Probing Quantum Geometry with Coupled Interferometers and Quantum Light

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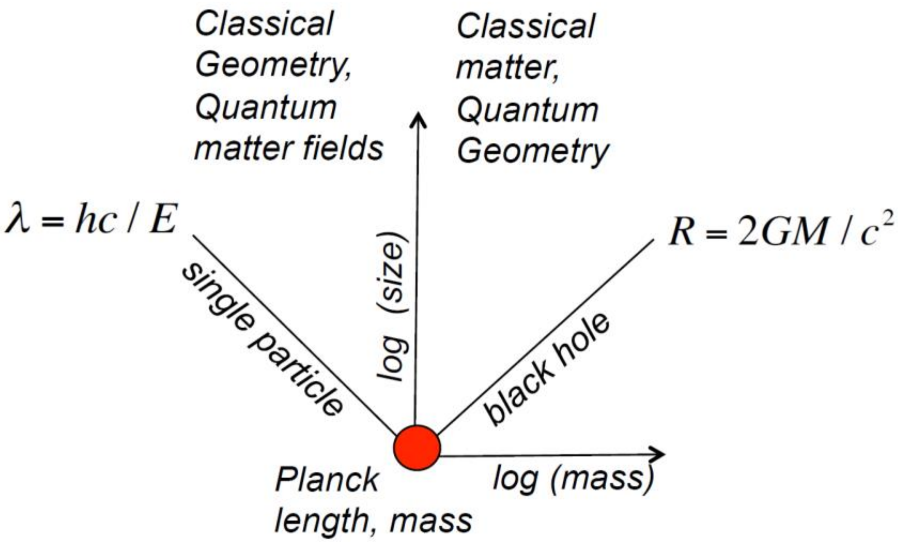
ME

Holographic Noise

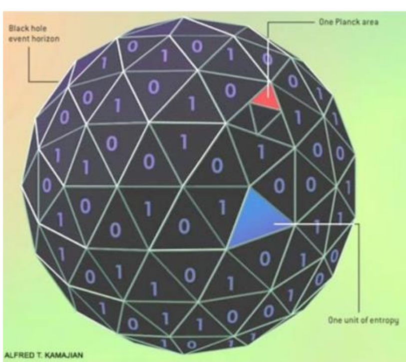


- Physics laws become non-consistent at the Planck length level ($l_p = c\, t_p = 1{,}6\ 10^{-35}\ m$)

- Quantum geometry postulates space-time and gravity emerge as an average over more fundamental degree of freedom existing at the Planck scale.

- The "emergent" space-time is said to be holographic

- Although quantum geometry approximates classical space-time on large scale, the Hogan's quantum geometry describes new quantum properties of collective positions of massive bodies

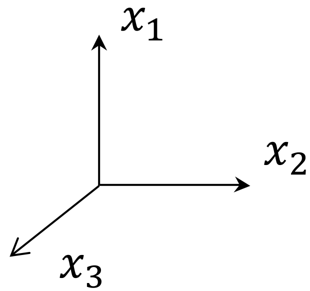


G. Hogan, Arxiv: 1204.5948

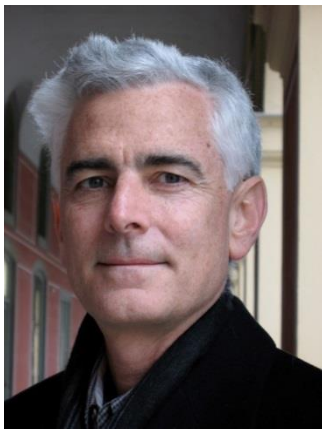
G. Hogan, Phys. Rev. D 85, 064007 (2012)

Holographic Noise

- Hogan's effective theory postulates that position operators in different directions do not commute



$$[\hat{x}_i, \hat{x}_j] = \hat{x}_k \epsilon_{ijk} i c t_P / \sqrt{4\pi}$$



(This heuristic approach allows to derive the holographic principle)

Sort of space-time uncertainty principle (*L*= radial separation)

$$\langle \hat{x}_\perp^2 \rangle = L c t_P / \sqrt{4\pi} = (2.135 \times 10^{-18}\mathrm{m})^2 (L/1\mathrm{m})$$

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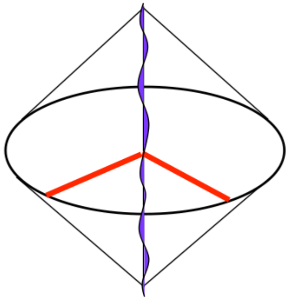
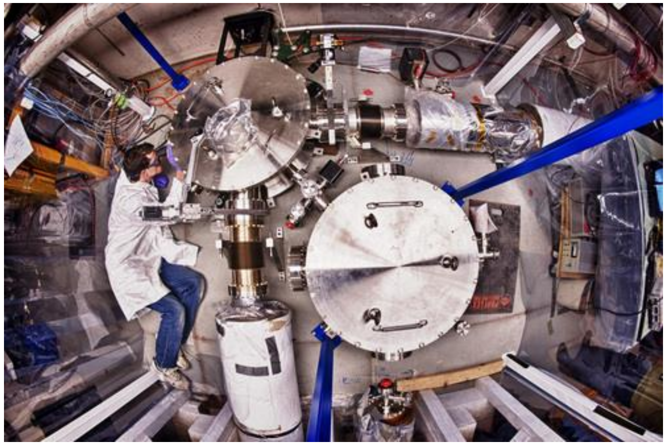
This new quantum uncertainty of space-time induces a slight random wandering of transverse position (called "holographic noise")

Holometer for HN observation

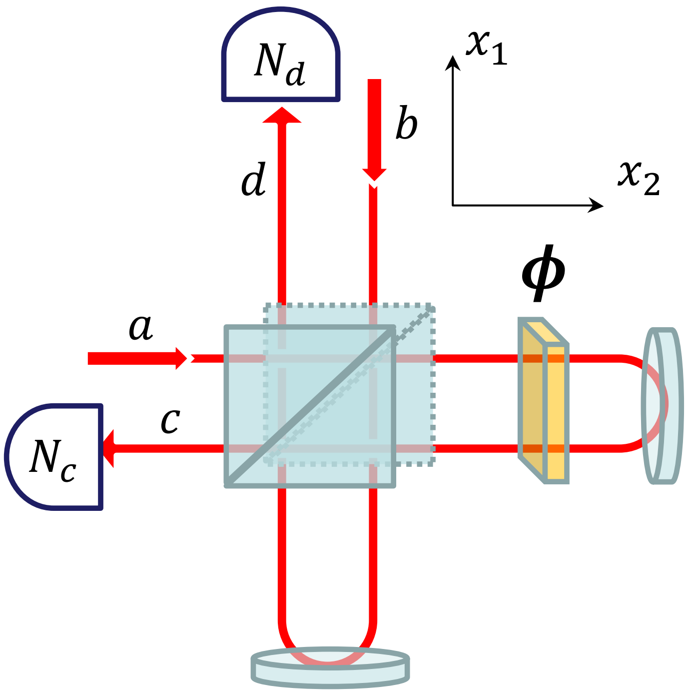
Holometer (**Holo**graphic Interfero**meter**) to measure the possible presence of a very slight random wandering of transverse position (the "holographic noise") over an extended volume of space-time is currently under construction @**Fermilab**



Holometer @**Fermilab:** two coupled ultra-sensitive Michelson interferometers (40 m arms)



http://holometer.fnal.gov/



In Michelson interferometer the *phase shift (ϕ)* can be seen as a simultaneous measurement of the position of the beam splitter ($x_1 - x_2$).

Holographic noise accumulates as a *random walk* becoming detectable

$$\langle [X(t) - X(t+\tau)]^2 \rangle = c^2 t_P \tau (2/\pi)$$
$$\tau << 2L/c$$

The random walk is bounded (an interferometer measures HN within the causal boundaries defined by a single light round trip)
($\tau = 2L/c$ the longest time over which differential random walk affects the measured phase)

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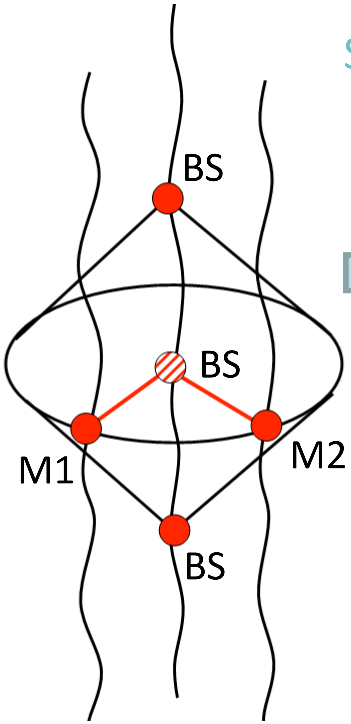
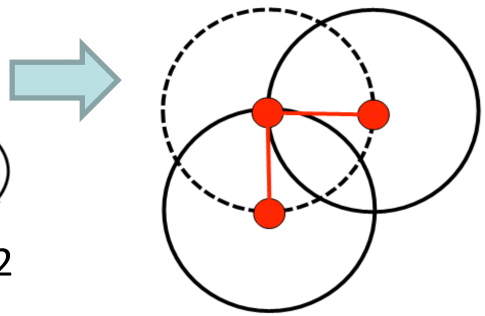
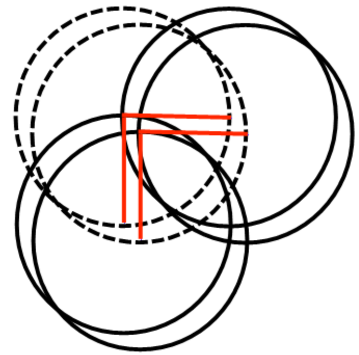
Holographic Noise and the Holometer

Single Interferometer

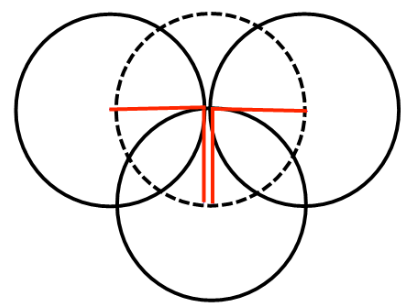




Holometer



«Overlapping» space-time volume



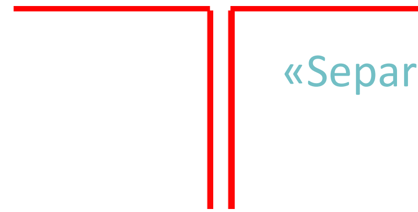
«Separated» space-time volume

HOLOMETER: *principles of operation*

- Evaluation of the cross-correlation between two equal Michelson interferometers occupying the same space-time volume
- Reference measurement: HN correlation «turned off» by separating the space-time volumes of the two interferometers

The model

«Overlapping»



«Separated»

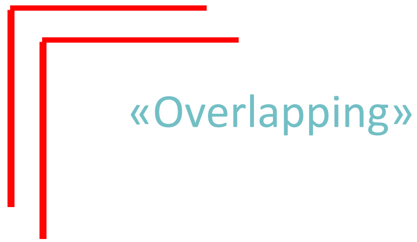
<u>AIM:</u> HN detected by measuring the phase covariance $\mathcal{E}_{\|}\left[\delta\phi_1\delta\phi_2\right]$ between the two interferometers of the holometer

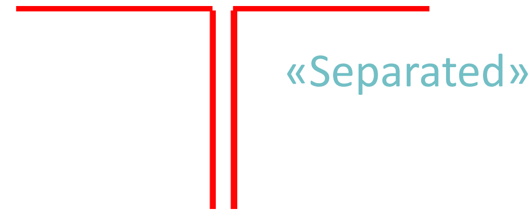
$$\delta\phi_k = \phi_k - \phi_{k,0}$$

$\widehat{C}(\phi_1,\phi_2)$: quantum observable measured at the output of the holometer

$$\boxed{\mathcal{E}_{\|}\left[\delta\phi_1\delta\phi_2\right] \approx \frac{\mathcal{E}_{\|}\left[\widehat{C}(\phi_1,\phi_2)\right] - \mathcal{E}_{\perp}\left[\widehat{C}(\phi_1,\phi_2)\right]}{\langle\partial^2_{\phi_1,\phi_2}\widehat{C}(\phi_{1,0},\phi_{2,0})\rangle}}$$

The model





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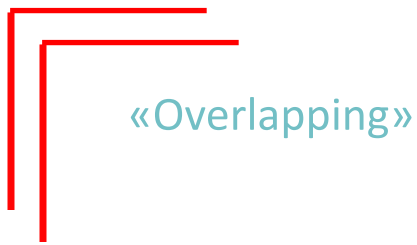
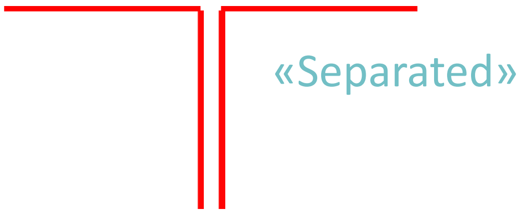
linearization $(\delta\phi_1, \delta\phi_2 \ll 1)$

Sensitivity Coefficient

The model





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linearization $(\delta\phi_1, \delta\phi_2 \ll 1)$

Sensitivity Coefficient

The uncertainty should be reduced as much as possible

$$\mathcal{U}(\delta\phi_1\delta\phi_2) \approx \sqrt{\frac{\mathrm{Var}_{\|}\left[\widehat{C}(\phi_1,\phi_2)\right] + \mathrm{Var}_{\perp}\left[\widehat{C}(\phi_1,\phi_2)\right]}{\left[\langle\partial^2_{\phi_1,\phi_2}\widehat{C}(\phi_{1,0},\phi_{2,0})\rangle\right]^2}}$$

PRL **110**, 213601 (2013)

The model

«Overlapping»

«Separated»

Phases covariance uncertainty

$$\mathcal{U}(\delta\phi_1\delta\phi_2) \approx \sqrt{\frac{\mathrm{Var}_{\parallel}\left[\widehat{C}(\phi_1,\phi_2)\right] + \mathrm{Var}_{\perp}\left[\widehat{C}(\phi_1,\phi_2)\right]}{\left[\langle\partial^2_{\phi_1,\phi_2}\widehat{C}(\phi_{1,0},\phi_{2,0})\rangle\right]^2}}$$

$$\mathrm{Var}_x\left[\widehat{C}(\phi_1,\phi_2)\right] \equiv \mathcal{E}_x\left[\widehat{C}^2(\phi_1,\phi_2)\right] - \mathcal{E}_x\left[\widehat{C}(\phi_1,\phi_2)\right]^2$$

$$\mathcal{E}_x\left[\widehat{O}(\phi_1,\phi_2)\right] \equiv \int \langle\widehat{O}(\phi_1,\phi_2)\rangle\ f_x(\phi_1,\phi_2)\ \mathrm{d}\phi_1\ \mathrm{d}\phi_2$$

«Overlapping»

«Separated»

Phases covariance uncertainty

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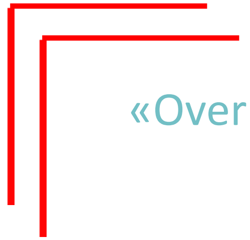
Quantum EV

$\mathrm{Tr}[\rho_{12}\widehat{C}(\phi_1,\phi_2)]$

The model



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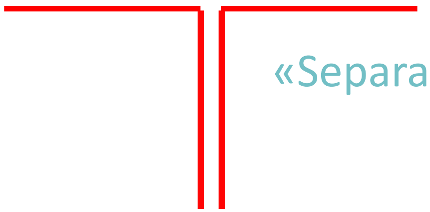

«Separated»

Phases covariance uncertainty

$$\mathcal{U}(\delta\phi_1\delta\phi_2) \approx \sqrt{\frac{\mathrm{Var}_{\parallel}\left[\widehat{C}(\phi_1,\phi_2)\right] + \mathrm{Var}_{\perp}\left[\widehat{C}(\phi_1,\phi_2)\right]}{\left[\langle\partial^2_{\phi_1,\phi_2}\widehat{C}(\phi_{1,0},\phi_{2,0})\rangle\right]^2}}$$

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$\boxed{f_x(\phi_1,\phi_2)}$ pdf of phase fluctuations due to HN

$x = \parallel, \perp$

Quantum EV

$\mathrm{Tr}[\rho_{12}\widehat{C}(\phi_1,\phi_2)]$

- $f_\perp(\phi_1,\phi_2) = \mathcal{F}_\perp^{(1)}(\phi_1)\mathcal{F}_\perp^{(2)}(\phi_2)$
- $\mathcal{F}_\parallel^{(k)}(\phi_k) = \mathcal{F}_\perp^{(k)}(\phi_k)$

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Phases covariance uncertainty $\mathcal{U}(\delta\phi_1\delta\phi_2) \approx \sqrt{\frac{\mathrm{Var}_{\parallel}\left[\widehat{C}(\phi_1,\phi_2)\right] + \mathrm{Var}_{\perp}\left[\widehat{C}(\phi_1,\phi_2)\right]}{\left[\langle\partial^2_{\phi_1,\phi_2}\widehat{C}(\phi_{1,0},\phi_{2,0})\rangle\right]^2}}$

linearization $(\delta\phi_1, \delta\phi_2 \ll 1)$

Var_x

$$\mathrm{Var}_x\left[\widehat{C}(\phi_1,\phi_2)\right] = \mathrm{Var}\left[\widehat{C}(\phi_{1,0},\phi_{2,0})\right] + \Sigma_k\, A_{kk}\, \mathcal{E}_x\left[\delta\phi_k^2\right] + A_{12}\, \mathcal{E}_x\left[\delta\phi_1\delta\phi_2\right] + \mathcal{O}(\delta\phi^3)$$

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$\boxed{f_x(\phi_1,\phi_2)}$ pdf of phase fluctuations due to HN

Quantum EV
$\mathrm{Tr}[\rho_{12}\widehat{C}(\phi_1,\phi_2)]$

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The model

«Overlapping»

«Separated»

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$$\mathrm{Var}_x\left[\widehat{C}(\phi_1,\phi_2)\right] = \underbrace{\boxed{\mathrm{Var}\left[\widehat{C}(\phi_{1,0},\phi_{2,0})\right]}}_{\text{0-th order}} + \Sigma_k\, A_{kk}\, \mathcal{E}_x\left[\delta\phi_k^2\right] + A_{12}\, \mathcal{E}_x\left[\delta\phi_1\delta\phi_2\right] + \mathcal{O}(\delta\phi^3)$$

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- *0-th order* independent from PSs fluctuations (i.e. HN)
- *0-th order* quantum light noise (shot-noise in the actual Holometer)

$\boxed{f_x(\phi_1,\phi_2)}$ pdf of phase fluctuations due to HN

Quantum EV
$\mathrm{Tr}[\rho_{12}\widehat{C}(\phi_1,\phi_2)]$

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$f_x(\phi_1,\phi_2)$ pdf of phase fluctuations due to HN

$x = \parallel, \perp$

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0-th order contribution to PSs covariance unc.:

$$\mathcal{U}^{(0)} = \frac{\sqrt{2\,\mathrm{Var}\left[\widehat{C}(\phi_{1,0},\phi_{2,0})\right]}}{\left|\langle \partial^2_{\phi_1,\phi_2}\widehat{C}(\phi_{1,0},\phi_{2,0})\rangle\right|}$$

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The model

Phases covariance uncertainty

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Exploiting quantum light to beat the “shot-noise” level!

PRL **110**, 213601 (2013)

Squeezed light in gravitational wave detectors!!

A sub-shot-noise PS measurement in a **single** interferometer (e.g. gravitational wave detector) was suggested exploiting squeezed light

*Caves, PRD **23**, 1693 (1981)*

*Kimble et al., PRD **65**, 022002 (2001)*

...

and recently realized at Ligo 600

*R. Schnabel et al., Nature Commun. **1**, 121 (2010)*

*Ligo, Nature Phys. **7**, 962 (2011)*

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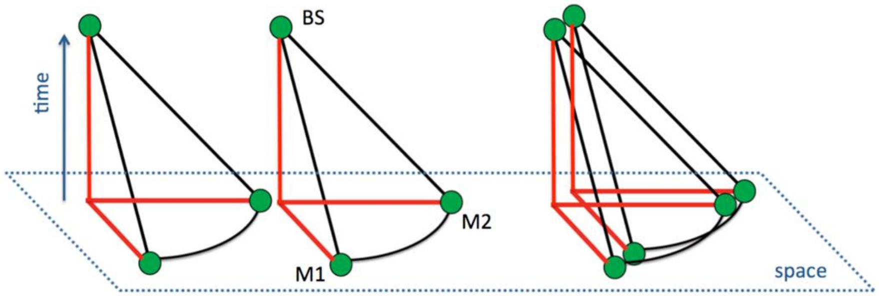
Does squeezed light help also in the case of the Holometer?



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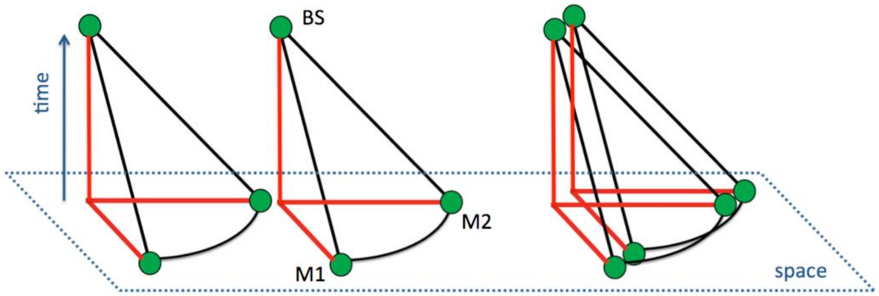

Before discussing it a quick overview of "relevant" Quantum Optics concepts

Quantization of the Electromagnetic Field

Classical → ***Quantum***

$$\mathbf{E}(\mathbf{r},t) = \sum_{\mathbf{k}} \hat{\epsilon}_{\mathbf{k}} \mathscr{E}_{\mathbf{k}} \alpha_{\mathbf{k}} e^{-i\nu_k t + i\mathbf{k}\cdot\mathbf{r}} + \text{c.c.},$$

$$\mathbf{H}(\mathbf{r},t) = \frac{1}{\mu_0} \sum_{\mathbf{k}} \frac{\mathbf{k}\times\hat{\epsilon}_{\mathbf{k}}}{\nu_k} \mathscr{E}_{\mathbf{k}} \alpha_{\mathbf{k}} e^{-i\nu_k t + i\mathbf{k}\cdot\mathbf{r}} + \text{c.c.}$$

$$\alpha_{\mathbf{k}} \quad \alpha_{\mathbf{k}}^*$$

Unitless Coefficients

$$\mathbf{E}(\mathbf{r},t) = \sum_{\mathbf{k}} \hat{\epsilon}_{\mathbf{k}} \mathscr{E}_{\mathbf{k}} a_{\mathbf{k}} e^{-i\nu_k t + i\mathbf{k}\cdot\mathbf{r}} + \text{H.c.},$$

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$$a_{\mathbf{k}} \quad a_{\mathbf{k}}^\dagger$$

Quantum Operators

$$[a_{\mathbf{k}}, a_{\mathbf{k}}^\dagger] = 1$$

Energy of a single mode quantum EM field

$$\mathscr{H}_{\mathbf{k}} = \hbar\nu_k \left(a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \frac{1}{2}\right)$$

$$\mathscr{H}_{\mathbf{k}}|n_{\mathbf{k}}\rangle = \hbar\nu_k \left(n_{\mathbf{k}} + \frac{1}{2}\right)|n_{\mathbf{k}}\rangle$$

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle$$

Quantization of the Electromagnetic Field

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$$\mathbf{E}(\mathbf{r},t)=\sum_{\mathbf{k}}\hat{\epsilon}_{\mathbf{k}}\mathscr{E}_{\mathbf{k}}\alpha_{\mathbf{k}}e^{-i\nu_k t+i\mathbf{k}\cdot\mathbf{r}}+\text{c.c.},$$

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Unitless Coefficients

$$\mathbf{E}(\mathbf{r},t)=\sum_{\mathbf{k}}\hat{\epsilon}_{\mathbf{k}}\mathscr{E}_{\mathbf{k}}a_{\mathbf{k}}e^{-i\nu_k t+i\mathbf{k}\cdot\mathbf{r}}+\text{H.c.},$$

$$\mathbf{H}(\mathbf{r},t)=\frac{1}{\mu_0}\sum_{\mathbf{k}}\frac{\mathbf{k}\times\hat{\epsilon}_{\mathbf{k}}}{\nu_k}\mathscr{E}_{\mathbf{k}}a_{\mathbf{k}}e^{-i\nu_k t+i\mathbf{k}\cdot\mathbf{r}}+\text{H.c.}$$

$$a_{\mathbf{k}}\quad a_{\mathbf{k}}^{\dagger}$$

Quantum Operators

$$[a_{\mathbf{k}},a_{\mathbf{k}}^{\dagger}]=1$$

Energy of a single mode quantum EM field

$$\mathscr{H}_{\mathbf{k}}=\hbar\nu_k\left(a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}+\frac{1}{2}\right)$$

$$\mathscr{H}_{\mathbf{k}}|n_{\mathbf{k}}\rangle=\hbar\nu_k\left(n_{\mathbf{k}}+\frac{1}{2}\right)|n_{\mathbf{k}}\rangle$$

$$|n\rangle=\frac{(a^{\dagger})^n}{\sqrt{n!}}|0\rangle$$

Quadrature Operators

$$X_1 = \frac{1}{2}(a + a^\dagger)$$ "Amplitude" or "Position"

$$X_2 = \frac{1}{2i}(a - a^\dagger)$$ "Phase" or "Momentum"

$$[X_1, X_2] = \frac{i}{2} \longrightarrow \Delta X_1 \Delta X_2 \geq \frac{1}{4}$$

Heisenberg's Unc. Relation

Coherent States

Coherent State: eigenstate of the annihilation operator

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

Displacement operator: $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$

$$|\alpha\rangle = D(\alpha)|0\rangle \qquad D^{-1}(\alpha) a D(\alpha) = a + \alpha$$

Mean photon number: $\langle\alpha|a^\dagger a|\alpha\rangle = |\alpha|^2$

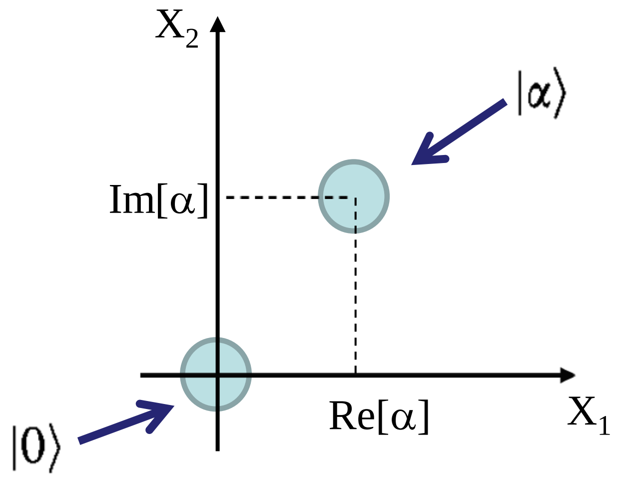
Photon number statistics: $p(n) = \langle n|\alpha\rangle\langle\alpha|n\rangle = \dfrac{\langle n\rangle^n e^{-\langle n\rangle}}{n!}$ $\qquad \langle n\rangle = |\alpha|^2$

Quadrature operators

$$(\Delta X_1)^2 = \langle\alpha|X_1^2|\alpha\rangle - (\langle\alpha|X_1|\alpha\rangle)^2 = \frac{1}{4}$$

$$(\Delta X_2)^2 = \frac{1}{4}$$

$$\boxed{\Delta X_1 \Delta X_2 = \frac{1}{4}}$$



$$X_1 = \frac{1}{2}(a + a^\dagger)$$
$$X_2 = \frac{1}{2i}(a - a^\dagger)$$

Squeezed States

Hamiltonian of a degenerate parametric process: $\mathscr{H} = i\hbar\left(g a^{\dagger 2} - g^* a^2\right)$

(Unitary) "Squeeze" Operator : $S(\xi) = \exp\left(\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{\dagger 2}\right)$ $\quad \xi = r\exp(i\theta)$

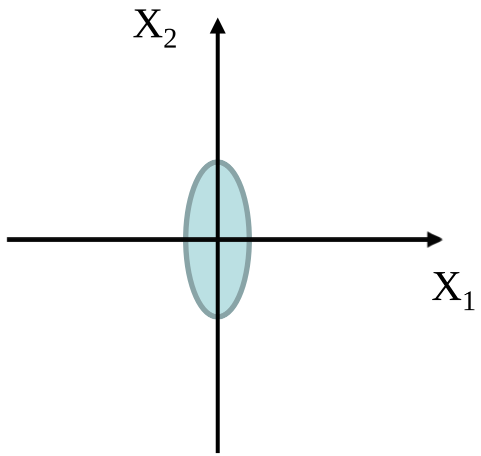
$$\begin{cases} S^\dagger(\xi) a S(\xi) = a\cosh r - a^\dagger e^{i\theta}\sinh r \\ S^\dagger(\xi) a^\dagger S(\xi) = a^\dagger\cosh r - a e^{-i\theta}\sinh r \end{cases}$$

Squeezed Vacuum: $|\xi\rangle = S(\xi)|0\rangle$

$$(\Delta X_1)^2 = \frac{1}{4}e^{-2r}$$

$$(\Delta X_2)^2 = \frac{1}{4}e^{2r}$$

$$\boxed{\Delta X_1 \Delta X_2 = \frac{1}{4}}$$



$$X_1 = \frac{1}{2}(a + a^\dagger)$$

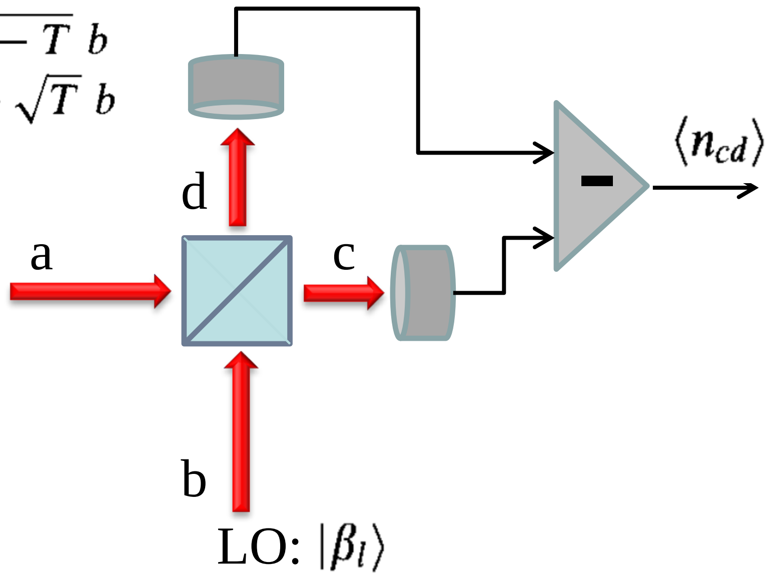
$$X_2 = \frac{1}{2i}(a - a^\dagger)$$

Squeezed Vacuum obtained with an OPO operating under threshold

How to measure Quadratures

BS transformation:

$$c = \sqrt{T}\, a + i\sqrt{1-T}\; b$$
$$d = i\sqrt{1-T}\; a + \sqrt{T}\; b$$



50:50 BS:

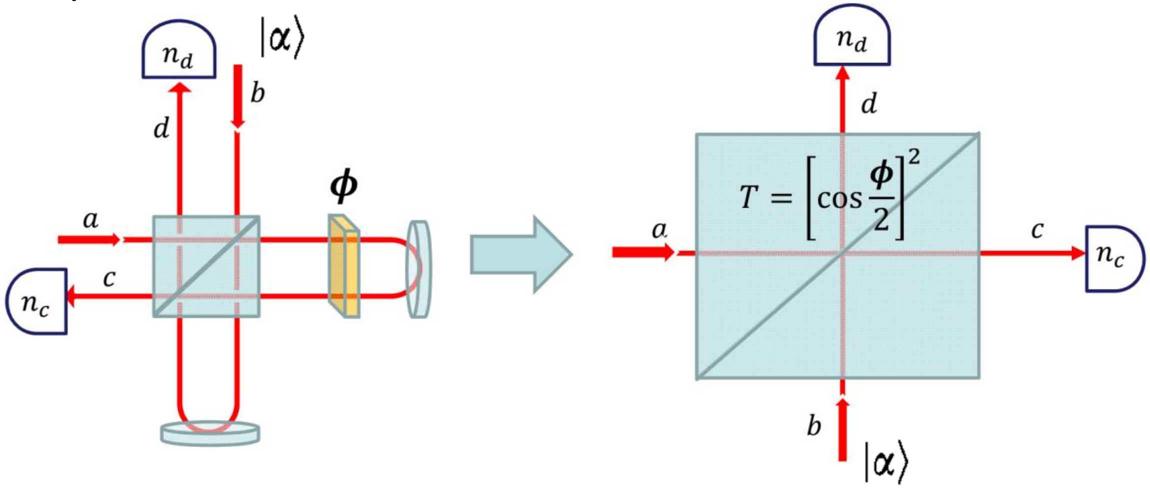
$$n_{cd} = c^\dagger c - d^\dagger d = -i(a^\dagger b - b^\dagger a)$$

$$\langle n_{cd}\rangle = -2|\beta_l|\langle X(\phi_l + \pi/2)\rangle$$
$$(\Delta n_{cd})^2 = 4|\beta_l|^2[\Delta X(\phi_l + \pi/2)]^2$$

$$X(\phi) \equiv X_\phi = \frac{1}{2}(ae^{-i\phi} + a^\dagger e^{i\phi})$$

Phase measurement in an interferometer

The input-output relations of the mode operators of an interferometer are the same of a BS with T (given by the the phase $\boldsymbol{\phi_p}$)



- $|0\rangle$ **in** a**-port,** $|\alpha\rangle$ **in** b**-port**

$$\langle n_{cd}\rangle = |\alpha|^2 \cos\phi_p$$

$$(\Delta n_{cd})^2 = |\alpha|^2$$

$$\Delta\phi = \frac{\Delta n_{cd}}{|\partial\langle n_{cd}\rangle/\partial\phi_p|} = \frac{1}{\sqrt{\langle n\rangle}}$$

Shot-Noise Limit
$\phi_p = \pi/2$

$$\langle n\rangle = |\alpha|^2$$

- $|\xi\rangle$ **in** a**-port,** $|\alpha\rangle$ **in** b**-port (** $\theta = 2\phi_l$ **)**

$$\langle n_{cd}\rangle = (\langle n\rangle + \sinh^2 r)\cos\phi_p \cong \langle n\rangle\cos\phi_p$$

$$(\Delta n_{cd})^2 = \langle n\rangle e^{-2r} + \sinh^2 r$$

Below the Shot-Noise Limit
$\phi_p = \pi/2$

$$\Delta\phi = \frac{\Delta n_{cd}}{|\partial\langle n_{cd}\rangle/\partial\phi_p|} = \frac{e^{-r}}{\sqrt{\langle n\rangle}}$$

Squeezed light in gravitational wave detectors!!

A sub-shot-noise PS measurement in a **single** interferometer (e.g. gravitational wave detector) was suggested exploiting squeezed light

*Caves, PRD **23**, 1693 (1981)*

*Kimble et al., PRD **65**, 022002 (2001)*

...

and recently realized at Ligo 600

*R. Schnabel et al., Nature Commun. **1**, 121 (2010)*

*Ligo, Nature Phys. **7**, 962 (2011)*

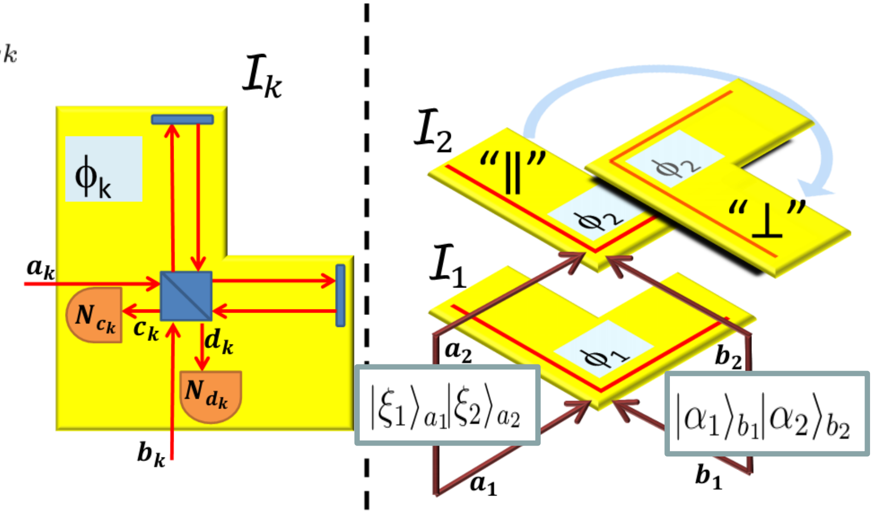
Does squeezed light help also in the case of the Holometer?

Squeezed light in the *a*'s ports: $|\xi_k\rangle_{a_k} = S_{a_k}(\xi_k)|0\rangle_{a_k}$

$$S_{a_k}(\xi_k) = \exp[\xi_k\ (a_k^\dagger)^2 - \xi_k^*\ (a_k)^2]$$

Coherent light in the *b*'s ports: $|\alpha_k\rangle_{b_k} = D_{b_k}(\alpha_k)|0\rangle_{b_k}$

$$D_{b_k}(\alpha_k) = \exp(\alpha_k\ b_k^\dagger - \alpha_k^*\ b_k)$$



Squeezed light in Coupled Interferometers

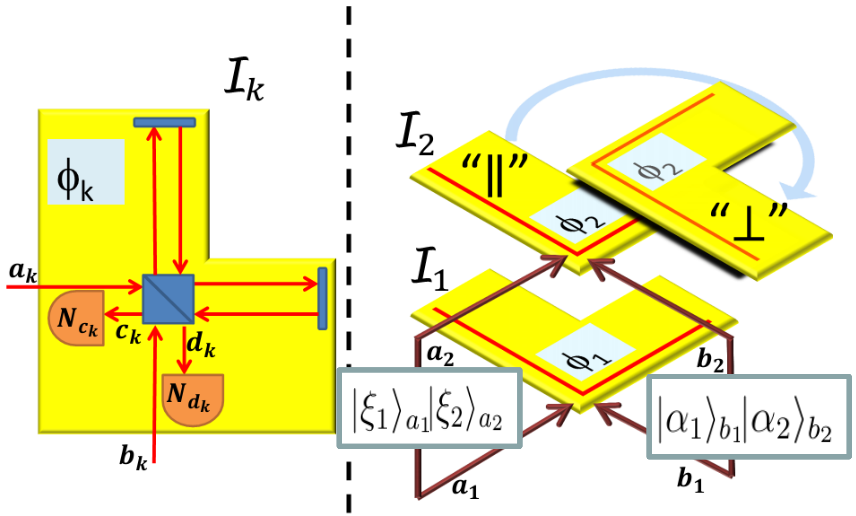
Does squeezed light help also in the case of the Holometer?

$\widehat{C}(\phi_1, \phi_2)$ is the covariance of photon # differences

$$\widehat{C}(\phi_1, \phi_2) = \Delta \hat{N}_{1-}(\phi_k)\ \Delta \hat{N}_{2-}(\phi_k)$$

$$\Delta \hat{N}_{k-}(\phi_k) = \hat{N}_{k-}(\phi_k) - \mathcal{E}\left[\hat{N}_{k-}(\phi_k)\right]$$

$$\hat{N}_{-}(\phi) = \hat{N}_c(\phi) - \hat{N}_d(\phi)$$



0-th order contribution to PSs covariance unc.:

$$\mathcal{U}^{(0)} = \frac{\sqrt{2\,\mathrm{Var}\left[\widehat{C}(\phi_{1,0}, \phi_{2,0})\right]}}{\left|\langle \partial^2_{\phi_1,\phi_2} \widehat{C}(\phi_{1,0}, \phi_{2,0})\rangle\right|}$$

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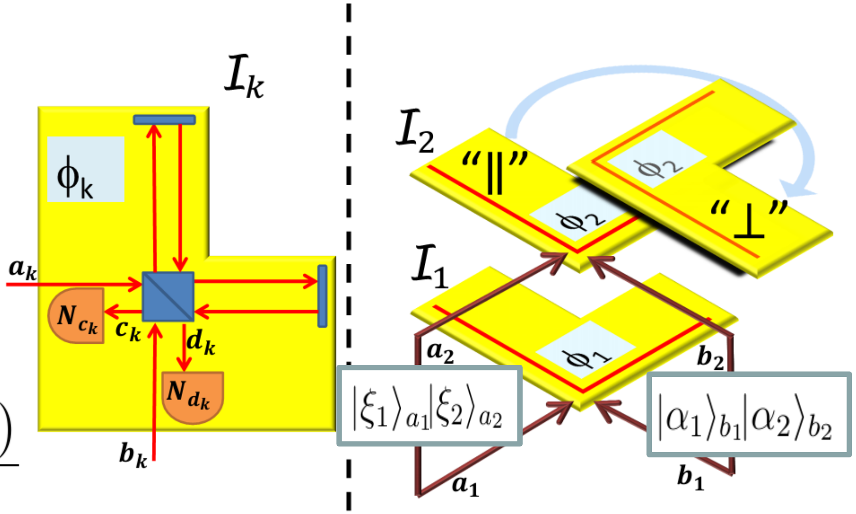
$$\mathcal{U}^{(0)}(\mu, \lambda, \underline{\phi_0 = \pi/2}) = \sqrt{2}\,\frac{\lambda + \mu\left(1 + 2\lambda - 2\sqrt{\lambda + \lambda^2}\right)}{(\lambda - \mu)^2}$$

$$\mathcal{U}^{(0)}_{\mathrm{SQ}} \approx \underset{\mu \gg \lambda \gg 1}{(2\sqrt{2}\lambda\mu)^{-1}}$$



μ : mean # photons coherent light

λ : mean # photons squeezed light

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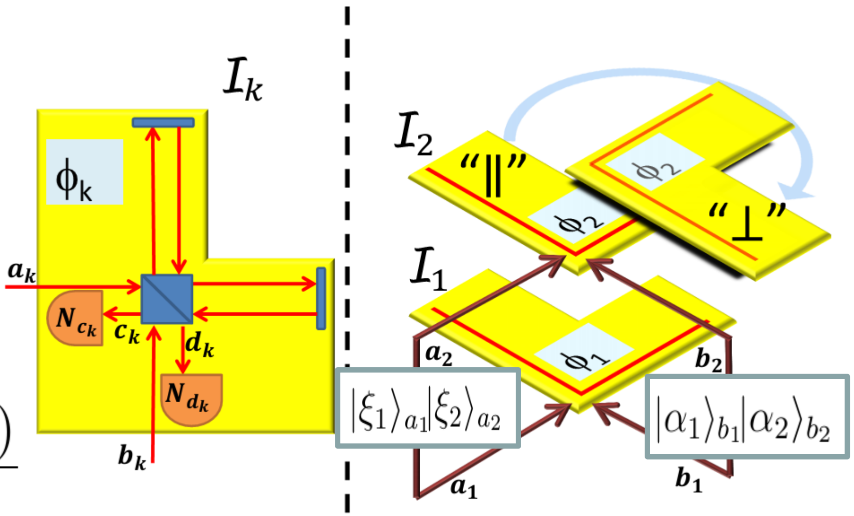

$$\mathcal{U}^{(0)}(\mu,\lambda,\phi_0=\pi/2) = \sqrt{2}\,\frac{\lambda+\mu\left(1+2\lambda-2\sqrt{\lambda+\lambda^2}\right)}{(\lambda-\mu)^2}$$

$$\mathcal{U}^{(0)}_{\mathrm{SQ}} \approx \left(2\sqrt{2}\lambda\mu\right)^{-1} \quad \mu \gg \lambda \gg 1$$

i.e. $(4\lambda)^{-1}$ better than the CL case $\mathcal{U}^{(0)}_{\mathrm{CL}} \approx \sqrt{2}/\mu$

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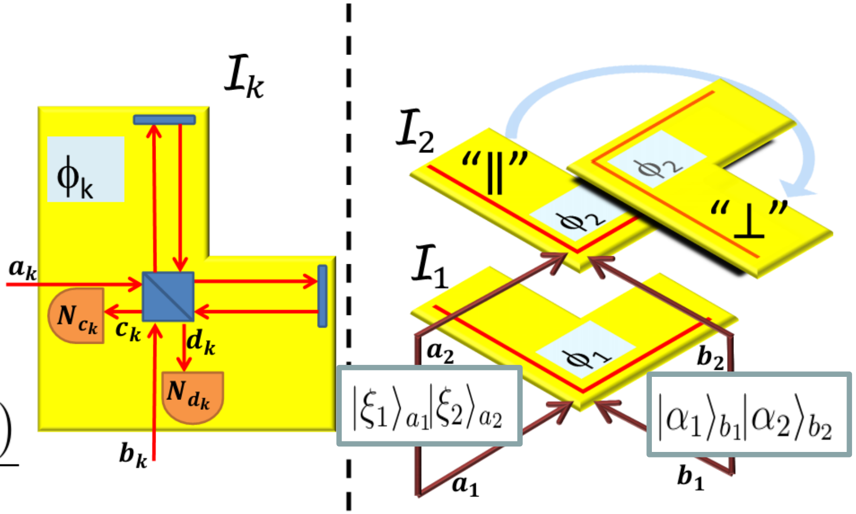

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μ : mean # photons coherent light
λ : mean # photons squeezed light

In the presence of losses η :

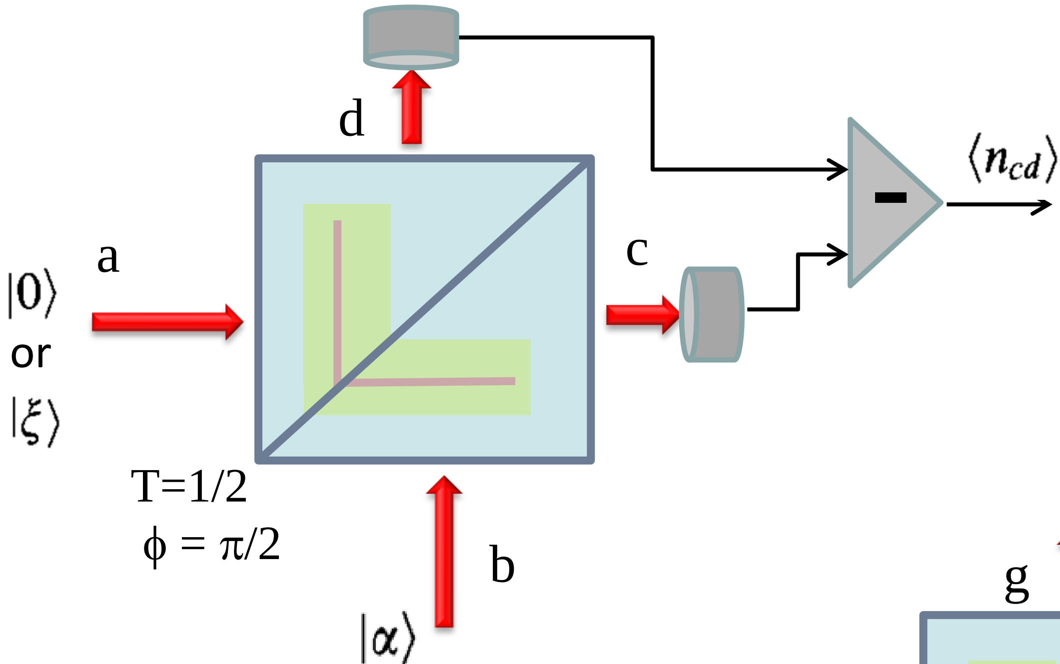
$$\mathcal{U}^{(0)}_{\mathrm{SQ}}/\mathcal{U}^{(0)}_{\mathrm{CL}} \approx (1-\eta) + \eta/(4\lambda) \qquad \mu \gg \lambda \gg 1$$

$$\mathcal{U}^{(0)}_{\mathrm{SQ}}/\mathcal{U}^{(0)}_{\mathrm{CL}} \approx 1 - 2\eta\sqrt{\lambda} \qquad \lambda \ll 1 \text{ and } \mu \gg 1$$

PRL **110**, 213601 (2013)

The "Dark-Port" configuration

What is done in practice in phase measurement (single interferometer)



Shot-Noise Limit

$$\Delta\phi = \frac{\Delta n_{cd}}{|\partial\langle n_{cd}\rangle/\partial\phi_p|} = \frac{1}{\sqrt{\langle n\rangle}}$$

$$\langle n\rangle = |\alpha|^2$$

Below the Shot-Noise Limit

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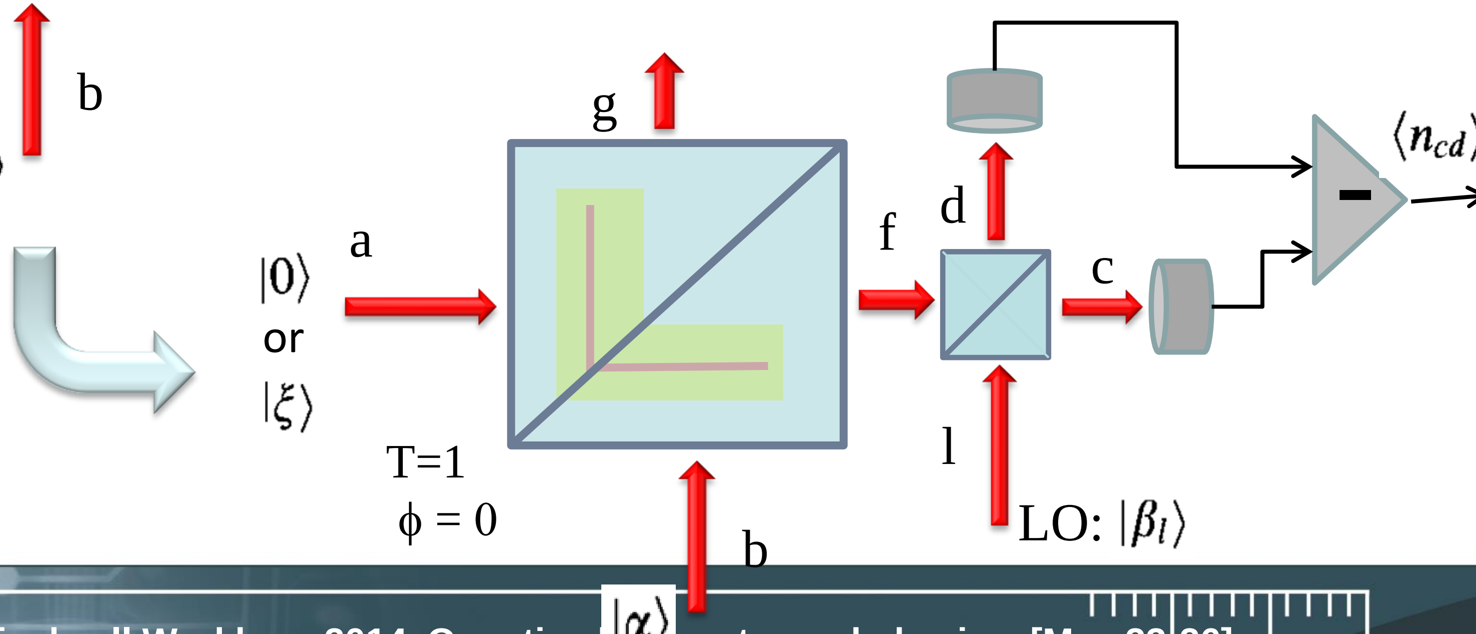


Does Q-correlated (Entangled) light help in coupled interferometers?

Twin-Beam state (or Two-mode squeezed vacuum)

Hamiltonian of a non-degenerate parametric process: $H \propto a^\dagger b^\dagger + h.c.$

(Unitary) Two-mode "Squeeze" Operator : $S_2(\xi) = \exp\left\{\xi a^\dagger b^\dagger - \xi^* ab\right\}$ $\quad \xi = re^{i\psi}$

$$S_2^\dagger(\xi)\begin{pmatrix} a \\ b^\dagger \end{pmatrix} S_2(\xi) = \boldsymbol{S}_{2\xi}\begin{pmatrix} a \\ b^\dagger \end{pmatrix}$$

$$\boldsymbol{S}_{2\xi} = \begin{pmatrix} \mu & \nu \\ \nu^* & \mu \end{pmatrix}$$

$$\mu = \cosh r$$
$$\nu = e^{i\psi}\sinh r$$

Twin Beam state: $$|\mathrm{TWB}\rangle\rangle = S_2(\xi)|\mathbf{0}\rangle = \frac{1}{\sqrt{\mu}}\sum_{k=0}^{\infty}\left(\frac{\nu}{\mu}\right)^k |k\rangle \otimes |k\rangle$$

TWB shows **perfect correlation** in the **photon number**, i.e TWB is an eigenstate of the photon number difference

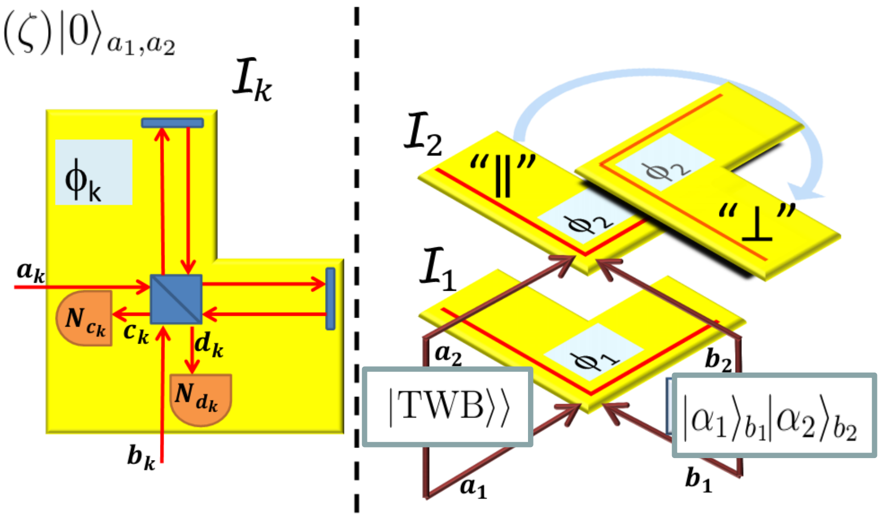
Does quantum correlated light help in coupled interferometers?

Twin-Beam light in the *a*'s ports: $|\mathrm{TWB}\rangle\rangle_{a_1,a_2} = S_{12}(\zeta)|0\rangle_{a_1,a_2}$

$$S_{12}(\zeta) = \exp(\zeta\ a_1^\dagger a_2^\dagger - \zeta^*\ a_1 a_2)$$

Coherent light in the *b*'s ports: $|\alpha_k\rangle_{b_k} = D_{b_k}(\alpha_k)|0\rangle_{b_k}$

$$D_{b_k}(\alpha_k) = \exp(\alpha_k\ b_k^\dagger - \alpha_k^*\ b_k)$$



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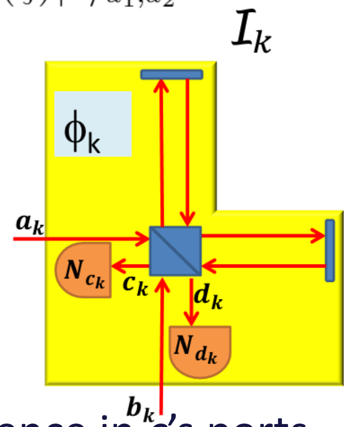

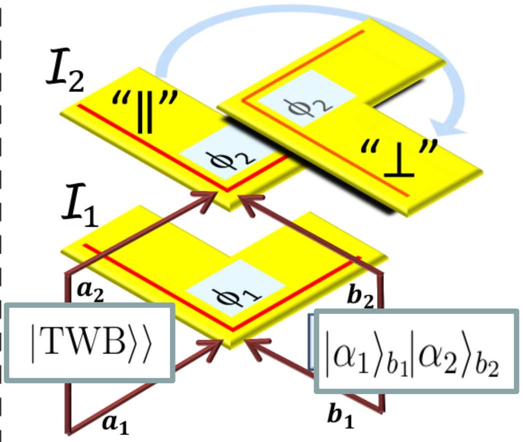

$\widehat{C}(\phi_1, \phi_2)$ is the fluctuations of the photon # difference in *c*'s ports

$$\widehat{C}(\phi_1, \phi_2) = \Delta^2\left[\hat{N}_{c_1} - \hat{N}_{c_2}\right]$$

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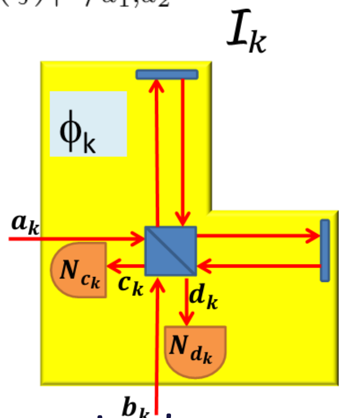

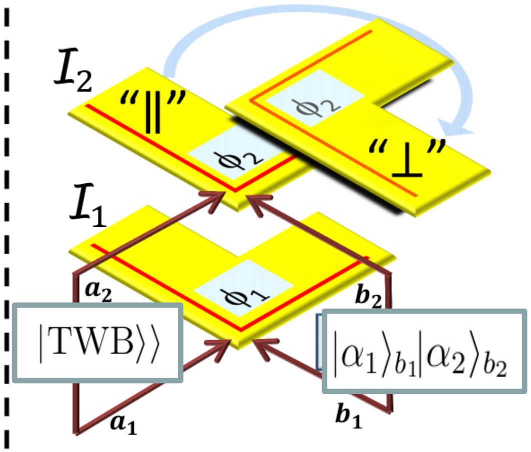

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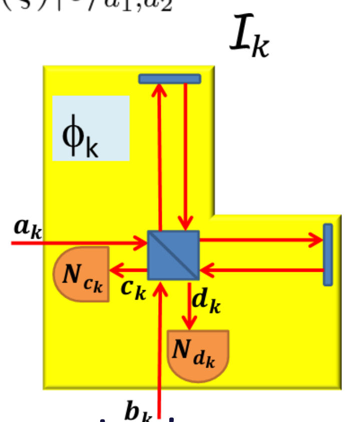

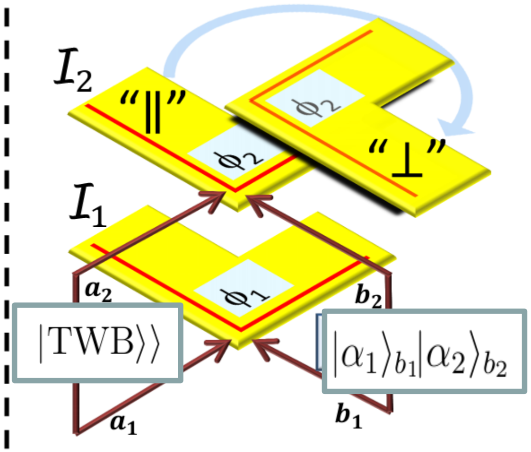

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$$\boxed{\phi_{k,0} = 0} \Rightarrow \boxed{\mathcal{U}^{(0)}_{\mathrm{TWB}} = 0}$$

$$\langle\langle \mathrm{TWB}|(\hat{N}_{a_1} - \hat{N}_{a_2})^M|\mathrm{TWB}\rangle\rangle = 0,\ \forall M > 0$$

$$\Rightarrow \mathrm{Var}\left\{\Delta^2\left[\hat{N}_{c_1}(0) - \hat{N}_{c_2}(0)\right]\right\} = 0$$

$$\left|\langle\partial^2_{\phi_1,\phi_2}\widehat{C}(\phi_{1,0},\phi_{2,0})\rangle\right| = -\frac{1}{2}\sqrt{\lambda(1+\lambda)}\mu\cos[2(\theta_\zeta - \theta_\alpha)]$$

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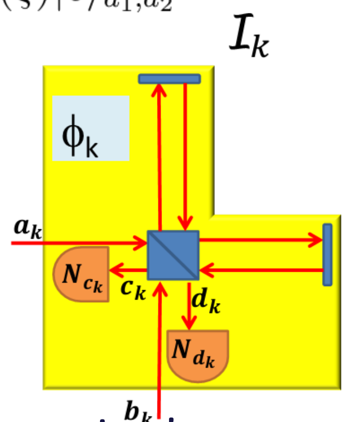

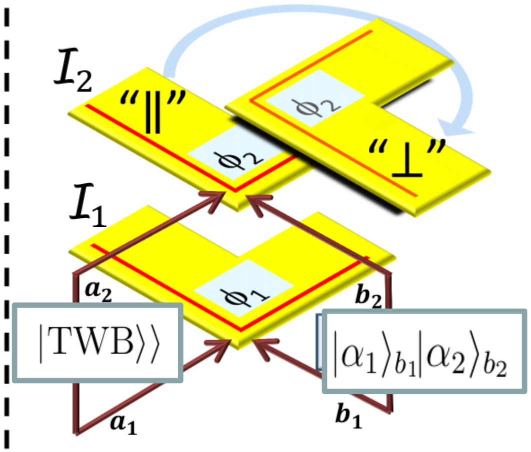

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In the presence of losses η :

$$\lambda \ll 1 \text{ and } \mu \gg 1$$

$$\mathcal{U}^{(0)}_{\mathrm{TWB}}/\mathcal{U}^{(0)}_{\mathrm{CL}} \approx \sqrt{2(1-\eta)/\eta}$$

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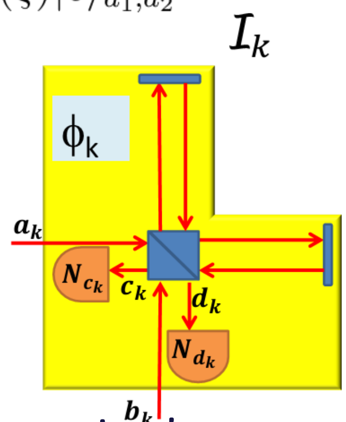

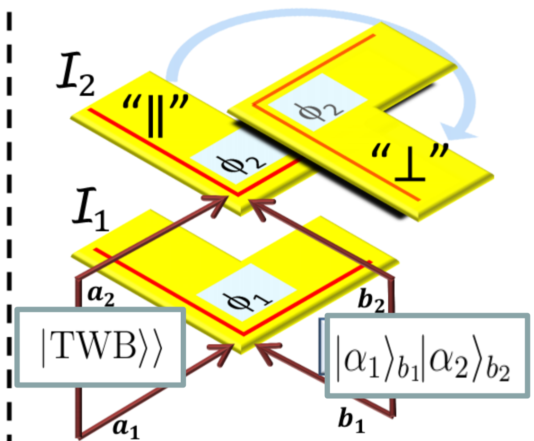

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$$\left|\langle\partial^2_{\phi_1,\phi_2}\widehat{C}(\phi_{1,0},\phi_{2,0})\rangle\right| = -\frac{1}{2}\sqrt{\lambda(1+\lambda)}\mu\cos[2(\theta_\zeta - \theta_\alpha)]$$

In the presence of losses η:

$$\lambda \ll 1 \text{ and } \mu \gg 1$$

$$\mathcal{U}^{(0)}_{\mathrm{TWB}}/\mathcal{U}^{(0)}_{\mathrm{CL}} \approx \sqrt{2(1-\eta)/\eta}$$

TWB is still advantageous in the presence of weak quantum light!!!

Squeezed light

$$\mathcal{U}^{(0)}_{\mathrm{SQ}}/\mathcal{U}^{(0)}_{\mathrm{CL}} \approx 1 - 2\eta\sqrt{\lambda}$$

Does quantum correlated light help in coupled interferometers?

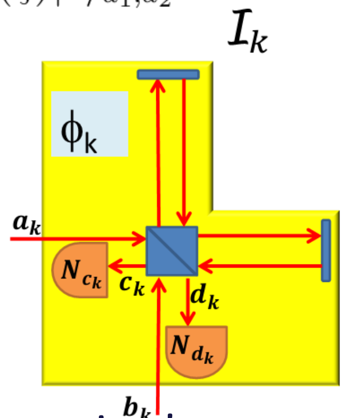
PRL **110**, 213601 (2013)

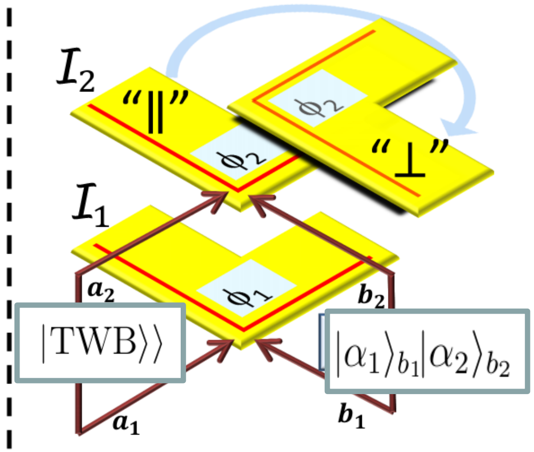
Twin-Beam light in the *a*'s ports: $|\mathrm{TWB}\rangle\rangle_{a_1,a_2} = S_{12}(\zeta)|0\rangle_{a_1,a_2}$

$$S_{12}(\zeta) = \exp(\zeta\, a_1^\dagger a_2^\dagger - \zeta^*\, a_1 a_2)$$

Coherent light in the *b*'s ports: $|\alpha_k\rangle_{b_k} = D_{b_k}(\alpha_k)|0\rangle_{b_k}$

$$D_{b_k}(\alpha_k) = \exp(\alpha_k\, b_k^\dagger - \alpha_k^*\, b_k)$$





$\widehat{C}(\phi_1,\phi_2)$ is the fluctuations of the photon # difference in *c*'s ports

$$\widehat{C}(\phi_1,\phi_2) = \Delta^2\left[\hat{N}_{c_1} - \hat{N}_{c_2}\right]$$

$\boxed{\phi_{k,0} = 0}$ ⇒ $\boxed{\mathcal{U}^{(0)}_{\mathrm{TWB}} = 0}$

$$\langle\langle \mathrm{TWB}|(\hat{N}_{a_1} - \hat{N}_{a_2})^M|\mathrm{TWB}\rangle\rangle = 0,\ \forall M > 0$$

$$\Rightarrow \mathrm{Var}\left\{\Delta^2\left[\hat{N}_{c_1}(0) - \hat{N}_{c_2}(0)\right]\right\} = 0$$

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Squeezed light

$$\mathcal{U}^{(0)}_{\mathrm{SQ}}/\mathcal{U}^{(0)}_{\mathrm{CL}} \approx 1 - 2\eta\sqrt{\lambda}$$

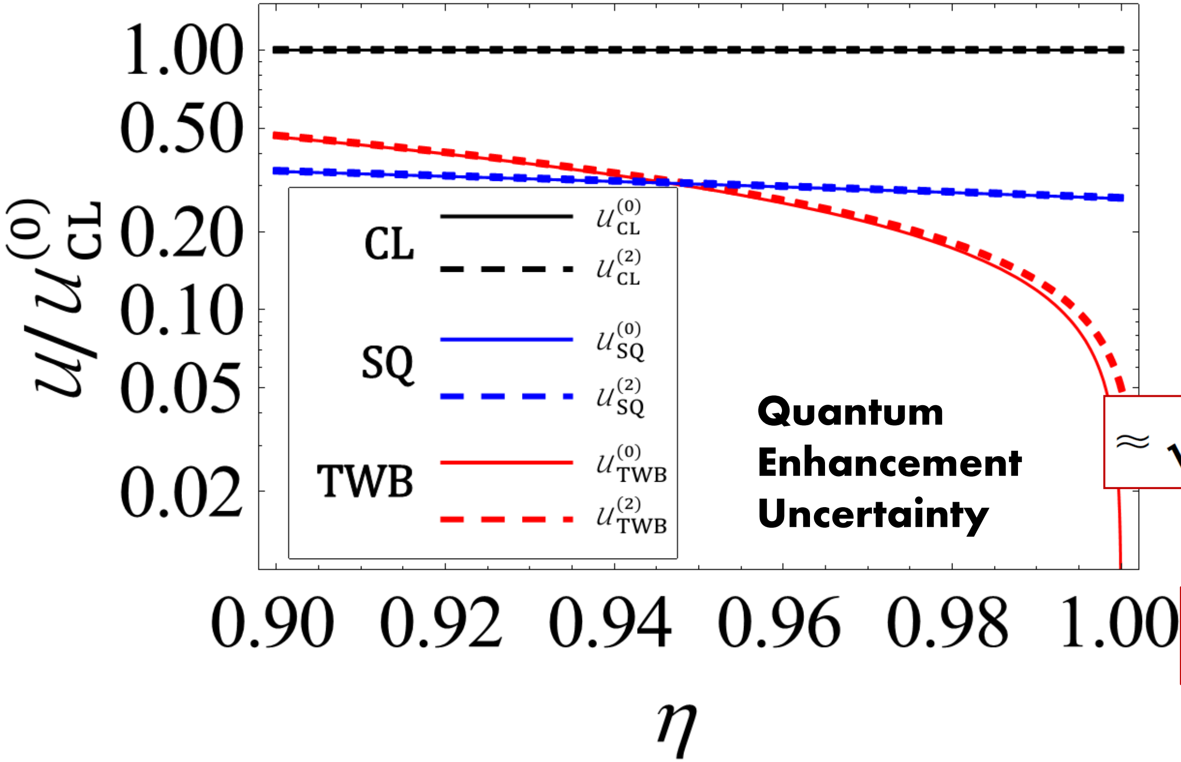
$\mu \gg \lambda \gg 1$

$$\mathcal{U}^{(0)}_{\mathrm{TWB}}/\mathcal{U}^{(0)}_{\mathrm{CL}} \approx 2\sqrt{5}\,(1-\eta)$$

Squeezed light

$$\mathcal{U}^{(0)}_{\mathrm{SQ}}/\mathcal{U}^{(0)}_{\mathrm{CL}} \approx (1-\eta) + \eta/(4\lambda)$$

Quantum light in Coupled Interferometers



$\lambda = 0.5$

$\mu = 10^{23}$

$Meas.time = 10^{-3}\ s$

$Wavelength = 600\ nm$

$Mirror\ mass = 10^{2} Kg$

> 10^7 W

$\approx \sqrt{2(1-\eta)/\eta}$

$\approx 0\ (\eta \approx 1)$

Fluctuations of the # of photons inside the interferometers arms induce phase fluctuations due to mirror recoil (Radiation Pressure Noise).

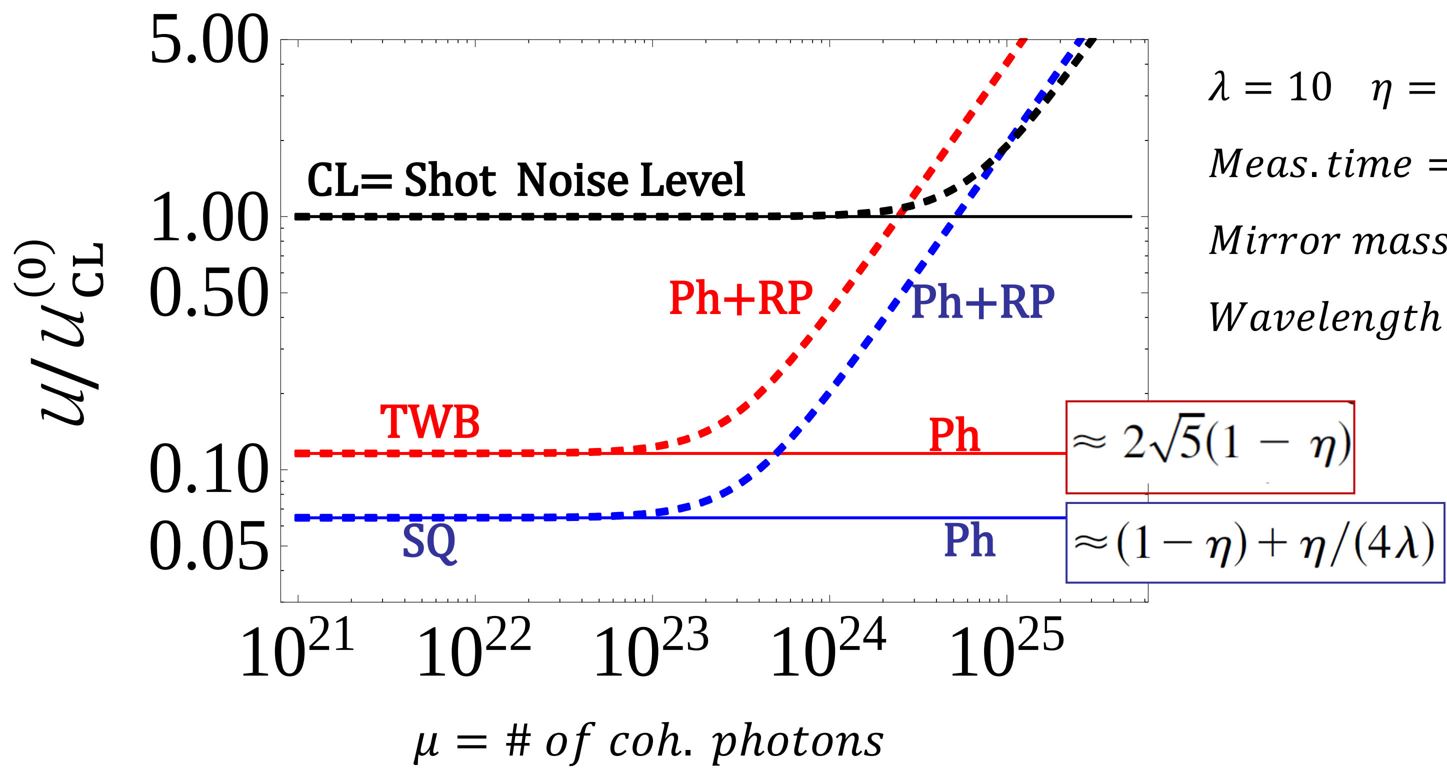
$$\delta\phi_{RP} = (\omega\tau/2mc)\mathcal{P}$$

$\mathcal{P}$ *photons momentum*

For shorter measurement time 10^{-6} s *(HN to be detected in the MHz region)* > 10^{13} W

Quantum light in Coupled Interferometers

"Strong" quantum light regime: $\mu \gg \lambda \gg 1$



$\lambda = 10 \quad \eta = 0{,}98$

$Meas.\,time = 10^{-3}\ s$

$Mirror\ mass = 10^{2} Kg$

$Wavelength = 600\ nm$

Radiation pressure (RP) noise is negligible for reasonable value of the optical power. It starts to appear at $P > 10^{7}\ \ W$

The TAKE-AWAY

- HN is due to the "possible" Quantum Geometric structure of the Space-Time at the Planck-length scale

- HN may have "observable" effect at the macroscopic scale $\rightarrow$ Holometer (2 coupled interferometers)

- Quantum light enhance the sensitivity of the Holometer below the "Shot-Noise" limit
  - Squeezed light provides an enhancement of the order of the mean number of photon of the squeezed light
  - Twin-Beam provides a complete suppression of the shot-noise contribution (0!!!!)
  - Losses (effectively) affect this enhancement
  - Radiation pressure is not an problem (for affordable light power level)

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