Noncommutativity and Physics: Spacetime Quantum Geometry

High energy cosmic rays experiments inspired by noncommutative quantum field theory

Josip Trampetić

Rudjer Bošković Institute, Zagreb, Croatia

Collaboration with

R. Horvat, A. Ilakovac, D. Kekez,
P. Schupp, J. You

Contents

1 Motivation for θ-exact covariant NCGFT: UHECR-ν in experiments versus theory at $E_\nu = 10^{10}$ to 10^{11} GeV.

$$[\hat{\mathbf{x}}^\mu, \hat{\mathbf{x}}^\nu] = [\mathbf{x}^\mu \overset{\star}{,} \mathbf{x}^\nu] = \mathbf{i}\theta^{\mu\nu}$$

2 Plasmon Physics: Plasmon Decay Rate
Neutrino charge radius

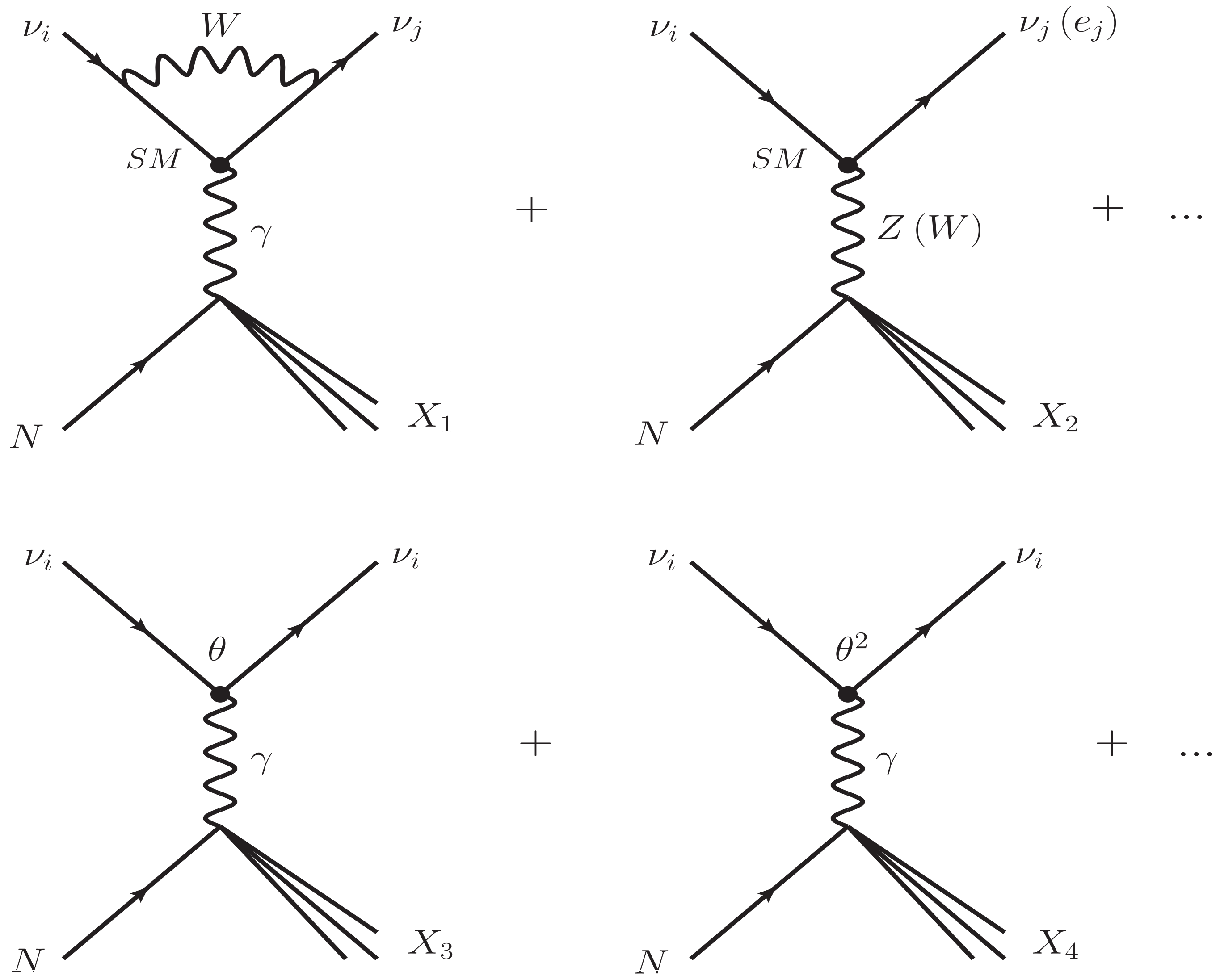
3 BBN from: Neutrino Scattering
Plasmon Decay Rate

4 Quantum properties: Holography and UV/IR mixing
Rehe. phase and UV/IR mixing
Neutrino propagation

5 Summary

Motivation: ν N $\to$ ν + *anything* cross sections

Spacetime NC and UHECR-ν experiments



Neutrino-photon NC interactions

[P. Schupp, J.Trampetic, J. Wess and G. Raffelt, *The photon neutrino interaction in non-commutative gauge field theory and astrophysical bounds,* Eur. Phys. J. C **36** (2004) 405]

Neutrino-photon interaction introduced via: $\star$–commutator with covariant derivative and Seiberg-Witten (SW) map

$$D_\mu \Psi = \partial_\mu \Psi - i\kappa e \left[A_\mu \star \Psi - \Psi \star A_\mu \right]$$

The SW action for a neutral fermion that couples to an Abelian gauge boson in the adjoint of NC $\mathrm{U}_\star(1)$,

$$\begin{aligned} S &= \int d^4x \left(\bar{\Psi} \star i\gamma^\mu D_\mu \Psi - m \bar{\Psi} \star \Psi \right) \\ \Psi &= \psi + e\theta^{\nu\rho} a_\rho \partial_\nu \psi + \mathcal{O}(\theta^2) \\ A_\mu &= a_\mu + e\theta^{\rho\nu} a_\nu \left[\partial_\rho a_\mu - \frac{1}{2} \partial_\mu a_\rho \right] + \mathcal{O}(\theta^2) \end{aligned}$$

$\nu\,\mathbf{N} \longrightarrow \nu$ + *anything*

[R. Horvat, D. Kekez and J. Trampetić, *Spacetime noncommutativity and ultrahigh energy cosmic ray experiments* Phys. Rev. D **83**, 065013 (2011)]

The gauge invariant action of order θ^1 and $\kappa = 1$

$$S = \frac{e}{2}\int d^4x\,\bar\psi f_{\mu\nu}(i\theta^{\mu\nu\rho}\partial_\rho - \theta^{\mu\nu}m)\psi,\;\; f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$$

Feynman rule $\Gamma^\mu_{\binom{\mathrm{L}}{\mathrm{R}}}(\nu\bar\nu\gamma) = ie\frac{1}{2}(1\mp\gamma_5)\theta^{\mu\nu\tau}k_\nu q_\tau, \quad m = 0$

Diagram 3 gives:

$$\frac{d^2\sigma_{\mathrm{NC}}}{dxdy} = \mathcal{I}\,\frac{2\pi\alpha^2}{E_\nu M_N (xy)^2}\left[(1-y)F_2^\gamma + y^2 x F_1^\gamma + y(1-y/2)xF_3^\gamma\right] .$$

$$\mathcal{I} = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\,(\frac{kck'}{2\Lambda^2_{\mathrm{NC}}})^2, \quad \text{with} \quad c^{\mu\nu} = \theta^{\mu\nu}\Lambda^2_{\mathrm{NC}}$$

$$\approx \left((c_{01}-c_{13})^2 + (c_{02}-c_{23})^2\right)\frac{E_\nu^3 M_N}{4\Lambda^4_{\mathrm{NC}}}\,x\,y\,(1-y) .$$

$\nu\, \mathbf{N} \to \nu$ + *anything*

Process reveal stronger energy dependence than expected

$$E_\nu^{1/2} s^{1/4} / \Lambda_{\rm NC} \lesssim 1\,, \quad s = 2 E_\nu M_N$$

Results are given for $(c_{01} - c_{13})^2 + (c_{02} - c_{23})^2 = 1$.
Employing $\sigma_{exp} = 4 \times 10^{-3}$ mb [for neutrino flux (FKRT-Fodor et al JCAP 11 (2003) 015] from RICE Collaboration search results at $E^\nu = 10^{11}$ GeV,

$$\sigma(\theta)/\sigma_{exp} \implies \Lambda_{\rm NC} \gtrsim 455 \ \ {\rm TeV}$$

$$\implies \left[\frac{\sigma(\theta^2)}{\sigma(\theta)} \right]_{\Lambda_{\rm NC} = 455 \ \ {\rm TeV}} \simeq 10^4 \quad {\rm UNACCEPTABLE!}$$

$\nu\,\mathbf{N} \to \nu$ + *anything*

The simplest possible modeling: first we approximate

$$\Psi \to \psi\,, \quad A_\mu \to a_\mu$$

$$\to S_{\rm NC}(\theta) = -ie\int d^4x\, \bar\psi\, \gamma^\mu (a_\mu \star \psi - \psi \star a_\mu) \quad \to$$

expansion/resummation of $\star$-product gives Feynman rule,

$\Gamma^{\mu}_{\binom{\rm L}{\rm R}}(\bar\nu\nu\gamma) = ie(1\pm\gamma_5)\gamma^\mu \sin(\frac{q\theta k}{2})$ and relevant integral

$$\mathcal{I} = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, 4\sin^2(\frac{kck'}{2\Lambda^2_{\rm NC}})$$

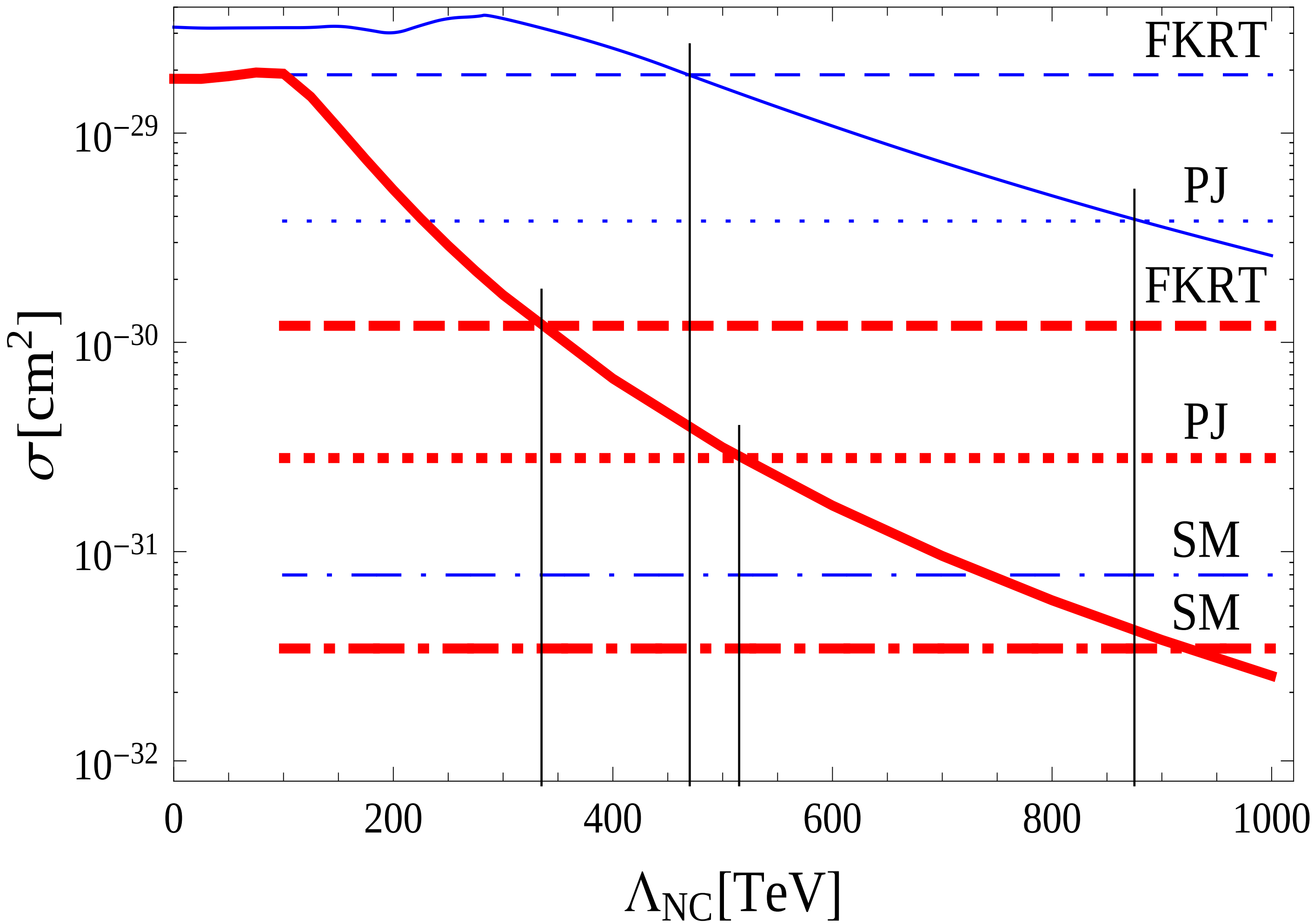
$$= 2\,(1-\cos(A)J_0(B))\ , \quad A = \frac{E_\nu E'_\nu}{\Lambda^2_{\rm NC}} c_{03}(\cos\vartheta - 1)\ ,$$

$$B = \frac{E_\nu E'_\nu}{\Lambda^2_{\rm NC}} \sin\vartheta\ \text{sign}(c_{01}-c_{03})\sqrt{(c_{01}-c_{13})^2+(c_{02}-c_{23})^2}.$$

giving total cross section $\sigma(\nu N \to \nu + \text{anything})$ as a function on NC scale and presented on next 2 Figures:

ν **N** $\longrightarrow \nu$ + *anything*

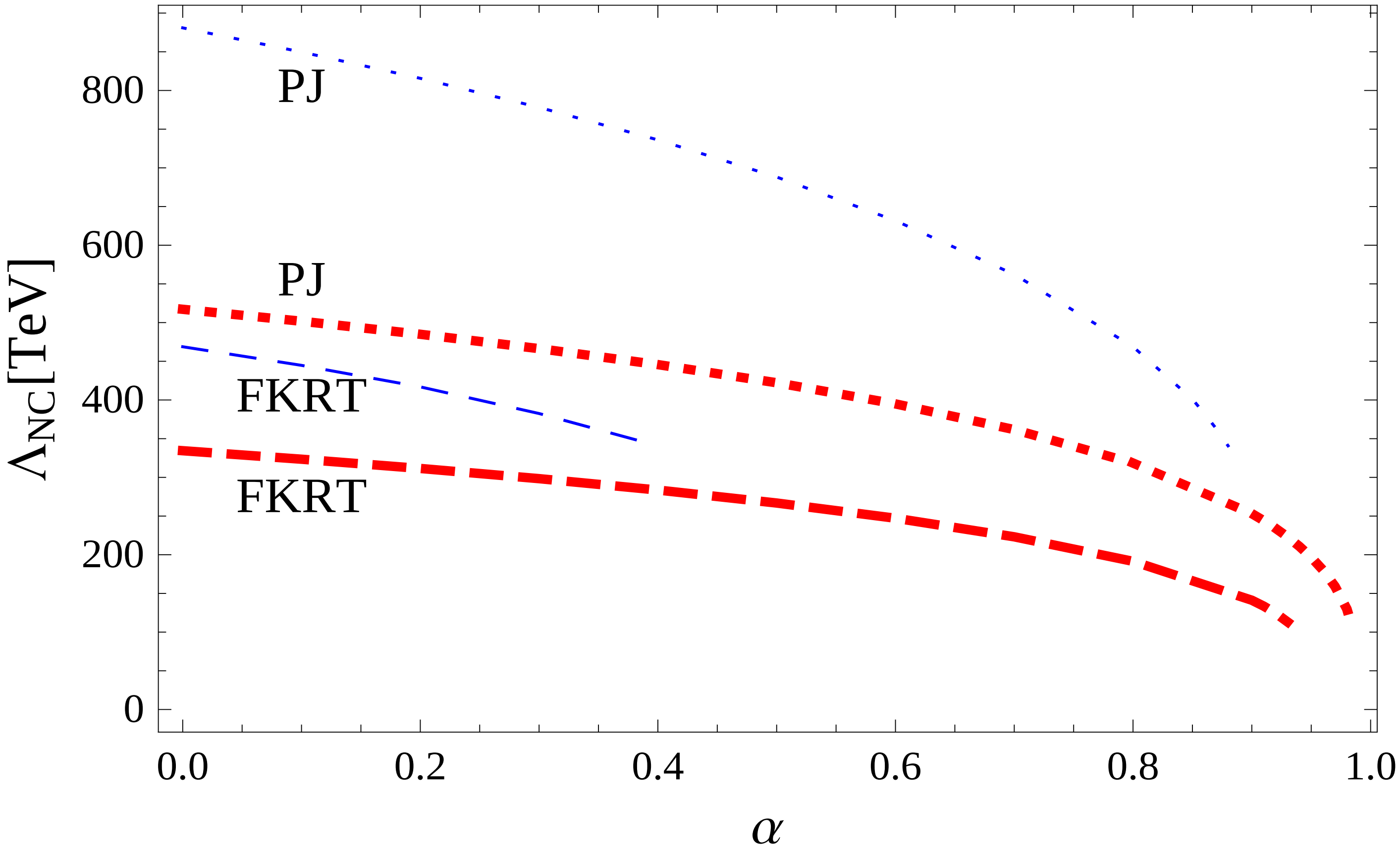
[R. Horvat, D. Kekez and J. Trampetić, *Spacetime NC and UHECR experiments* PRD 83, 065013 (2011)]



$\sigma[cm^2]$ versus $\Lambda_{\rm NC}$ for $E_\nu = 10^{10}$ GeV (thick lines) and $E_\nu = 10^{11}$ GeV (thin lines).

ν **N** $\longrightarrow \nu$ + *anything*

[R. Horvat, D. Kekez and J. Trampetić, *Spacetime NC and UHECR experiments* PRD 83, 065013 (2011)]



The intersections of our curves with the RICEn results (cf. Fig.1) as a function of the fraction of Fe nuclei in the UHE cosmic rays. The terminal point on each curve represents the highest fraction of Fe nuclei above which no useful information on Λ_{NC} can be inferred with our method.

θ-exact model properties / What do we want? / Wishing list?

* Direct neutrino-photon coupling in θ-exact NCFT
* Model contains enormous freedom due to the SW map
* No charge quantization problem
* Any gauge group and arbitrary matter repres.
* Covariant NCSM Yukawa couplings OK
* Unitarity is OK for: $\theta^{ij} \neq 0$, $\theta^{0i} = 0$;
* Covariant generalization of $\theta^{0i} = 0$ to:

$$\theta_{\mu\nu}\theta^{\mu\nu} \;=\; -\theta^2 = \frac{2}{\Lambda_{\rm NC}^4}\left(\vec{B}_\theta^2 - \vec{E}_\theta^2\right) > 0$$

* UV/IR mixing and/or Renormalisability $\leftrightarrow$ Quantum Gravity
* Holography distinct UV/IR connection $\rightarrow \Lambda_{\rm IR}/\Lambda_{\rm NC}/M_{Pl}/\Lambda_{\rm UV}$
* Neutrino dispersion relations in NC spacetime

Photon-neutrino interaction in θ-exact covariant NCFT

[P. Schupp and J. You, *UV/IR mixing in NC QED defined by Seiberg-Witten map*, *JHEP* **08** (2008) 107

$$S=\int\left(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+i\bar{\Psi}\left(\not{D}-m_\nu\right)\Psi\right)\,d^4x$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-i[A_\mu\stackrel{\star}{,}A_\nu];\ D_\mu\Psi=\partial_\mu\Psi-i[A_\mu\stackrel{\star}{,}\Psi]$$

At least three known methods for θ-exact computations:

- The closed formula derived using deformation quantization based on Kontsevich formality maps ,
- the relationship between open Wilson lines in the commutative and noncommutative picture and
- direct recursive computations using consistency conditions

direct deduction from the recursion and consistency relations:

$$\delta_\Lambda A_\mu\equiv i[\Lambda\stackrel{\star}{,}A_\mu]=A_\mu[a_\mu+\delta_\lambda a_\mu]-A_\mu[a_\mu]+\mathcal{O}(\lambda^2)\,,$$

$$\delta_\Lambda\Psi\equiv i[\Lambda\stackrel{\star}{,}\Psi]=\Psi[a_\mu+\delta_\lambda a_\mu,\psi+\delta_\lambda\psi]-\Psi[a_\mu,\psi]+\mathcal{O}(\lambda^2)\,,$$

$$\Lambda[[\lambda_1,\lambda_2],a_\mu]=$$

$$[\Lambda[\lambda_1,a_\mu]\stackrel{\star}{,}\Lambda[\lambda_2,a_\mu]]+i\delta_{\lambda_1}\Lambda[\lambda_2,a_\mu]-i\delta_{\lambda_2}\Lambda[\lambda_1,a_\mu]\,,$$

Photon-neutrino interaction in θ-exact covariant NCFT

With the ansatz

$$\Lambda = \hat{\Lambda}[a_\mu]\lambda = (1 + \hat{\Lambda}^1[a_\mu] + \hat{\Lambda}^2[a_\mu] + \mathcal{O}(a^3))\lambda\,,$$

$$\Psi = \hat{\Psi}[a_\mu]\psi = (1 + \hat{\Psi}^1[a_\mu] + \hat{\Psi}^2[a_\mu] + \mathcal{O}(a^3))\psi\,,$$

starting with the fermion field Ψ, at lowest order we have

$i[\lambda \stackrel{\star}{,} \psi] = \hat{\Psi}[\partial\lambda]\psi$ from

$$[f \stackrel{\star}{,} g] = i\theta^{ij}\left(\frac{\partial f(x)}{\partial x^i}\right)\frac{\sin(\frac{\partial_x\theta\partial_y}{2})}{\frac{\partial_x\theta\partial_y}{2}}\left(\frac{\partial g(y)}{\partial y^i}\right)\Bigg|_{x=y}$$

we observe that

$\hat{\Psi}[a_\mu] = -\theta^{ij}a_i \star_2 \partial_j$ where $f \star_2 g = f(x)\frac{\sin\frac{\partial_x\theta\partial_y}{2}}{\frac{\partial_x\theta\partial_y}{2}}g(y)\Big|_{x=y}$.

Gauge transformation Λ similar $\hat{\Lambda}^1 = -\frac{1}{2}\theta^{ij}a_i \star_2 \partial_j$.

Lowest order consistency relation:

$$-\partial_\mu(\tfrac{1}{2}\theta^{ij}a_i \star_2 \partial_j\lambda) - i[\lambda \stackrel{\star}{,} a_\mu] = A^2_\mu[a_\mu + \partial_\mu\lambda] - A^2_\mu[a_\mu]$$

Photon-neutrino interaction in θ-exact covariant NCFT

We obtain NC fields θ-exact SW maps expanded in terms of number of gauge fields

$$A_\mu = a_\mu - \frac{1}{2}\theta^{ij} a_i \star_2 (\partial_j a_\mu + f_{j\mu}) + \mathcal{O}(a^3)\,,$$
$$\Psi = \psi - \theta^{ij} a_i \star_2 \partial_j \psi + \mathcal{O}(a^2)\psi\,,$$
$$\Lambda = \lambda - \frac{1}{2}\theta^{ij} a_i \star_2 \partial_j \lambda + \mathcal{O}(a^2)\lambda\,,$$
$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$$
$$\mathcal{L} = \bar{\psi}\gamma^\mu [a_\mu \stackrel{\star}{,} \psi]$$
$$-(\theta^{ij} a_i \star_2 \partial_j \bar{\psi})(i\partial\!\!\!/ - m_\nu)\psi - \bar{\psi}(i\partial\!\!\!/ - m_\nu)(\theta^{ij} a_i \star_2 \partial_j \psi) + \bar{\psi}\mathcal{O}(a^2)\psi\,.$$

To extract Feynman rules in an appropriate form, we use the $\star$-product arithmetic property

$$[f \stackrel{\star}{,} g] = i\theta^{ij}\partial_i f \star_2 \partial_j g$$

to obtain the effective neutrino-photon Lagrangian density

Photon-neutrino interaction in θ-exact covariant NCFT

[R. Horvat, D. Kekez, P. Schupp, J. Trampetić and J. You, *Photon-neutrino interaction in θ-exact covariant noncommutative field theory*, Phys. Rev. **D84** (2011) 045004]

$$\mathcal{L} = -(\theta^{ij} a_i \star_2 \partial_j \bar{\psi})(i\not{\partial} - m_\nu)\psi - \bar{\psi}(i\not{\partial} - m_\nu)(\theta^{ij} a_i \star_2 \partial_j \psi) + i\bar{\psi}\gamma^\mu(\theta^{ij}\partial_i a_\mu \star_2 \partial_j \psi) + \mathcal{O}(a^2\bar{\psi}\psi)\,.$$

Generalized star product $\star_2$ turns into a function,

$$F(q,k) = \frac{\sin\frac{q\theta k}{2}}{\frac{q\theta k}{2}}$$

$$F(q,k) = F(k,q) = F(k,k') = F(k',k)\,, \quad q = k - k'$$

$$\Gamma^\mu = iF(q,k)\left[(\not{k} - m_\nu)\tilde{q}^\mu + (q\theta k)\gamma^\mu - \not{q}\tilde{k}^\mu\right], \;\; \tilde{k}^\mu = (\theta k)^\mu\,.$$

Test of SW map dependences of the action:

Start with the action for a neutral massless free fermion field

$$S_f = \int \bar{\psi}\gamma^\mu \partial_\mu \psi \, d^4x = \int \bar{\psi} \star \gamma^\mu \partial_\mu \psi \, d^4x \,,$$

and we lift the factors in the action via generalized SW maps $\hat{\Psi}[a_\mu]$ and $\hat{\Phi}[a_\mu]$ to NC status

$$S_f \rightarrow S_{f_{alt}} = \int \hat{\Psi}(\bar{\psi})\gamma^\mu \hat{\Phi}(\partial_\mu \psi) \, d^4x = \int \hat{\Psi}(\bar{\psi}) \star \gamma^\mu \hat{\Phi}(\partial_\mu \psi) \, d^4x \,.$$

and satisfy

$$\delta_\lambda(\hat{\Psi}(\bar{\psi})) = i[\hat{\Lambda}(\lambda) \overset{\star}{,} \hat{\Psi}(\bar{\psi})], \quad \delta_\lambda(\hat{\Phi}(\partial_\mu \psi)) = i[\hat{\Lambda}(\lambda) \overset{\star}{,} \hat{\Phi}(\partial_\mu \psi)]$$

$\hat{\Psi}(\psi) = \psi - \theta^{ij} a_i \star_2 \partial_j \psi$, and neutral fields $\delta\psi = \delta(\partial_\mu \psi) = 0$,

we notice that we can use the same map also for $\hat{\Phi}$:

$$\hat{\Phi}_{alt}(\partial_\mu \psi) = \hat{\Psi}(\partial_\mu \psi) = \partial_\mu \psi - \theta^{ij} a_i \star_2 (\partial_j \partial_\mu \psi) + \mathcal{O}(a^2)\psi \,.$$

Photon-neutrino interaction in θ-exact covariant NCFT

This action construction is quite unusual from the point of gauge theory, as it yields a covariant derivative term without introducing a covariant derivative:

$$S_{f_{alt1}} = \int \Big(i\bar{\psi}\gamma^\mu\partial_\mu\psi - i\left(\theta^{ij}\partial_j\bar{\psi}\star_2 a_i\right)\gamma^\mu\partial_\mu\psi + i\bar{\psi}\gamma^\mu\left(\theta^{ij}a_i\star_2\partial_\mu\partial_j\psi\right)\Big)\, d^4x \; + \mathcal{O}(a^2) \;\longrightarrow$$

$\Gamma^\mu_{f_{alt1}} = iF(q,k)\tilde{q}^\mu \not{k} \rightarrow$ drasticaly simplify loop computations !

Second choice for $\hat{\Phi}$:

$$\hat{\Phi}(\partial_\mu\psi) = D^\star_\mu\hat{\Psi}(\psi) = \partial_\mu\hat{\Psi}(\psi) - i[A_\mu \stackrel{\star}{,} \hat{\Psi}(\psi)] = \partial_\mu\psi - \theta^{ij}a_i\star_2\partial_j\partial_\mu\psi + \theta^{ij}f_{i\mu}\star_2\partial_j\psi + O(a^2)\psi\,,$$

based on the well-known NC QED-type covariant derivative gives

$$\Gamma^\mu_{f_{alt2}} = iF(q,k)\left[\not{k}\tilde{q}^\mu + (q\theta k)\gamma^\mu - \not{q}\tilde{k}^\mu\right].$$

Photon dispersion relation in a stellar plasma

$$q^2 \equiv \mathrm{E}_\gamma^2 - \mathbf{q}_\gamma^2 \overset{def.}{=} \omega_{pl}^2 \overset{calc.}{=} \mathcal{R}e\, \Pi_T(q_0, |\vec q| = 0) = \frac{e^2T^2}{9}\,,$$

$$|M_{\rm NC}(\gamma_{pl.} \to \bar\nu\nu)|^2 = 4e^2(F(q,k))^2(q\theta k)^2\left(q^2 + 2m_\nu^2\right)\,,$$

$$\Gamma_{\rm NC}(\gamma_{pl.} \to \bar\nu_{\binom{L}{R}}\nu_{\binom{L}{R}}) = \frac{\alpha\,\omega_{pl}}{4\pi}\int_0^\pi \sin\vartheta d\vartheta \int_0^{2\pi} d\phi \sin^2\frac{q\theta k}{2}$$

$$= \frac{1}{4}\alpha\,\omega_{pl}\int_{-1}^{1} dx\left[1 - (\cos Ax)J_0(B\sqrt{1-x^2})\right],$$

$$A \equiv \frac{c_{03}\,\omega_{pl}^2}{2\Lambda_{\rm NC}^2}, \quad B \equiv \frac{\omega_{pl}^2}{2\Lambda_{\rm NC}^2}\sqrt{c_{01}^2 + c_{02}^2}\,.$$

Rate for $\gamma_{pl} \to \bar{\nu}\nu$

Partial width:

$$\Gamma_{\rm NC}(\gamma_{\rm pl} \to \bar{\nu}_{\binom{\rm L}{\rm R}} \nu_{\binom{\rm L}{\rm R}}) = \frac{\alpha}{2}\omega_{pl}\left(1 - \frac{\sin\xi}{\xi}\right), \quad \xi = \frac{\omega_{pl}^2}{2\Lambda_{\rm NC}^2}$$

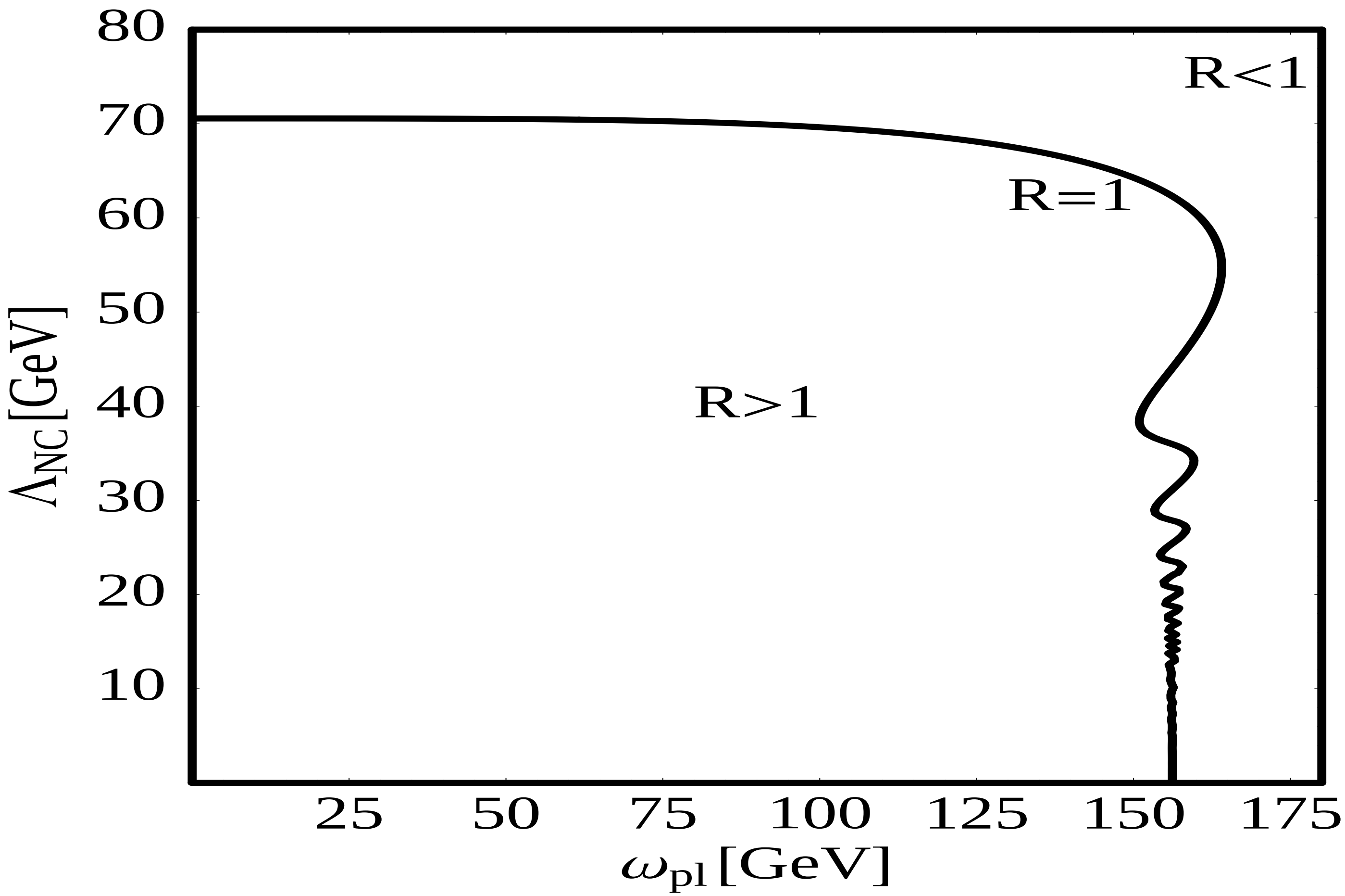
$$R \equiv \frac{\sum_{flavors} \Gamma_{\rm NC}(\gamma_{pl.} \to \bar{\nu}_L\nu_L + \bar{\nu}_R\nu_R)}{\sum_{flavors} \Gamma_{\rm SM}(\gamma_{pl.} \to \bar{\nu}_L\nu_L)}$$

$$= \frac{3 \cdot 48\pi^2\alpha^2}{(c_{\nu_e}^2 + c_{\nu_\mu}^2 + c_{\nu_\tau}^2)\, G_F^2\, \omega_{pl}^4}\left(1 - \frac{\sin\xi}{\xi}\right),$$

For ν_e, we have $c_{\rm v} = \frac{1}{2} + 2\sin^2\Theta_W$, while for ν_μ and ν_τ we have $c_{\rm v} = -\frac{1}{2} + 2\sin^2\Theta_W$. For $1 - \frac{\sin\xi}{\xi} = \frac{1}{6}\xi^2 - \frac{1}{120}\xi^4 + \ldots$, the ω_{pl} dependence vanishes.

$R/\Lambda_{\rm NC}/\omega_{pl}$

[R. Horvat, D. Kekez, P. Schupp, J. Trampetić and J. You, *Photon-neutrino interaction in θ-exact covariant noncommutative field theory*, Phys. Rev. **D84** (2011) 045004]



The plot of scale $\Lambda_{\rm NC}$ versus the plasmon frequency ω_{pl} with $R = 1$

Neutrino charge radius

[P. Minkowski, P. Schupp, and J. Trampetic, *Neutrino dipole moments and charge radii in NC spacetime*, EPJC 37 (2004) 123]; [R. Horvat, D. Kekez, P. Schupp, J. Trampetić and J. You, *Photon-neutrino interaction in θ-exact covariant NCFT*, Phys. Rev. **D84** (2011) 045004]

$$\Gamma(\gamma_{pl.} \to \bar{\nu}_{\rm L}\nu_{\rm L}) = \frac{\alpha}{144}\frac{q^6}{E_\gamma}\left|\langle r_\nu^2\rangle\right|^2 \to \left|\langle r_\nu^2\rangle\right| = \lim_{\omega_{pl}\to 0}\frac{6\sqrt{2}}{\omega_{pl}^2}\sqrt{1-\frac{\sin\xi}{\xi}}\,.$$

The limit $\omega_{pl} \to 0$ picks up only the first term that corresponds to the θ^1 result. This implies that there are no θ-exact corrections to the θ^1 charge radius which was obtained earlier

$$\left|\langle r_\nu^2\rangle\right| = \frac{\sqrt{3}}{\Lambda_{\rm NC}^2}\,.$$

Very stringent bound on $\langle r_{\nu_R}^2\rangle$ based on SN1987A . With $\langle r_{\nu_R}^2\rangle \lesssim 2\times 10^{-33}{\rm cm}^2$, one obtains $\Lambda_{\rm NC} \gtrsim 0.6$ TeV.

BBN from $f^{\pm}\,\nu_R \to f^{\pm}\,\nu_R$

[R. Horvat and J. Trampetic, *Constraining spacetime noncommutativity with primordial nucleosynthesis*, Phys. Rev. D **79** (2009) 087701]

Energy density of 3 light ν_R at nucleosynthesis time ($T \sim 1$MeV) is equivalent to the effective additional number of doublet neutrino specis ΔN_ν ($\lesssim 1$):

$$3\left(\frac{T_{\nu_R}}{T_{\nu_L}}\right)^4 \lesssim \Delta N_{\nu,max}\,, \qquad \frac{T_{\nu_R}}{T_{\nu_L}} = \left[\frac{g_{*S}(T_{\nu_L})}{g_{*S}(T_{dec})}\right]^{1/3}\,,$$

here g_{*S} are degrees of freedom specifying the entropy of the still interacting species

$$\sigma_{scatt}(f^{\pm}\,\nu_R \to f^{\pm}\,\nu_R) \simeq 36\,\alpha^2 E^2/\Lambda_{\rm NC}^4, \quad E \simeq 9T\,.$$

ν_R decouple at T_C when thermally averaged scatt. rate Γ_{scatt} and $\rm Hubble$ -expansion rate of the Universe in radiation dominated epoch are about equal $\Gamma_{scatt}(T_{dec}) \simeq H(T_{dec})$.

BBN from $f^{\pm}\,\nu_R \to f^{\pm}\,\nu_R$

$$\Gamma_{scatt}(T_{dec}) = < n_{scatt}\,\sigma_{scatt}\,v >, \quad n_{scatt} \simeq 0.18T^3$$
$$H(T_{dec}) \simeq 1.66 g_*^{1/2} T^2/M_{Pl}, \quad g_* \simeq g_{*S}\ .$$

This and σ_{scatt} gives

$$T_{dec} \simeq 0.5\,\alpha^{-2/3} M_{Pl}^{-1/3} \Lambda_{\rm NC}^{4/3}\ .$$

Imposing conservative bound $\Delta N_{\nu,max} = 1, (e, \mu, s)$ enforces constraint $T_{dec} > T_C$ (- critical temperature for deconfinment restoration phase transition);

$$T_{dec} \lesssim 200{\rm MeV} \Longrightarrow \Lambda_{\rm NC} \gtrsim 3{\rm TeV}.$$

For $\Delta N_{\nu,max} < 0.2,$ (all charged lepton and quarks) we have

$$T_{dec} \lesssim 300{\rm GeV} \Longrightarrow \Lambda_{\rm NC} \gtrsim 10^3{\rm TeV}.$$

BBN from $\Gamma(\gamma_{pl} \to \bar{\nu}\nu)$

The RH neutrino is commonly considered to decouple at the temperature T_{dec} satisfying the condition with the Hubble expansion rate

$$\Gamma(\gamma_{pl.} \to \bar{\nu}_R \nu_R) \simeq H(T_{dec}) \simeq 1.66\, g_* \frac{T_{dec}^2}{M_{Pl}}, \quad \omega_{pl} = \frac{eT_{dec}}{3}\, g_*^{ch},$$

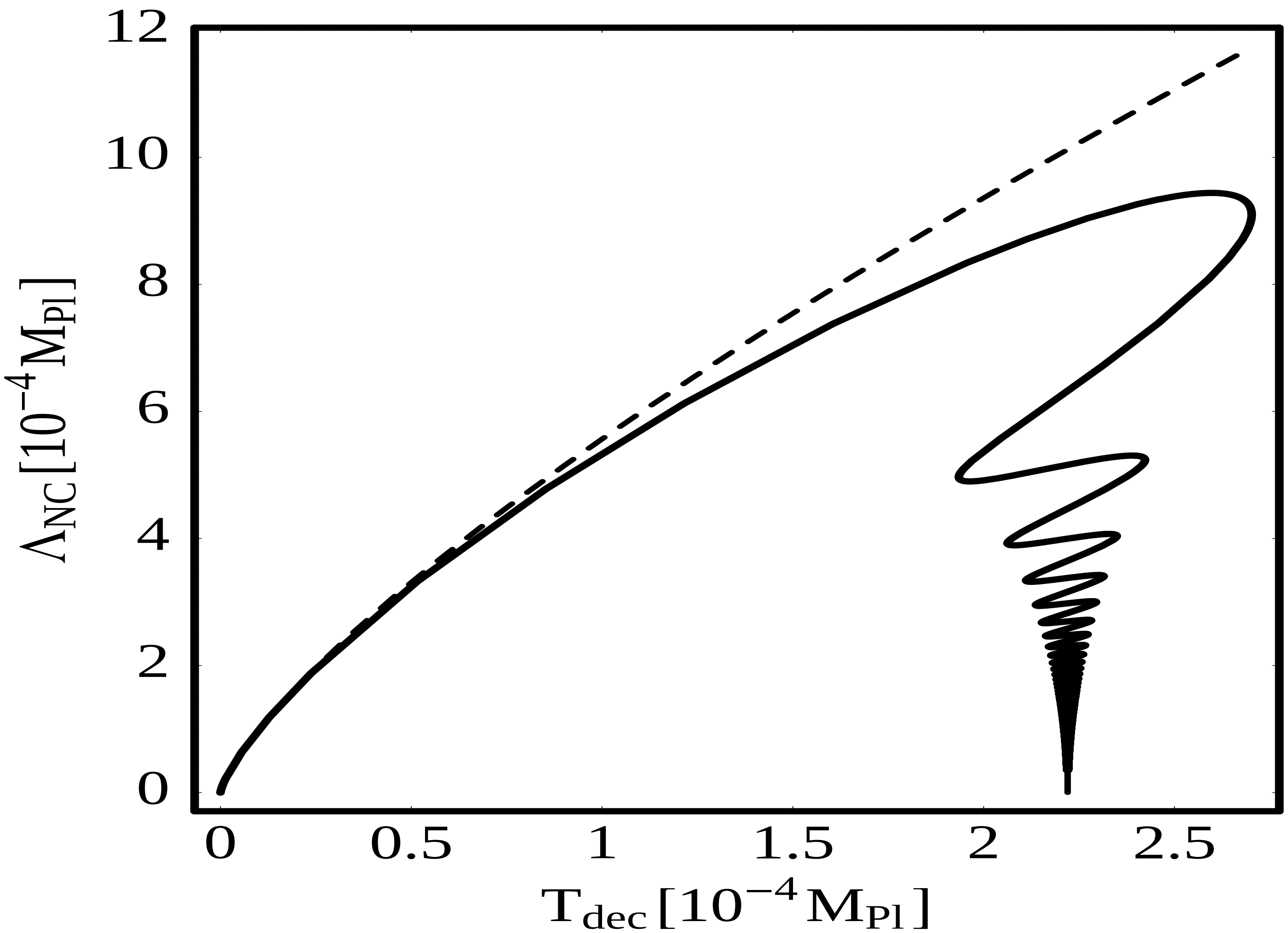
Computing the decoupling temperature T_{dec} based on the assumption that the decay rate is solely due to the NC effects and comparing with lower bounds on T_{dec} that can be inferred from observational data, we can determine lower bounds on the NC scale $\Lambda_{\rm NC}$ from

$$T_{dec} \simeq \frac{M_{pl} e^3 g_*^{ch}}{39.84 \pi g_*} \left(1 - \frac{\sin\xi}{\xi}\right), \quad \xi = \frac{e^2 (g_*^{ch})^2 T_{dec}^2}{18 \Lambda_{\rm NC}^2}$$

$$T_{dec} \simeq 2.22 \times 10^{-4} M_{Pl} \left(1 - \frac{\sin\xi}{\xi}\right) \stackrel{\xi \to 0}{\Longrightarrow} \Lambda_{\rm NC} > 3.68(887) \mathrm{TeV}.$$

BBN from $\Gamma(\gamma_{pl} \rightarrow \bar{\nu}\nu)$

[R. Horvat, D. Kekez, P. Schupp, J. Trampetić and J. You, *Photon-neutrino interaction in θ-exact covariant noncommutative field theory*, Phys. Rev. **D84** (2011) 045004]



The plot of the scale $\Lambda_{\rm NC}$ versus T_{dec} for perturbative/exact solution (dashed/full curve).

Quantum properties I: Holography and UV/IR mixing

[R. Horvat and J. Trampetić, *Constraining NCFT with holography*, *JHEP* **01** (2011) 112]

UV/IR mixing effects: an interpretation that a quantum in NC GFT gauge theory is a straight string connecting two opposite charges. Phenomenological effects of the UV completion (for a large class of more general QFTs above the UV cutoff $\Lambda_{\rm UV}$) can be quite successfully modeled by a threshold value $\Lambda_{\rm UV}$. Combine uncertainty relations:

$$\Delta x^\mu \Delta x^\nu > \theta^{\mu\nu} \;\;\&\;\; \Delta x^i \Delta p^j \geq (1/2)\delta^{ij}$$

and switching to the language of effective QFT with

$$(\Delta p)_{max} \sim \Lambda_{\rm UV},\, (\Delta x)_{max} \sim \Lambda_{\rm IR}^{-1},\, \Lambda_{\rm NC}^{-2} \sim |\theta|,$$

immediately produces: $\Lambda_{\rm UV}\Lambda_{\rm IR} \sim \Lambda_{\rm NC}^2$, (*)

Theory thus becomes an effective QFT with UV and IR cutoffs.

Quantum properties I: Holography and UV/IR mixing

From the absolute Bekenstein-Hawking bound $S_{\rm BH} \sim L^2 M_{Pl}^2$, and properties of effective QFT in a box of size L (providing an IR cutoff), with respect to black hole physics, a stringent constraint is obeyed

$$\Lambda_{\rm UV}^3 \Lambda_{\rm IR}^{-3} \lesssim M_{Pl}^{3/2} \Lambda_{\rm IR}^{-3/2} \sim S_{\rm BH}^{3/4}. \qquad (**)$$

$$(*)\,\&\,(**) \rightarrow \Lambda_{\rm IR} \gtrsim \Lambda_{\rm NC} \left(\frac{\Lambda_{\rm NC}}{M_{Pl}}\right)^{1/3}, \; \Lambda_{\rm UV} \lesssim \Lambda_{\rm NC} \left(\frac{M_{Pl}}{\Lambda_{\rm NC}}\right)^{1/3} \; (***)$$

Considering the muon

$$\Delta(g_\mu - 2) \sim \frac{\alpha}{\pi} \left[\left(\frac{m_\mu}{\Lambda_{\rm UV}}\right)^2 + \left(\frac{\Lambda_{\rm IR}}{m_\mu}\right)^2 \right]$$

Note that because of $(***)$ one is no longer allowed to set $\Lambda_{\rm UV}$ and $\Lambda_{\rm IR}$ independently, that is, no longer simultaneously

$\Lambda_{\rm UV} \rightarrow \infty$ and $\Lambda_{\rm IR} \rightarrow 0$!

Quantum properties I: Holography and UV/IR mixing

Some numerics:

For $\Lambda_{\rm NC} \gtrsim m_\mu \longrightarrow \Delta(g_\mu - 2)_{\rm IR} \sim \frac{\alpha}{\pi}\left(\frac{\Lambda_{\rm IR}}{m_\mu}\right)^2$, this together with $\frac{g_\mu - 2}{2}(\mathrm{Exp} - \mathrm{SM}) = (22 - 26) \times 10^{-10}$, gives

$$m_\mu \lesssim \Lambda_{\rm NC} \lesssim 0.1 \text{ TeV}$$

$\Lambda_{\rm IR} \lesssim 10^{-1}$ MeV and 10^5 MeV $\lesssim \Lambda_{\rm UV} \lesssim 10^5$ TeV

For $\Lambda_{\rm NC} \lesssim m_\mu \longrightarrow \Delta(g_\mu - 2)_{\rm IR} \sim \frac{\alpha}{\pi}\left(\frac{m_\mu}{\Lambda_{\rm UV}}\right)^2$, gives

$$10^{-4} \text{ MeV} \lesssim \Lambda_{\rm NC} \lesssim \mathrm{m}_\mu$$

$\Lambda_{\rm UV} \gtrsim 10^2$ GeV and 10^{-1} MeV $\lesssim \Lambda_{\rm IR} \lesssim 10^{-13}$ MeV

Q. properties I: Reheating phase after inflation and UV/IR mixing

[R. Horvat and J. Trampetić, *A bound on the scale of spacetime noncommutativity from the reheating phase after inflation.*, *Phys. Lett.* **B 710** (2012) 210-222]

The effective theory QFT with the UV and IR cutoffs obey

$$\Lambda_{\rm UV}\Lambda_{\rm IR} \sim \Lambda^2_{\rm NC}. \quad (*)$$

To describe a system at a temperature T in a box of size L we employ a specific form of UV/IR relationship and derive an upper bound on the NC parameter. Choosing θ to lie in the (1, 2) plane, $\theta^{1,2} = -\theta^{2,1} \equiv \theta$, a particle moving inside the NC plane with momentum P along the one axis, has a spatial extension of size $|\theta P|$ along the other. The FT of NC spacetime with (*) describe a thermal system L if

$$|\theta p|_{max} = \frac{1}{\sqrt{2}}\frac{\Lambda_{\rm UV}}{\Lambda^2_{\rm NC}} \lesssim L, \;\; \Lambda_{\rm UV} \gtrsim T, \;\; T \gg L^{-1}.$$

giving upper bound on $\Lambda_{\rm NC}$

$$\Lambda_{\rm NC} \gtrsim (2)^{-1/4} L^{-1/2} T^{1/2} \; .$$

Q. properties I: Reheating phase after inflation and UV/IR mixing

With Hubble distance choice $H = L^{-1}$ the bound becomes

$$\Lambda_{\rm NC} \gtrsim \left(\frac{4\pi^3}{90} g_*(T_{RH})\right)^{1/4} \frac{T_{RH}^{3/2}}{M_{Pl}^{1/2}} \, .$$

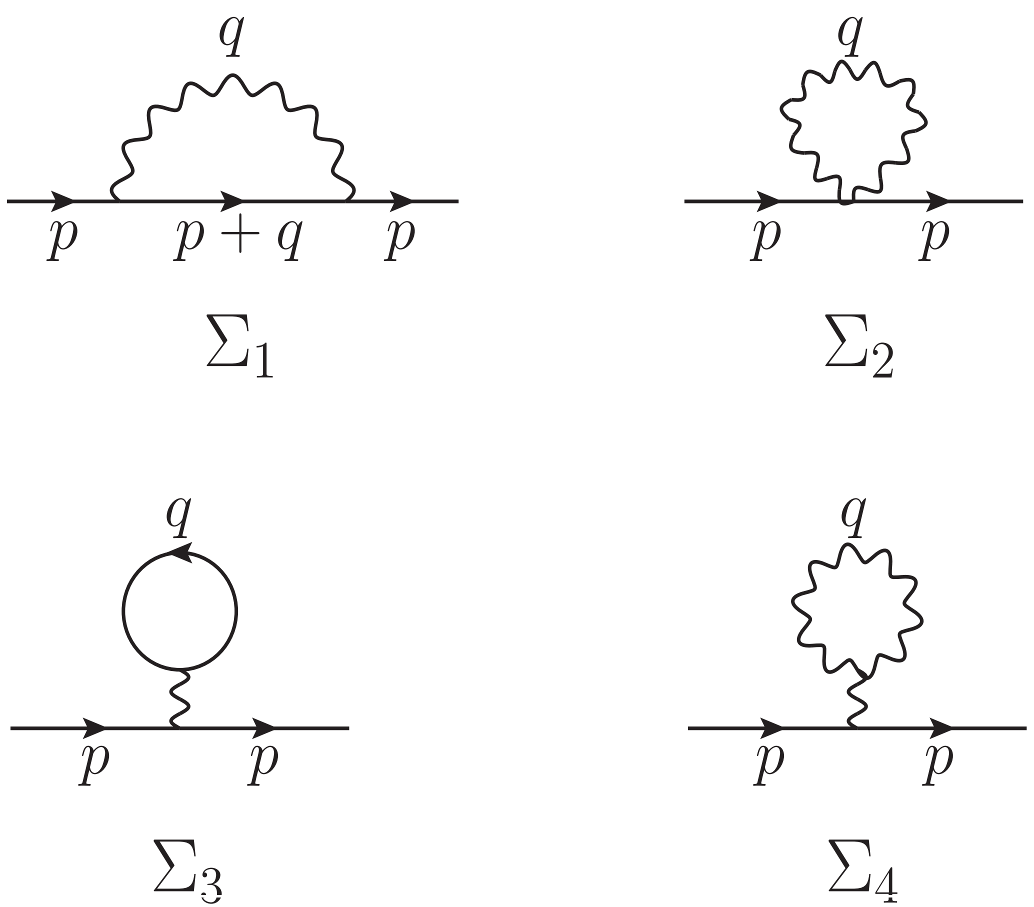
The main reason of why the reheating temperature should not be too high (thus weakening our bounds) is that one inevitably overproduces gravitinos in supergravity theories. The limit from gravitino overproduction is $T_{RH} \lesssim 10^9 - 10^{10}$ GeV. Taking the effective number of degrees of freedom at the reheating temperature as for the MSSM $(g_*(T_{RH}) = 915/4)$ one obtains for the maximum T_{RH}

$$\Lambda_{\rm NC} \gtrsim 500 \ {\rm TeV} \, .$$

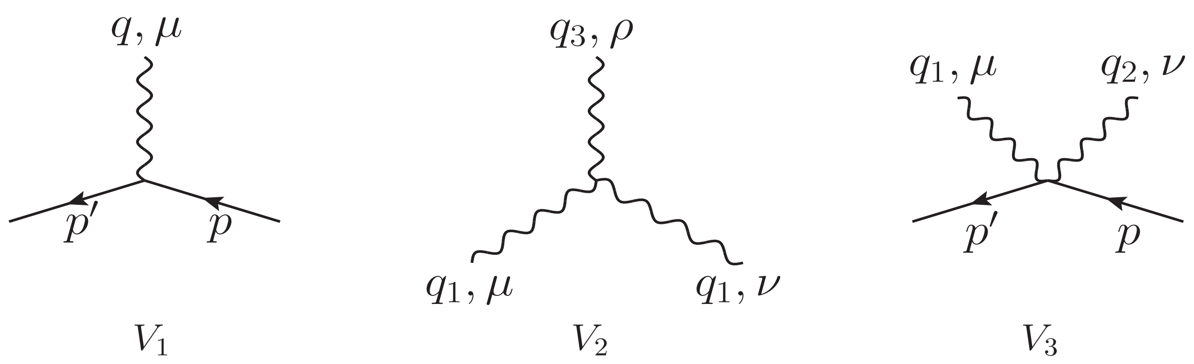
Quantum properties II: Neutrino self energy

[R. Horvat, A. Ilakovac, J. Trampetić and J. You, *On UV/IR mixing in noncommutative gauge field theories*, JHEP **12** (2011) 081], [R. Horvat, A. Ilakovac, P. Schupp, J. Trampetić and J. You, *Neutrino propagation in noncommutative spacetimes*, JHEP 04 (2012) 108]

Neutrino self energy in the θ-exact $U_\star(1)$ NCGFT/SW map.



Quantum properties II: Neutrino self energy



$q = p - p'$ $\quad$ q_1, q_2, q_3 incoming $\quad$ q_1, q_2 incoming

$$
\begin{aligned}
V_1^{\mu} &= -iF(q,p)[\gamma^\mu q\theta p + \not{p}\tilde{q}^\mu - \not{q}\tilde{p}^\mu]P_{L,R}, \quad F(q,p) = \frac{\sin\frac{1}{2}q\theta p}{\frac{1}{2}q\theta p}, \quad \tilde{q}^\mu = (\theta q)^\mu \\
V_2^{\mu\nu\rho} &= -2\sin\frac{1}{2}q_1\theta q_2[(q_1-q_2)^\rho g^{\mu\nu} + (q_2-q_3)^\mu g^{\nu\rho} + (q_3-q_1)^\nu g^{\rho\mu}] \\
&- 2F(q_1,q_2)\Big[\theta^{\mu\nu}(q_2q_3q_1^\rho - q_1q_3q_2^\rho) + \theta^{\mu\rho}(q_2q_3q_1^\nu - q_1q_2q_3^\nu) \\
&\quad +\theta^{\nu\rho}(q_1q_3q_2^\mu - q_1q_2q_3^\mu) \\
&- g^{\mu\nu}(q_2^2\tilde{q}_1^\rho + q_1^2\tilde{q}_2^\rho) - g^{\mu\rho}(q_1^2\tilde{q}_3^\nu + q_3^2\tilde{q}_1^\nu) - g^{\nu\rho}(q_3^2\tilde{q}_2^\mu + q_2^2\tilde{q}_3^\mu) \\
&+ q_3^\rho(\tilde{q}_2^\mu q_3^\nu + \tilde{q}_1^\nu q_3^\mu) + q_2^\nu(\tilde{q}_1^\rho q_2^\mu + \tilde{q}_3^\mu q_2^\rho) + q_1^\mu(\tilde{q}_2^\rho q_1^\nu + \tilde{q}_3^\nu q_1^\rho)\Big]
\end{aligned}
$$

Quantum properties II: Neutrino self energy

$$
\begin{aligned}
V_3^{\mu\nu}(p_1,p_2,q_1,q_2) &= 4i\frac{\sin\frac{p_1\theta q_1}{2}\sin\frac{p_2\theta q_2}{2}}{p_1\theta q_1}\tilde{q}_1^\mu\gamma^\nu - 4i\frac{\sin\frac{p_1\theta q_1}{2}\sin\frac{p_2\theta q_2}{2}}{p_2\theta q_2}\tilde{q}_2^\nu\gamma^\mu \\
&- 2i\frac{\sin\frac{q_1\theta q_2}{2}\sin\frac{p_1\theta p_2}{2}}{p_1\theta p_2}(2\gamma^\nu\tilde{p}_2^\mu - \not{p}_2\theta^{\mu\nu}) - 4i\frac{\sin\frac{p_1\theta q_1}{2}\sin\frac{p_2\theta q_2}{2}}{p_1\theta q_1 p_2\theta q_2}(\not{p}_2+\not{q}_2)\tilde{q}_1^\mu\tilde{q}_2^\nu \\
&+ 2i\not{q}_2\Bigg[\frac{\sin\frac{q_1\theta q_2}{2}\sin\frac{p_1\theta p_2}{2}}{p_1\theta p_2 k_1\theta q_2}(p_2\theta q_1\theta^{\mu\nu} - 2\tilde{p}_2^\mu\tilde{q}_1^\nu) \\
&- \frac{\sin\frac{p_1\theta q_2}{2}\sin\frac{p_2\theta q_1}{2}}{p_1\theta q_2 p_2\theta q_1}2(\tilde{p}_2-\tilde{q}_1)^\mu\tilde{q}_1^\nu + \frac{\sin\frac{p_1\theta q_2}{2}\sin\frac{p_2\theta q_1}{2}}{p_1\theta q_2}\theta^{\mu\nu} \\
&+ \left(\frac{\sin\frac{p_2\theta q_1}{2}\sin\frac{p_1\theta q_2}{2}}{p_2\theta q_2 p_1\theta q_2} + \frac{\sin\frac{p_1\theta p_2}{2}\sin\frac{q_1\theta q_2}{2}}{p_2\theta q_2 q_1\theta q_2}\right)(2\tilde{q}_1^\nu\tilde{p}_2^\mu + \theta^{\mu\nu}q_1\theta p_2 - \tilde{q}_1^\mu\tilde{q}_1^\nu)\Bigg] \\
&+ 2i\not{q}_1\Bigg[\frac{\sin\frac{q_2\theta q_1}{2}\sin\frac{p_1\theta p_2}{2}}{p_1\theta p_2 q_2\theta q_1}(2\tilde{p}_2^\mu\tilde{q}_2^\nu - p_2\theta q_2\theta^{\mu\nu}) \\
&+ \frac{\sin\frac{p_1\theta q_1}{2}\sin\frac{p_2\theta q_2}{2}}{p_1\theta q_1 p_2\theta q_2}2(\tilde{p}_2+\tilde{q}_2)^\mu\tilde{q}_2^\nu - \frac{\sin\frac{p_1\theta q_1}{2}\sin\frac{p_2\theta q_2}{2}}{p_1\theta q_1}\theta^{\mu\nu} \\
&- \left(\frac{\sin\frac{p_2\theta q_2}{2}\sin\frac{p_1\theta q_1}{2}}{p_2\theta q_1 p_1\theta q_1} + \frac{\sin\frac{p_2\theta p_1}{2}\sin\frac{q_2\theta q_1}{2}}{p_2\theta q_1 q_2\theta q_1}\right)(2\tilde{q}_2^\nu\tilde{p}_2^\mu + \theta^{\mu\nu}q_2\theta p_2 + \tilde{q}_2^\mu\tilde{q}_2^\nu)\Bigg] \\
&+ \{p_1 \leftrightarrow p_2 \text{ and } \mu\leftrightarrow\nu\}
\end{aligned}
$$

Quantum properties II: Neutrino self energy

Schwinger parameterization:

$$\frac{1}{a^n} = \frac{1}{\Gamma(n)} \int_0^\infty e^{a\alpha} \alpha^{n-1} d\alpha$$

$$\frac{1}{a_1^{n_1} a_2^{n_2}} = \frac{1}{\Gamma(n_1)\Gamma(n_2)} \int_0^\infty \int_0^\infty e^{-a_1\alpha_1 - a_2\alpha_2} \alpha_1^{n_1-1} \alpha_2^{n_2-1} d\alpha_1 d\alpha_2$$

Feynman parameterization:

$$\frac{1}{a_1^{n_1} a_2^{n_2}} = \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} \int_0^1 \frac{x^{n_1-1}(1-x)^{n_2-1} dx}{[a_1 x + a_2(1-x)]^{n_1+n_2}}$$

"HQEFT" parameterization (Grozin):

Used to simplify a product of propagator denominators linear (a_1) and quadratic in loop momenta. Obtained from Schwinger parameterization putting $\alpha_1 = y\alpha$ and $\alpha_2 = \alpha$ (now both y and α are dimensionfull parameters) and integrating over α

$$\frac{1}{a_1^{n_1} a_2^{n_2}} = \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} \int_0^\infty \frac{y^{n_1-1} dy}{a_1 y + a_2}$$

Quantum properties II: Neutrino self energy

Amplitudes and results in model 1 for $D = 4 - \epsilon$:

$$\begin{aligned}\Sigma_1 &= \int \frac{\mu^{4-D} d^D q}{(2\pi)^D} \underbrace{\frac{(2 - e^{iq\theta p} - e^{-iq\theta p})}{(q\theta p)^2}}_{(F(p,q))^2} \frac{1}{q^2} \frac{1}{(q+p)^2} \\ &\times \Big[(q\theta p)^2[(4-D)2(\not{p}+\not{q})] + (q\theta p)[\not{\tilde{q}}(2p^2+2pq) - \not{\tilde{p}}(2q^2+2pq)] \\ &+[\not{p}(\tilde{q}^2(p^2+2qp) - q^2(\tilde{p}^2+2\tilde{p}\tilde{q})) + \not{q}(\tilde{p}^2(q^2+2pq) - p^2(\tilde{q}^2+2\tilde{p}\tilde{q}))]P_{L,R}\Big]\end{aligned}$$

$$\Sigma_1 = \gamma_\mu \left[p^\mu A + (\theta\theta p)^\mu \frac{p^2}{(\theta p)^2} B \right],$$

$$\tilde{q}^\mu = (\theta q)^\mu = \theta^{\mu\nu} q_\nu, \qquad \tilde{\tilde{q}}^\mu = (\theta\theta q)^\mu = \theta^{\mu\nu}\theta_{\nu\rho} q^\rho$$

Quantum properties II: Neutrino self energy

$$\Sigma_1 = -\frac{1}{(4\pi)^{\frac{D}{2}}} 2\tilde{\not{p}} p^2 \left(\frac{\mathrm{tr}\theta\theta}{\tilde{p}^2} + \frac{2\tilde{\tilde{p}}^2}{\tilde{p}^4}\right) \Bigg[(p^2)^{\frac{D}{2}-2} \Gamma\left(2 - \frac{D}{2}\right) B\left(\frac{D}{2} - 1, \frac{D}{2}\right)$$

$$- 2 \int_0^1 dx (1-x) \left(x(1-x)p^2\right)^{\frac{D}{4}-1} 2^{\frac{D}{2}-2} (\tilde{p}^2)^{1-\frac{D}{4}} K_{2-\frac{D}{2}} \left[\left(x(1-x)p^2\tilde{p}^2\right)^{\frac{1}{2}} \right] \Bigg]$$

$$- \frac{1}{(4\pi)^{\frac{D}{2}}} \Bigg\{ 2 \left(\not{p} \left(1 - \frac{D}{2}\right) + \frac{p^2 \tilde{\tilde{\not{p}}}}{\tilde{p}^2} - \frac{\mathrm{tr}\theta\theta}{2} \frac{p^2 \tilde{\not{p}}}{\tilde{p}^2} \right) - \frac{\tilde{\not{p}}}{\tilde{p}^4} \left(\tilde{\tilde{p}}^2 p^2 - \tilde{p}^4\right) \Bigg\}$$

$$\cdot \frac{\pi}{2 \sin \frac{D\pi}{2}} \int_0^1 dx (1-x) (\tilde{p}^2)^{2-\frac{D}{2}}$$

$$\cdot \Bigg[\left(x(1-x)p^2\tilde{p}^2\right)^{\frac{D}{2}-1} \Gamma\left(\frac{1}{2}\right) {}_1\tilde{F}_2 \left(\frac{1}{2}; \frac{3}{2}, \frac{D}{2}; \frac{x(1-x)p^2\tilde{p}^2}{4}\right)$$

$$- 2^{D-2} \Gamma\left(\frac{3-D}{2}\right) {}_1\tilde{F}_2 \left(\frac{3-D}{2}; \frac{4-D}{2}, \frac{5-D}{2}; \frac{x(1-x)p^2\tilde{p}^2}{4}\right) \Bigg].$$

Quantum properties II: Neutrino self energy

$$
\begin{aligned}
A &= \frac{-1}{(4\pi)^2}\left[p^2\left(\frac{\mathrm{tr}\theta\theta}{(\theta p)^2}+2\frac{(\theta\theta p)^2}{(\theta p)^4}\right)A_1+\left(1+p^2\left(\frac{\mathrm{tr}\theta\theta}{(\theta p)^2}+\frac{(\theta\theta p)^2}{(\theta p)^4}\right)\right)A_2\right], \\
A_1 &= \frac{2}{\epsilon}+\ln(\mu^2(\theta p)^2)+\ln(\pi e^{\gamma_E})+\sum_{k=1}^{\infty}\frac{\left(p^2(\theta p)^2/4\right)^k}{\Gamma(2k+2)}\left(\ln\frac{p^2(\theta p)^2}{4}+2\psi_0(2k+2)\right), \\
A_2 &= -\frac{(4\pi)^2}{2}B=-2 \\
+ & \sum_{k=0}^{\infty}\frac{\left(p^2(\theta p)^2/4\right)^{k+1}}{(2k+1)(2k+3)\Gamma(2k+2)}\left(\ln\frac{p^2(\theta p)^2}{4}-2\psi_0(2k+2)-\frac{8(k+1)}{(2k+1)(2k+3)}\right),
\end{aligned}
$$

Denote with $s_{1,2}$ the scale-independent θ-ratios

$$
s_1 = p^2\frac{\mathrm{tr}\theta\theta}{(\theta p)^2}, \quad s_2 = p^2\frac{(\theta\theta p)^2}{(\theta p)^4},
$$

From above clearly we do need non-local counter terms to eliminate divergences!

Quantum properties II: Neutrino self energy

The series are always convergent for finite arguments:

$$\begin{aligned}
A_1 &\simeq \frac{2}{\epsilon} + \ln\left(\pi e^{\gamma_{\rm E}} \mu^2 (\theta p)^2\right) \\
&- \frac{11}{72} p^2 (\theta p)^2 \left(1 + \frac{137}{8800}(p^2(\theta p)^2) + \frac{33}{313600}(p^2(\theta p)^2)^2 + \frac{7129}{17882726400}(p^2(\theta p)^2)^3 + \ldots\right) \\
&+ \gamma_{\rm E} \left(1 + \ln\left(\frac{p^2(\theta p)^2}{4}\right)^{\frac{1}{2\gamma_E}}\right) \frac{p^2(\theta p)^2}{12}\left(1 + \frac{(p^2(\theta p)^2)}{80} + \frac{(p^2(\theta p)^2)^2}{13440} + \frac{(p^2(\theta p)^2)^3}{3870720} + \ldots\right), \\
A_2 &= -8\pi^2 B \\
&\simeq 2 + \frac{7}{18} p^2 (\theta p)^2 \left(1 + \frac{71}{8400} p^2 (\theta p)^2 + \frac{1103}{21952000}(p^2(\theta p)^2)^2 + \frac{3587}{19914854400}(p^2(\theta p)^2)^3 + \ldots\right) \\
&- 2\gamma_{\rm E} \left(1 + \ln\left(\frac{p^2(\theta p)^2}{4}\right)^{\frac{1}{2\gamma_{\rm E}}}\right) \frac{p^2(\theta p)^2}{12}\left(1 + \frac{p^2(\theta p)^2}{120} + \frac{(p^2(\theta p)^2)^2}{22400} + \frac{(p^2(\theta p)^2)^3}{6773760} + \ldots\right),
\end{aligned}$$

where $\gamma_{\rm E} \simeq 0.577216$ is Euler's constant. It is to be noted here that the spinor structure proportional to $\gamma_\mu (\theta p)^\mu$ is missing in the final result,

Elimination of divergences in model 1:

Since $\theta^{0i} \neq 0$ makes a NC theory nonunitary, we can, without loss of generality, chose θ to lie in the (1, 2) plane

$$\theta^{\mu\nu} = \frac{1}{\Lambda_{\rm NC}^2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \frac{\text{tr}\theta\theta}{(\theta p)^2} + 2\frac{(\theta\theta p)^2}{(\theta p)^4} = 0, \; \forall p,$$

than Σ_1 free of divergences becomes:

$$\Sigma_1 = \frac{-1}{(4\pi)^2}\gamma_\mu \left[p^\mu \left(1 + \frac{\text{tr}\theta\theta}{2}\frac{p^2}{(\theta p)^2} \right) - 2(\theta\theta p)^\mu \frac{p^2}{(\theta p)^2} \right] A_2.$$

showing spontaneous breaking of Lorentz symmetry!

Quantum properties II: Neutrino self energy

To remind on the form of UV/IR mixing term in model 1:

$$\Sigma_{\rm UV/IR} = -\not{p}\, p^2 \left(\frac{{\rm tr}\theta\theta}{(\theta p)^2} + 2\frac{(\theta\theta p)^2}{(\theta p)^4} \right) \cdot \frac{2}{(4\pi)^2} \ln |\mu(\theta p)|.$$

General properties of neutrino self-energy in model 1:

⋆ Free from the $1/\epsilon$ and the UV/IR mixing divergences,

⋆ well-behaved in the infrared, in the $\theta \to 0$,

⋆ well-behaved in the $\theta p \to 0$ limit,

⋆ the first example where this two limits are not degenerate

⋆ UV/IR mixing receives a new form where the commutative limit and the limit of zero size of the extended object are fully disentangled.

⋆ we have obtained the final result in an analytic, closed-form!

Quantum properties II: Neutrino self energy

Neutrino dispersion relation in model 1

$$\frac{1}{\not\Sigma} = \frac{1}{\not p - \Sigma_{1-loop_M}} = \frac{\not\Sigma}{\Sigma^2} \, .$$

$$\Sigma^2 = p^2 \left[\hat{A}_2^2 \left(\frac{p^4}{p_r^4} + 2\frac{p^2}{p_r^2} + 5 \right) - \hat{A}_2 \left(6 + 2\frac{p^2}{p_r^2} \right) + 1 \right] = p^2 \, \Sigma' \, ,$$

$$\Sigma^2 = 0 \longrightarrow p^2 = 0 \text{ \& } \Sigma' = 0; \;\; (p_r^2 = p_1^2 + p_2^2),$$

$$\frac{p^2}{p_r^2} = \frac{1}{\hat{A}_2} \left[\left(1 - \hat{A}_2\right) \pm 2 \left(\hat{A}_2 - \hat{A}_2^2 \right)^{\frac{1}{2}} \right], \, \hat{A}_2 = e^2 A_2/(4\pi)^2 = -B/2,$$

are direction dependent, i.e. birefringent.
Two limits of $y := p^2 p_r^2 / \Lambda_{\mathrm{NC}}^4$, i.e. $y \rightarrow 0$, and $y \rightarrow \infty$.

Quantum properties II: Neutrino self energy

The low-energy regime: $p^2 p_r^2 \ll \Lambda_{\rm NC}^4$

For $y \ll 1$ we simply set $\hat{A}_2$ to its zeroth order value $e^2/8\pi^2$

$$p^2 \sim \left(\left(\frac{8\pi^2}{e^2} - 1 \right) \pm 2 \left(\frac{8\pi^2}{e^2} - 1 \right)^{\frac{1}{2}} \right) \cdot p_r^2 \simeq (859 \pm 59) \cdot p_r^2 \,,$$

defines two (approximate) positive and real zero points.

Dispersion relations in model 1 and speed of light?

The maximal attainable velocity of neutrinos - quantum effect

$$\frac{\mathrm{v}_{max}}{c} = \frac{dE}{d|\vec{p}|} \sim \sqrt{1 + (859 \pm 59) \sin^2 \vartheta} \;\; \longrightarrow$$

Spontaneous breaking of causality at quantum level!

Quantum properties II: Neutrino self energy

The high-energy regime: $p^2 p_r^2 \gg \Lambda_{\rm NC}^4$

$$
\begin{aligned}
A_2 &= \frac{\pi}{2\sin\frac{\epsilon}{2}\pi}\left(\frac{p_r^2}{\Lambda_{\rm NC}^4}\right)^{\frac{\epsilon}{2}}\int dx\,(1-x) \\
&\cdot \left[\left(-\frac{x(1-x)p^2p_r^2}{\Lambda_{\rm NC}^4}\right)^{1-\frac{\epsilon}{2}}\Gamma\left(\frac{1}{2}\right){}_1\tilde{F}_2\left(\frac{1}{2};\frac{3}{2},2-\frac{\epsilon}{2};-\frac{x(1-x)p^2p_r^2}{4\Lambda_{\rm NC}^4}\right)\right. \\
&- \left. 2^{2-\epsilon}\Gamma\left(-\frac{1}{2}+\frac{\epsilon}{2}\right){}_1\tilde{F}_2\left(-\frac{1}{2}+\frac{\epsilon}{2};\frac{\epsilon}{2},\frac{1}{2}+\frac{\epsilon}{2};-\frac{x(1-x)p^2p_r^2}{4\Lambda_{\rm NC}^4}\right)\right] \\
&= \left(\frac{p_r^2}{\Lambda_{\rm NC}^4}\right)^{\frac{\epsilon}{2}}2^{-1-\epsilon}\frac{\pi^{\frac{3}{2}}}{\sin\frac{\epsilon}{2}\pi}\left[\left(-\frac{p^2p_r^2}{16\Lambda_{\rm NC}^4}\right)^{1-\frac{\epsilon}{2}}\Gamma\left(\frac{1}{2}\right){}_1\tilde{F}_2\left(\frac{1}{2};\frac{3}{2},\frac{5}{2}-\frac{\epsilon}{2};-\frac{p^2p_r^2}{16\Lambda_{\rm NC}^4}\right)\right. \\
&- \left.\Gamma\left(-\frac{1}{2}+\frac{\epsilon}{2}\right){}_2\tilde{F}_3\left(-\frac{1}{2}+\frac{\epsilon}{2},1;\frac{\epsilon}{2},\frac{1}{2}+\frac{\epsilon}{2},\frac{3}{2};-\frac{p^2p_r^2}{16\Lambda_{\rm NC}^4}\right)\right].
\end{aligned}
$$

The leading and next-to-leading asymptotic orders of A_2 when $y \to \infty$ reads

$$
A_2 \sim \frac{i\pi^2}{8}y^{\frac{1}{2}}\left(1-16i\pi^{-1}y^{-1}e^{-\frac{i}{2}y^{\frac{1}{2}}}\right) \to z \sim -1\pm 2i \to p_0^2 \sim p_3^2 \pm 2ip_r^2.
$$

Quantum properties II: Neutrino self energy

Alternative action 2 shows SW map dependent results

$$\Sigma_{1_{alt2}} = \frac{\not p}{(4\pi)^2}\left[\frac{8}{3}\frac{1}{(\theta p)^2}\left(\frac{\text{tr}\theta\theta}{(\theta p)^2} + 4\frac{(\theta\theta p)^2}{(\theta p)^4}\right)\right].$$

Here we have again $\Sigma_{1-loop_{alt2}} = \Sigma_{1_{alt2}}$. There is no alternative dispersion relation in degenerate case, since the factor that multiplies $\not p$, does not dependent on the time-like component p_0 (energy).
There are no hard $1/\epsilon$ UV divergent and no logarithmic UV/IR mixing terms, and the finite terms like in A_1 and A_2 are also absent. Thus the subgraph Σ_1 does not require any counter-term. However, the result of the subgraph Σ_1 evaluation, from alternative action 2, does express powerful UV/IR mixing effect due to scale dependent θ-ratios. Namely, in terms of scales only, the $\Sigma_{1_{alt2}}$ experience the forth-power of the *NC-scale/momentum-scale* ratios $\sim |p|^{-2}|\theta p|^{-2}$, i.e. we are dealing with the $\Sigma_{1_{alt2}} \sim \not p\,(\Lambda_{\rm NC}/p)^4$ within the ultraviolet and infrared limits for $\Lambda_{\rm NC}$ and p, respectively.
The absence of new spinor structure in the alternative neutrino self-energy further suggests the possibility of an appropriate field strength renormalization with suitable divergence cancellation for $\theta \to 0$ limit.

Quantum properties II: Neutrino self energy

- A constant $\theta \neq 0$ background is for simplicity. The results will, however, still hold for a NC background that is varying sufficiently slowly with respect to Λ_{NC}. There is no physics reason to expect θ to be a globally constant background *ether*. In fact, if the θ background is only nonzero in tiny regions (NC bubbles and/or slots of NC) the effects of modified dispersion relation will be suppressed macroscopically. Understanding of possible sources of NC is needed.

Quantum properties II: Neutrino self energy

- In our computation we considered only the purely noncommutative neutrino-photon coupling, it has been pointed out that modified neutrino dispersion relation could open decay channels within the commutative standard model framework. In our case this would further provide decay channel(s) which can bring superluminal neutrinos to normal ones.

Quantum properties II: Neutrino self energy

- Finally, model 1 is not the only allowed deformed model with NC neutrino-photon coupling. And as we have shown for our model 2, there could be no modified dispersion relation(s) for deformation(s) other than 1, therefore it is reasonable to conjecture that SW map freedom may also serve as one possible remedy to this issue.

SUMMARY

1. The θ-exact NCGFT, motivated by UHECR-ν Figs. 1-3.

2. Action, SW map based, is covariant and gauge invariant.

3. Physical quantities as a functions of energy behaves correctly, in a closed form, for full energy scale.

4. In plasmon physics: Decay rate becomes finite - good defined function of $\Lambda_{\rm NC}$ and ω_{pl} (see Fig. 4).

5. Neutrino ch. radius: No θ-exact corrections to θ^1-results.

6. In BBN from plasmon decay, the $\Lambda_{\rm NC}$ becomes finite - good defined function of $T_{\rm dec}$ (see Fig. 5).

SUMMARY

7. Quantum property: Connection of effective NCFT with Holography via $\Lambda_{\rm IR}/\Lambda_{\rm NC}/M_{Pl}/\Lambda_{\rm UV} \longrightarrow$

   No longer simultaneously $\Lambda_{\rm UV} \to \infty$ and $\Lambda_{\rm IR} \to 0$!

8. Quantum property 1: Neutrino 2-point function NEW behavior: simultaneous presence of SOFTENED - UV/IR mixing and HARD - $1/\epsilon$ UV-divergences !

9. Quantum property 1: NEW property Modified dispersion relations: Spontaneous Lorentz Symmetry Breaking $\rightarrow$ $\mathbf{v}_\nu \sim \mathbf{30c}$ as pure Quantum effect $\rightarrow$ Causality breaking at Quantum level Remember the Quantum tunnel effect!

10. Quantum property 2: NO modified dispersion relations, but powerful UV/IR mixing $\Longrightarrow$ Quantum Gravity?