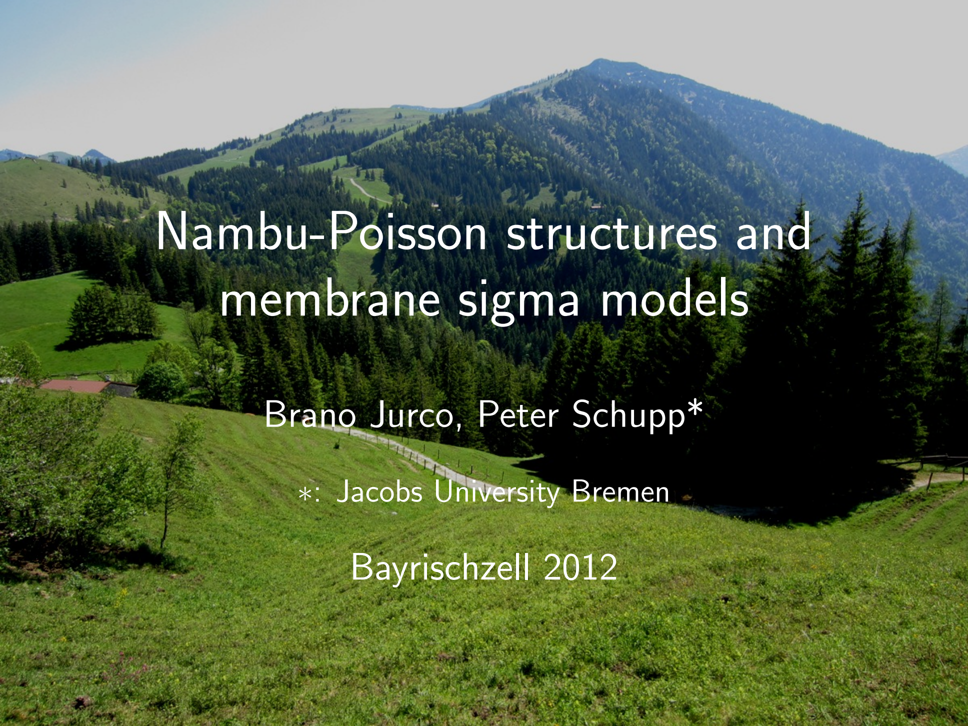

Nambu-Poisson structures and membrane sigma models

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Nambu and membrane sigma models & effective actions

Motivation

- Non-commutative $\rightarrow$ "Nambuian" geometry
- Nambu-Poisson deformation quantization
- DBI-like effective action for open brane systems

Outline

- Nambu-Poisson structures
- Poisson σ-model
- Nambu σ-model
- Membrane actions
- Nambu-DBI action
- brief remarks on quantization

Nambu-Poisson structures

Nambu mechanics

multi-Hamiltonian dynamics with generalized Poisson brackets

e.g. Euler's equations for spinning top

$$I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2) = 0 \qquad \text{etc.}$$

$$\Rightarrow \dot{L}_i = \epsilon_{ijk} L_j L_k / I_j = \{L_i, T, \frac{1}{2}\vec{L}^2\}$$

with $\vec{L} = I \cdot \vec{\omega}$, $T = \frac{1}{2}\vec{L} \cdot \vec{\omega}$ and Nambu-Poisson bracket

$$\{f, g, h\} = \det\left[\frac{\partial(f, g, h)}{\partial(L_1, L_2, L_3)}\right] = \epsilon^{ijk}\,\partial_i f\,\partial_j g\,\partial_k h$$

Nambu-Poisson (NP) bracket

more generally: skew-symmetric, multi-linear, derivation

$$\{f, h_1, \ldots, h_p\} = \Pi^{i\, j_1 \ldots j_p}(x)\, \partial_i f\, \partial_{j_1} h_1 \cdots \partial_{j_p} h_p$$

+ Fundamental Identity (FI)

$$\begin{aligned} \{\{f_0, \cdots, f_p\}, h_1, \cdots, h_p\} &= \{\{f_0, h_1, \cdots, h_p\}, f_1, \cdots, f_p\} + \ldots \\ &\quad \ldots + \{f_0, \ldots, f_{p-1}, \{f_p, h_1, \cdots, h_p\}\} \end{aligned}$$

Nambu-Poisson structures

Alternative viewpoint

- Nambu tensor $\Pi \in TM \otimes \Lambda^p TM$ maps a time-evolution p-form η "Nambuian" ($\eta = dH$ for $p = 1$) to a time-evolution vector field

$$\Pi(\eta) = \frac{1}{p!}\Pi^{i\, j_1 \ldots j_p}\, \eta_{j_1 \ldots j_p} \partial_i \equiv \Pi^{iJ} \eta_J \partial_i \in TM$$

  with $J = (j_1, \ldots, j_p)$: ordered multi-index

- Canonical transformation property

$$d\eta = 0 \quad \Rightarrow \quad \mathcal{L}_{\Pi(\eta)}\, \Pi = 0$$

- Conservation law property

$$\eta = dh_1 \wedge \ldots \wedge dh_p \quad \Rightarrow \quad \mathcal{L}_{\Pi(\eta)}\, \eta = 0$$

Nambu-Poisson structures

For $p = 1$:
ordinary Poisson structure, differential constraint (Jacobi identity)

For $p > 1$:
Nambu-Poisson structure, differential & algebraic constraint

$\Leftrightarrow$ Π factorizes into wedge product of vector fields

$$\Pi = V_0 \wedge V_1 \wedge \ldots \wedge V_p = |\Pi(x)|^{\frac{1}{p+1}} \, e_0 \wedge \ldots \wedge e_p$$

- foliation into $(p+1)$-dimensional submanifolds
- $|\Pi(x)|^{\frac{1}{p+1}}$ is a scalar density of weight -1
- $|\Pi(x)|$ is the generalized determinant of the rectangular matrix Π^{iJ} and a scalar density of weight $-(p+1)$

Poisson σ-model

Nonlinear gauge theory/Poisson σ-model (Ikeda; Schaller, Strobel)

$$S[A,X] = \int_\Sigma \left(A_i \wedge dX^i - \frac{1}{2}\Pi^{ij} A_i \wedge A_j \right) \qquad \Pi = \frac{1}{2}\Pi^{ij}(X)\partial_i \wedge \partial_j$$

$X : \Sigma \to M$ (Σ: 2D world sheet, M: target space)

$A(\sigma) =$ 1-form on Σ with values in $T^*_{X(\sigma)}M$

equations of motion

$$dX^i - \Pi^{ij} A_j = 0 \qquad dA_i + \frac{1}{2}\partial_i \Pi^{kl} A_k \wedge A_l = 0$$

consistency of eom requires

$$[\Pi,\Pi]_S^{ijk} = \frac{1}{3}\left(\Pi^{il}\partial_l \Pi^{jk} + \text{cycl}\right) = 0 \quad \Rightarrow (M,\Pi) \text{ must be Poisson}$$

Poisson σ-model

Generalized (non-topological) Poisson σ-model

$$S = \int_\Sigma \left(A_i \wedge dX^i - \frac{1}{2}\Pi^{ij} A_i \wedge A_j - \frac{1}{2}(G^{-1})^{ij} A_i \wedge *A_j \right)$$

$A_i = A_{i\alpha}(\sigma) d\sigma^\alpha$ are auxiliary fields $\rightarrow$ integrate out

$$S' = -\int_\Sigma \frac{1}{2}\left(g_{ij} dX^i \wedge *dX^j + B_{ij} dX^i \wedge dX^j \right)$$

$\Rightarrow$ closed-open string relations

$$\frac{1}{g+B} = G^{-1} + \Pi \quad \Rightarrow \quad G = g - Bg^{-1}B\,, \quad \theta = -G^{-1}Bg^{-1}$$

quick & 'dirty' derivation...

$$\sum_{\alpha,\beta} (G^{-1} + \Pi)^{ij} A_{i\alpha} A_{j\beta} \qquad \sum_{\alpha,\beta} (g+B)_{ij} \partial_\alpha X^i \partial_\beta X^j$$

Nambu σ-model

Let $\eta_i = \eta_i(\sigma) d\sigma^1 := -A_{i1}(\sigma) d\sigma^1$ and $\tilde{\eta}_j = \tilde{\eta}_j(\sigma) d\sigma^0 := A_{j0}(\sigma) d\sigma^0$

Generalized Poisson σ-model

$$S = \int_{\Sigma_{1+1}} \left(dX^i \wedge \eta_i + \tilde{\eta}_j \wedge dX^j - \Pi^{ij} \tilde{\eta}_j \wedge \eta_i - \frac{1}{2} G^{ij} \eta_i \wedge * \eta_j - \frac{1}{2} G^{ij} \tilde{\eta}_i \wedge * \tilde{\eta}_j \right)$$

p-brane version $\rightarrow$ Nambu σ model

$$S = \int_{\Sigma_{1+p}} \left(dX^i \wedge \eta_i + \tilde{\eta}_J \wedge d^p X^J - \Pi^{iJ} \tilde{\eta}_J \wedge \eta_i - \frac{1}{2} G^{ij} \eta_i \wedge * \eta_j - \frac{1}{2} \tilde{G}^{IJ} \tilde{\eta}_I \wedge * \tilde{\eta}_J \right)$$

Nambu sigma model

Notation

$$X^i(\sigma) \qquad \text{embedding fn's (scalar fields)}$$

$$I, J \qquad \text{ordered } p\text{-tuple multi-indices}$$

$$I = (i_1, \ldots, i_p) \qquad 0 \le i_1 < \ldots < i_p \le D-1$$

$$\widetilde{\partial X}^I \equiv \sum_{a_1,\ldots,a_p=1}^{p} \epsilon^{a_1\ldots a_p} \partial_{a_1} X^{i_1} \cdots \partial_{a_p} X^{i_p}$$

$$\alpha, \beta = 0, 1, \ldots, p \qquad \text{world volume indices}$$

$$a, b = 1, \ldots, p \qquad \text{spatial components}$$

A tilde distinguishes fields that carry multi indices.

Nambu sigma model (in components)

$$S[\eta, \tilde{\eta}, X] = \int d^{p+1}\sigma \, \Big[-\frac{1}{2}(G^{-1})^{ij}\eta_i\eta_j + \frac{1}{2}(\tilde{G}^{-1})^{IJ}\tilde{\eta}_I\tilde{\eta}_J + \eta_i\partial_0 X^i + \tilde{\eta}_I\widetilde{\partial X}^I - \Pi^{iJ}\eta_i\tilde{\eta}_J \Big]$$

Nambu gauge theory

Nambu-Poisson map

add fluctuations: p-form gauge potential A with field strength $F = dA$

gauge action of F on Π:

$$\Pi \mapsto \Pi^F = (I - \Pi F^T)^{-1}\Pi = (1 - \langle \Pi, F\rangle)^{-1}\Pi$$

with inner product $\langle \Pi, F\rangle \equiv \mathrm{tr}\,\Pi F^T$

Nambu-Poisson map $\rho_{[A]}$ (change of coordinates) relates Π and Π^F

gauge tr. $\delta A = d\lambda \;\Rightarrow \delta\rho_{[A]}$ generated by $X_{[\lambda,A]} = \Pi^{iJ}(d\hat{\lambda}_{[\lambda,A]})_J\partial_i$

$$\hat{\lambda}_{[\lambda,A]} = \sum_k \frac{(-\mathcal{L}_{\Pi^{tF}(A)} + \partial_t)^k(\lambda)}{(k+1)!}\Big|_{t=0}.$$

Nambu gauge theory

Covariant functions and coordinates:

$$\hat{f} = \rho_{[A]}(f) \quad \rightsquigarrow \quad \delta \hat{f} = \mathcal{L}_{\Pi(d\hat{\lambda})} \hat{f} = \sum \{\hat{f}, \hat{\lambda}^{(1)}, \ldots, \hat{\lambda}^{(p)}\}$$

$$\hat{x}^i = \rho_{[A]}(x^i) = x^i + \hat{A}^i \quad \rightsquigarrow \quad \delta \hat{A}^i = \sum \{\hat{x}^i + \hat{A}^i, \hat{\lambda}^{(1)}, \ldots, \hat{\lambda}^{(p)}\}$$

$(d\hat{\lambda} \equiv \sum d\hat{\lambda}^{(1)} \wedge \ldots \wedge d\hat{\lambda}^{(p)})$

Jacobian of $\rho_{[A]} : x^i \mapsto \hat{x}^i$

Using the decomposability of Π for $p > 1$ and fact that the degenerate matrix $F\Pi^T$ acts non-trivially only on a $(p+1)$-dimensional subspace (via multiplication by $\langle \Pi, F \rangle$):

$$\det(1 - F\Pi^T) = (1 - \langle \Pi, F \rangle)^{p+1} = \frac{|\Pi(\hat{x})|}{|\Pi(x)|} \cdot \left| \frac{\partial x}{\partial \hat{x}} \right|^{p+1} .$$

Nambu sigma model

Nambu sigma model in block matrix form

$$S[\eta,\tilde{\eta},X] = \int d^{p+1}\sigma \, \left[\frac{1}{2} \begin{pmatrix} \eta_i \\ \tilde{\eta}_I \end{pmatrix}^T \begin{pmatrix} (G^{-1})^{ij} & -i\Pi^{iJ} \\ -i(\Pi^T)^{Ij} & (\tilde{G}^{-1})^{IJ} \end{pmatrix} \begin{pmatrix} \eta_j \\ \tilde{\eta}_J \end{pmatrix} \right. \\ \left. -i \begin{pmatrix} \eta_i \\ \tilde{\eta}_I \end{pmatrix}^T \begin{pmatrix} \partial_0 X^i \\ \widetilde{\partial X}^I \end{pmatrix} \right]$$

equations of motion $\rightarrow$ eliminate auxiliary fields η, $\tilde{\eta}$

$$\begin{pmatrix} (G^{-1})^{ij} & -i\Pi^{iJ} \\ -i(\Pi^T)^{Ij} & (\tilde{G}^{-1})^{IJ} \end{pmatrix} \begin{pmatrix} \eta_j \\ \tilde{\eta}_J \end{pmatrix} = -i \begin{pmatrix} \partial_0 X^i \\ \widetilde{\partial X}^I \end{pmatrix}$$

solve for η, $\tilde{\eta}$ and plug back into action

$$S[X] = \int d^{p+1}\sigma \, \left[\frac{1}{2} \begin{pmatrix} \partial_0 X^i \\ \widetilde{\partial X}^I \end{pmatrix}^T \begin{pmatrix} (G^{-1})^{ij} & -i\Pi^{iJ} \\ -i(\Pi^T)^{Ij} & (\tilde{G}^{-1})^{IJ} \end{pmatrix}^{-1} \begin{pmatrix} \partial_0 X^j \\ \widetilde{\partial X}^J \end{pmatrix} \right]$$

Nambu sigma model

→ membrane σ model with C-field background

$$S[X] = \int d^{p+1}\sigma \left[g_{ij}\partial_0 X^i \partial_0 X^j + \tilde{g}_{IJ}\widetilde{\partial X}^I \widetilde{\partial X}^J - i \int d^{p+1}\sigma \sum_{i,J} C_{iJ}\partial_0 X^i \widetilde{\partial X}^J \right]$$

where we identify

$$\begin{pmatrix} g & -iC \\ -iC^T & \tilde{g} \end{pmatrix} = \begin{pmatrix} G^{-1} & -i\Pi \\ -i\Pi^T & \tilde{G}^{-1} \end{pmatrix}^{-1}$$

more generally:

$$\begin{pmatrix} g & -iC \\ -iC^T & \tilde{g} \end{pmatrix}^{-1} = \begin{pmatrix} G & -i\Phi \\ -i\Phi^T & \tilde{G} \end{pmatrix}^{-1} + \begin{pmatrix} 0 & -i\Pi \\ -i\Pi^T & 0 \end{pmatrix}$$

Membrane actions

M-Brane with a closed membrane C-field background:

Nambu-Goto p-brane action

$$S[X] = T_p \int_\Sigma d^{p+1}\sigma \, \sqrt{\det(g_{ij}\partial_\alpha X^i \partial_\beta X^j)}$$

classically equivalent: p-brane sigma model action

$$S[X,h] = \frac{T'_p}{2} \int_\Sigma d^{p+1}\sigma \, \sqrt{\det h} \left[g_{ij} h^{\alpha\beta} \partial_\alpha X^i \partial_\beta X^j - (p-1)\lambda \right]$$

where $T'_p = \lambda^{\frac{p-1}{2}} T_p$ and $\lambda > 0$

Membrane actions

gauge fix

$$h_{a,0} = h_{0,b} = 0 \text{ and } h_{00} = \lambda^{p-1} \det(h_{ab})$$

(valid globally for Σ of form $\Sigma_p \times \mathbb{R}$, $\Sigma_p \times I$ or $\Sigma_p \times S^1$)

eliminate h_{ab} $\Rightarrow$

$$S_{\text{gf}}[X] = \frac{T_p}{2} \int d^{p+1}\sigma \left[g_{ij} \partial_0 X^i \partial_0 X^j + \det(g_{ij} \partial_a X^i \partial_b X^j) \right]$$

introduce multi-index notation

$$\tilde{g}_{IJ} \equiv \sum_{\pi \in \mathfrak{S}_p} \text{sgn}(\pi) g_{i_{\pi(1)} j_1} \cdots g_{i_{\pi(p)} j_p}$$

Membrane actions

gauge-fixed p-brane action in multi-index notation

$$S_{\text{gf}}[X] = \frac{T_p}{2} \int d^{p+1}\sigma \, \left[g_{ij} \partial_0 X^i \partial_0 X^j + \tilde{g}_{IJ} \widetilde{\partial X}^I \widetilde{\partial X}^J \right]$$

add background C_{p+1}-field

$$\frac{1}{(p+1)!} C_{ij_1 \dots j_p} dx^i dx^{j_1} \dots dx^{j_p}$$

with field strength $H = dC$ $\rightarrow$ membrane σ model

$$S[X] = \int d^{p+1}\sigma \, \Big[g_{ij} \partial_0 X^i \partial_0 X^j + \tilde{g}_{IJ} \widetilde{\partial X}^I \widetilde{\partial X}^J \\ - i \int d^{p+1}\sigma \, \sum_{i,J} C_{iJ} \partial_0 X^i \widetilde{\partial X}^J \Big]$$

Membrane versus Nambu sigma model

Closed-open membrane relations

$$
\begin{aligned}
g + C\tilde{g}^{-1}C^T &= G + \Phi\tilde{G}^{-1}\Phi^T \\
\tilde{g} + C^T g^{-1} C &= \tilde{G} + \Phi^T G^{-1}\Phi \\
g^{-1}C &= G^{-1}\Phi - \Pi(\tilde{G} + \Phi^T G^{-1}\Phi) \\
C\tilde{g}^{-1} &= \Phi\tilde{G}^{-1} - (G + \Phi\tilde{G}^{-1}\Phi^T)\Pi
\end{aligned}
$$

$$
\frac{1}{g + C} = \frac{1}{G + \Phi} + \Pi \qquad (\text{for } p = 1)
$$

Born Infeld action

$p = 1$ open strings, effective action

Born-Infeld action (ignoring dilaton)

$$S_{DBI} = \frac{1}{g_s}\int d^n x \det{}^{\frac{1}{2}}[g+B] = \frac{1}{g_s}\int d^n x \det{}^{\frac{1}{4}}[g] \det{}^{\frac{1}{4}}\big[\underbrace{g - Bg^{-1}B}_{G}\big]$$

add fluctuations $B \mapsto \mathcal{F} = B + F$:
$\theta \mapsto \theta' = (1-\theta F)^{-1}\theta$ and $x^i \mapsto \rho^*(x^i) = \hat{x}^i = x^i + \theta^{ij}\hat{a}_j$,
where $\hat{a}_j$ is a "semi-noncommutative" gauge field.

The flow ρ^* (SW map) is generated by the vector field $\theta^{ij} a_j \partial_i$ with
$f_{ij} = \partial_i a_j - \partial_j a_i$.

Born Infeld action

$p = 1$ noncommutative description

$$S_{DBI} = \int d^n x \frac{1}{g_s} \det{}^{\frac{1}{2}} [g + B + F] = \int d^n x \frac{1}{\widehat{G}_s} \frac{\widehat{|\theta|}^{\frac{1}{2}}}{|\theta|^{\frac{1}{2}}} \det{}^{\frac{1}{2}} \left[\hat{G} + \hat{\Phi} + \widehat{F}'\right]$$

The proof of this exact identity is based on matrix manipulations starting from

$$\frac{1}{g+B} = \theta + \frac{1}{G+\Phi} \,,$$

using the flow ρ^*, and appropriate relations for the string couplings.

exact identity only with *semi-classical* noncommutative description

Born Infeld action

$p > 1$ effective action (?)

$\det[g + B]$ makes no sense, but $\det[g + B\tilde{g}^{-1}B^T]$ does.

$$S_{DBI} = \frac{1}{g_m} \int d^{p'+1}x \det{}^x[g] \det{}^y \Big[\underbrace{g + B\tilde{g}^{-1}B^T}_{G} \Big]$$

$$= \frac{1}{g_m} \int d^{p'+1}x \det{}^{\frac{1}{2}}[g] \det{}^y \left[1 + g^{-1}B\tilde{g}^{-1}B^T\right]$$

$y =?$

Effective open p-brane action

Miraculous identity

$$\det[g+(B+F)\tilde{g}^{-1}(B+F)^T] = \det{}^2[1-F\Pi^T]\cdot\det[G+(\Phi+F')\tilde{G}^{-1}(\Phi+F')^T]$$

where $F' = (I - F\Pi^T)^{-1}F$, holds for all p.

The Jacobian of the Nambu-Poisson map fixes the appropriate power:

$\Rightarrow$ Effective action (conjecture)

$$S_{p\text{-DBI}} = \int d^{p'+1}x \, \frac{1}{g_m} \det{}^{\mathrm{x}}(g) \cdot \det{}^{\mathrm{y}}\left[g + (C+F)\tilde{g}^{-1}(C+F)^T\right]$$

with $\mathrm{x} = \frac{p}{2(p+1)}$, $\mathrm{y} = \frac{1}{2(p+1)}$

Effective open p-brane action

NC Dual

$$S_{p\text{-NCDBI}} = \int d^{p'+1}x \, \frac{1}{\widehat{G}_m} \frac{\widehat{|\Pi|}^{\frac{1}{p+1}}}{|\Pi|^{\frac{1}{p+1}}} \det{}^{\mathrm{x}}(\widehat{G})$$
$$\cdot \det{}^{\mathrm{y}}\left[\widehat{G} + (\widehat{\Phi} + \widehat{F}')\widehat{\tilde{G}}^{-1}(\widehat{\Phi} + \widehat{F}')^T\right]$$

$\widehat{\ }$ denotes objects evaluated at covariant coordinates

$\widehat{F}'$ is the Nambu (NC) field strength

open-closed membrane coupling constants

$$G_m = g_m \left(\frac{\det G}{\det g}\right)^{\frac{p}{2(p+1)}}$$

Effective open p-brane action

Expansion of action

ignore a cosmological constant term and let $\mathcal{F} = C + F$

$$\mathcal{S}_{p\text{-DBI}} = \frac{1}{2(p+1)g_m} \det{}^{\frac{1}{2}}(g)\,\mathrm{tr}\left[g^{-1}\mathcal{F}\tilde{g}^{-1}\mathcal{F}^T\right] + \dots$$

the coupling constant g_m is dimensionless for:

- strings on D3 with 2-form field strength (Maxwell/Yang-Mills)
- 2-brane on 5-brane with 3-form field strength ($\rightsquigarrow$ M2-M5 system)
- p-brane on $2(p+1)$-brane with $p+1$ form field strength

consider $p = 2$, $p' = 5$ and expand further ($k = \mathcal{F}_i^{kl}\mathcal{F}_{jkl}$):

$$det^{\frac{1}{6}}(1+k) = \sqrt{1 + \frac{1}{3}\mathrm{tr}k - \frac{1}{6}\mathrm{tr}k^2 + \frac{1}{36}(\mathrm{tr}\,k)^2 + \dots}.$$

$\Rightarrow$ exact match with κ-symmetry computation of
Cederwall, Nilsson, Sundell, “An Action for the superfive-brane” (1998)

Effective open p-brane action

Background independent gauge

Focus on $(p+1)$-dimensional maximal NC subspace:

$\Rightarrow \Pi^{iJ}$, C_{iJ} are square matrices

The open-closed membrane relations imply

$$\Pi = -(C^T)^{-1} \qquad G = C\tilde{g}^{-1}C^T \qquad \tilde{G} = C^T g^{-1} C$$

The relevant part of the action becomes

$$S_{\mathrm{M}} = \int d^{p+1}x \, \frac{1}{|\Pi|^{\frac{1}{p+1}}} \, \frac{1}{\widehat{g}_m} \, \det{}^{y}\left[1 + \widehat{\Pi}'^T \widehat{g} \, \widehat{\Pi}' \widehat{\tilde{g}}\,\right]$$

Effective open p-brane action

From higher gauge theory to matrix model...

Expanding to lowest order (ignoring a non-cosmological constant) $\Rightarrow$ semi-classical/infinite-dimensional version of a matrix model

$$\int d^{p+1}x \, \frac{1}{|\Pi|^{\frac{1}{p+1}}} \, \frac{1}{2(p+1)\widehat{g}_m} \cdot \hat{g}_{i_0 j_0} \cdots \hat{g}_{i_p j_p} \{\widehat{X}^{j_0}, \ldots, \widehat{X}^{j_p}\}\{\widehat{X}^{i_0}, \ldots, \widehat{X}^{i_p}\}$$

quantize:

$$\rightsquigarrow \frac{1}{2(p+1)\widehat{g}_m} \, \mathrm{Tr}\left(\hat{g}_{i_0 j_0} \cdots \hat{g}_{i_p j_p} \left[\widehat{X}^{j_0}, \ldots, \widehat{X}^{j_p}\right] \left[\widehat{X}^{i_0}, \ldots, \widehat{X}^{i_p}\right]\right)$$

Quantization

Quantization I: auxiliary fields

path integral based on AKSZ-style action (Jurco/Bouwknegt)

$$
\begin{aligned}
S_{c,\pi} = & \int_X (\mathbf{F}_i D\mathbf{X}^i + \psi^i D\chi_i + \mathbf{G}_I D\mathbf{H}^I + \boldsymbol{\eta}^I D\mathbf{A}_I) \\
& + \int_X (-\boldsymbol{\psi}^i \mathbf{F}_i + \frac{1}{(p-1)!}\mathbf{G}_{i_1\dots i_{p-1}}(\boldsymbol{\eta}^{i_1\dots i_{p-1}} - \boldsymbol{\psi}^{i_1}\dots\boldsymbol{\psi}^{i_{p-1}})) \\
& + \frac{1}{(p+1)!}\int_X \mathbf{c}_{i_1\dots i_{p+1}}\boldsymbol{\psi}^{i_1}\dots\boldsymbol{\psi}^{i_{p+1}} + \int_{\partial X}\frac{1}{(p-1)!}\boldsymbol{\pi}^{i_1 i_2\dots i_p}\mathbf{A}_{i_1\dots i_{p-1}}\boldsymbol{\chi}_{i_p}
\end{aligned}
$$

Quantization II: extended target space

deformation quantization of twisted Poisson structure on phase space

$$\Theta(x,p) = \begin{pmatrix} [\Pi(x)]^{ijk} p_k & \delta^i_j \\ -\delta_i^j & 0 \end{pmatrix}$$

$\rightsquigarrow$ non-trivial Jacobiators, non-associative $\star$ product

relevant for closed strings in R-flux background, “non-associative gravity”

joint work in Edinburgh with Richard Szabo and Dionysis Mylonas

Conclusion

Summary

- Nambu-Sigma model related to p-branes in C-field background
- open-closed relations for p-branes
- Nambu versions of: NC gauge theory and Seiberg-Witten map
- commutative $\leftrightarrow$ non-commutative "duality"
- DBI-type effective action for branes ending on branes (conjecture)
- 2 limits: higher form gauge theory and covariant matrix model

To do's

- add fermions
- checks: κ-symmetry, dualities, dimensional reduction
- direct path integral derivation (unclear how to do this)
- quantization
- ...

Thank you for listening