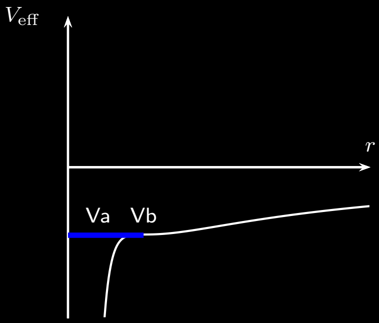
finite parabolic spiral — infinite parabolic spiral

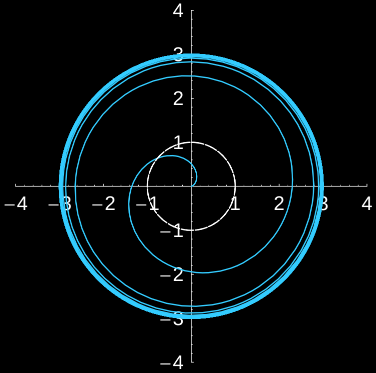
Orbits in Schwarzschild space–time



spiral — double spiral (Poincarés double circle limit)

Orbits in Schwarzschild space–time





spiral — circle (ISCO)

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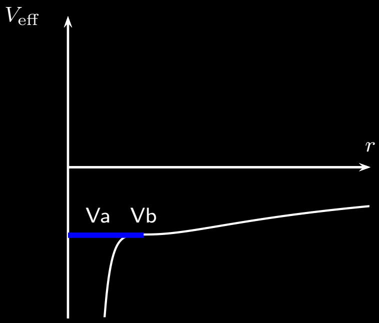

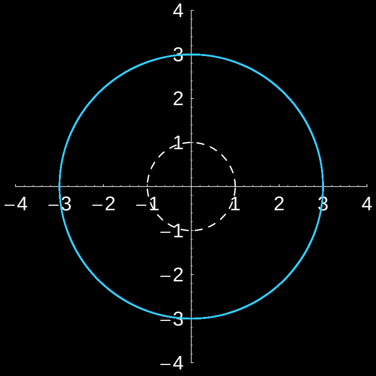

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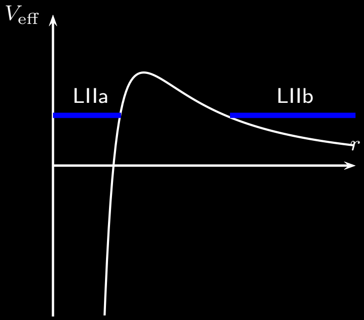
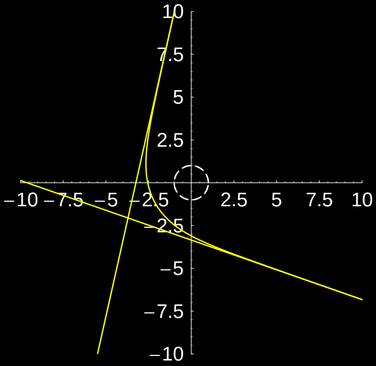
Light rays in Schwarzschild space–time





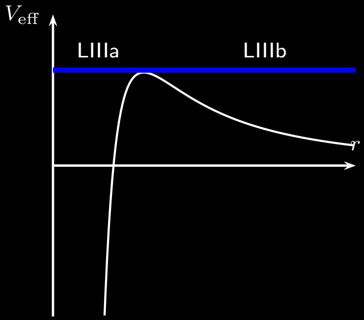
pseudo hyperbolic — quasi–hyperbolic

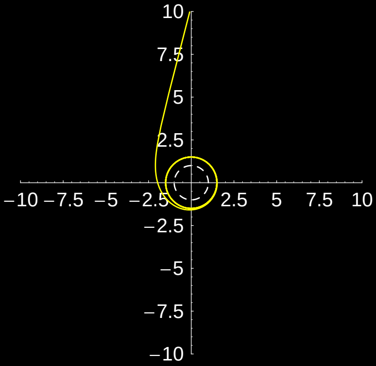
Light deflection

Deflection angle

$$\Delta\varphi = \frac{4}{\sqrt{e_1 - e_3}} F(\alpha, k)\,, \qquad \sin\alpha = \sqrt{-\frac{e_3 + \frac{1}{3}}{e_2 - e_3}}\,, \quad k = \sqrt{\frac{e_2 - e_3}{e_1 - e_3}}$$

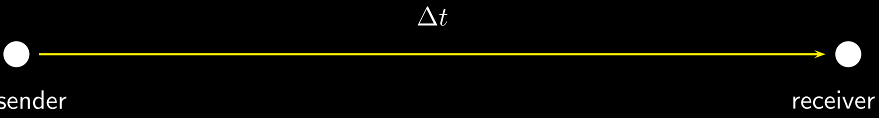
Light rays in Schwarzschild space–time





finite pseudo–hyperbolic spiral — infinite quasi-hyperbolic spiral — ISCO

Gravitational time delay



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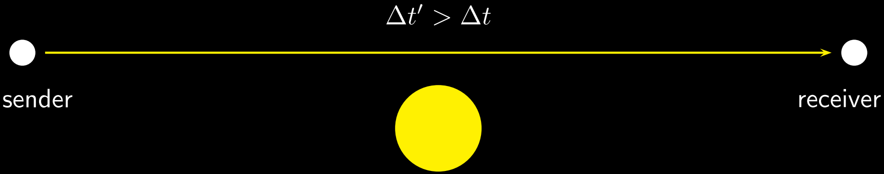

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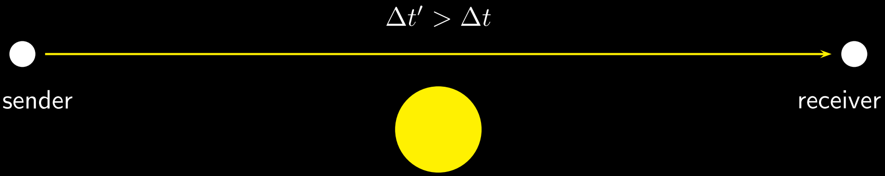

Caution

- Within an exact framework for gravitational effects there is no definition or identification of points with and without gravitational field
  $\Rightarrow$ there is no notion of a gravitational time delay
- Within exact treatment there is only a combined effect due to gravitational time delay, redshift, kinematical time delay (Doppler effect) and light bending
- There is no way to isolate a gravitational time delay – only possible asymptotically, in weak field approximation

Further solutions

Analytic solution of geodesic equation in

- Schwarzschild space–time (Hagihara, JJAG 1931)
- Kerr space–time (Chandrasekhar 1983)
- Reissner–Nordström (Chandrasekhar 1983)
- Schwarzschild–de Sitter (Hackmann & C.L., PRL 2008, PRD 2008)
- Kerr–de Sitter (Hackmann & C.L. 2009)
- Reissner–Nordström–de Sitter (Hackmann, Kagramanova, Kunz, & C.L. 2008)
- NUT–de Sitter (Hackmann, Kagramanova, Kunz, & C.L. 2008)
- Schwarzschild / Kerr with string (Hackmann, Hartmann, et al. 2010a, 2010b)
- higher dimensions (Enolskii, Hackmann et al. 2011)
- Observables (Hackmann & C.L. 2012)

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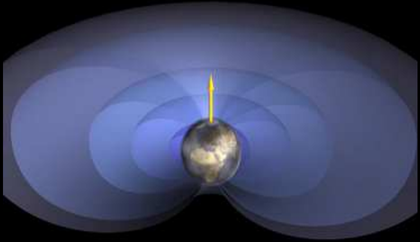
astrophysically relevant

The gravitomagnetic field



The gravitomagnetic field

- Kerr metric
- orbits in Kerr metric, analytical expressions for:
- Perihelion shift
- Lense–Thirring effect
- combined effects
- ...

Applications in:

- motion of stars around black holes
- EMRI
- accretion discs

Gravitomagnetic clock effect

- rotating gravitating mass ⇒ Kerr solution

$$ds^2 = \ldots + \frac{2Mr}{r^2 + a^2\cos^2\vartheta} a \sin^2\vartheta \, d\varphi \, dt + \ldots$$

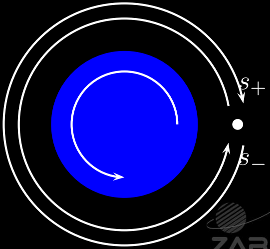
- geodesic equation for circular orbits in equatorial plane

$$\frac{d\varphi}{dt} = \pm\Omega_0 + \Omega_{\text{Lense-Thirring}}$$

- proper time difference of two counterpropagating clocks

$$s_+ - s_- = 4\pi \frac{J}{M} \sim 10^{-7}\ \text{s}$$

does not depend on G and on r
decreases with inclination, vanishes for polar orbits



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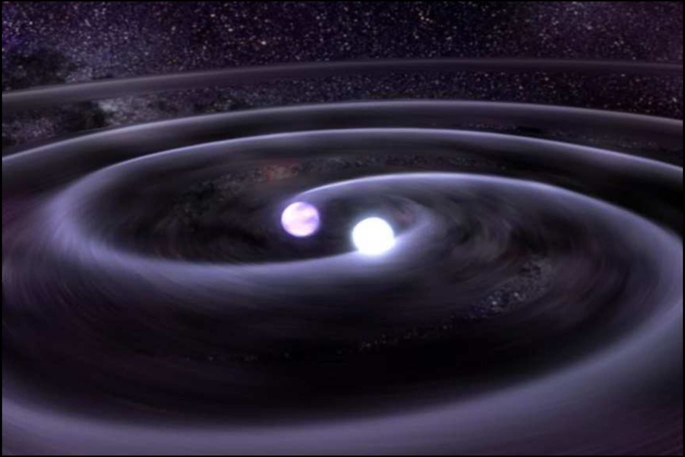
Binary systems

Strong field regime ⟶ access to more PPN parameters, nonlinearities, spin–spin effects (Kramer et al, Science 2006) ...



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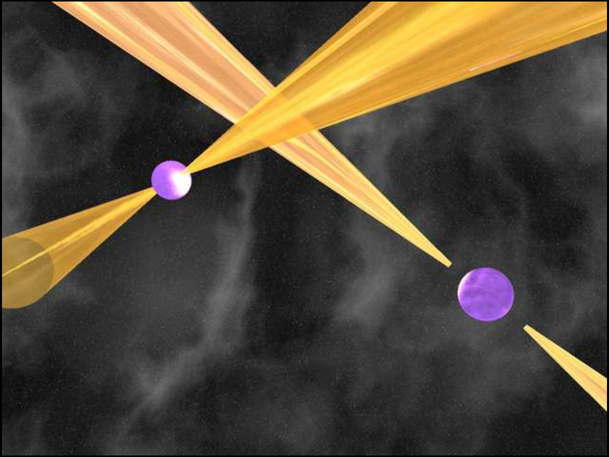

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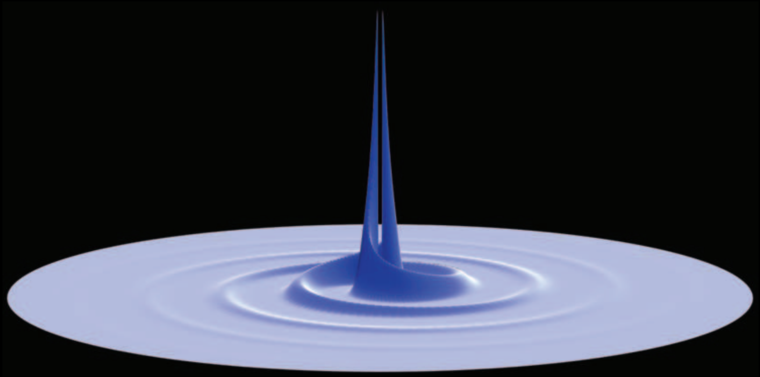
Gravitational waves

Radiation properties ⟶ mass of graviton, ...



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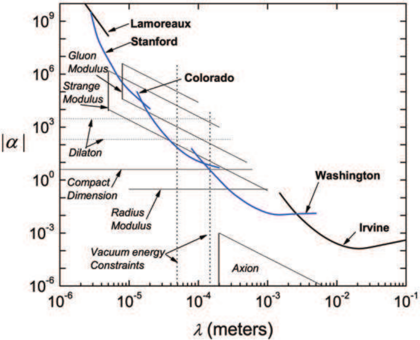
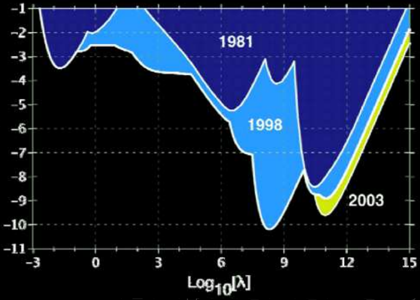
Test of Newton potential I

$$U = \frac{GM}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$



short range limits

long range limits



Test of Newton potential I

Kostelecky framework (Kostelecky, PRD 2005): anisotropy of Newtonian potential

$$U = \frac{MG}{r}\left(1 + \frac{r^i c_{ij} r^j}{r^2}\right)$$

Experiments

- Atomic interferometry (Müller et al, PRL 2007)
- LLR (Battat, Chandler & Stubbs, PRL 2007)

Result

$$|c_{ij}| \leq 10^{-5} \dots 10^{-9}$$

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Unexplained observations

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Needed to describe galactic rotation curves, lensing, structure formation

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Needed to describe galactic rotation curves, lensing, structure formation

Dark Energy (Turner 1999)
Needed to describe the accelerated expansion of our universe

Pioneer Anomaly (Anderson et al 1998, 2002)
Non–Newtonian acceleration of Pioneer spacecraft toward the Sun – resolved

Flyby anomaly (ESA and NASA communications)
Satellite velocities are too large by a few mm/s after flybys

Increase of Astronomical Unit (Krasinski & Brumberg 2005, Pitjeva in Standish 2005) Distance Earth–Sun increases by ~ 10 m/cy

Quadrupole and octupole anomaly (Tegmark et al 2004, Schwarz et al 2005)
Quadrupole and octupole of CMB are correlated with Solar system ecliptic

Is the gravitational physics in the Solar system really well understood?

But: Dark clouds over General Relativity?

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Is the gravitational physics in the Solar system really well understood?
Gravity at large distances? Weak gravity? Small accelerations?

Fate of Einstein Equations?

very likely to be modified due to Quantum Gravity ...

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Thank you!

- H. Dittus
- E. Hackmann
- D. Lorek
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- DFG
- Center of Excellence QUEST
- Research Training Group *Models of Gravity*
- DLR