

Review of Double and Exceptional Field Theory

Olaf Hohm

Based on work with:

- C. Hull, S. Ki Kwak, B. Zwiebach arXiv: 1003.5027, 1006.4823, 1111.7293
- W. Siegel, B. Zwiebach arXiv: 1306.2970
- H. Samtleben arXiv: 1307.0039, 1307.0509, 1308.1673, 1312.0614, 1312.4542
- D. Lüst, B. Zwiebach arXiv: 1309.2977

Earlier work:

- W. Siegel hep-th/9305073

Bayrischzell, May 2014

Plan of the talk:

Part I: Two-derivative DFT

- Efficient reformulation of supergravity ('generalized geometry')
- Gauge symmetry of DFT: generalized coordinate transformations
- non-geometric backgrounds

Part II: Higher-derivative α' corrections

- deformed gauge structure
- exact action

Part III: Exceptional Field Theory

- Gauge fields for 'generalized geometry brackets'
- $E_{n(n)}$ covariant form of $D = 11$ supergravity
- Conclusions and Outlook

Part I: Two-derivative Double Field Theory

Reformulation (Extension?) of spacetime action for massless string fields:

$$S_{\text{NS}} = \int d^D x \sqrt{-g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12} H^{ijk} H_{ijk} + \frac{1}{4} \alpha' R^{ijkl} R_{ijkl} + \dots \right]$$

generalized metric and doubled coordinates $X^M = (\tilde{x}_i, x^i)$,

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik} b_{kj} \\ b_{ik} g^{kj} & g_{ij} - b_{ik} g^{kl} b_{lj} \end{pmatrix} \in O(D, D)$$

DFT Action (dilaton density $e^{-2d} = e^{-2\phi} \sqrt{-g}$):

$$S_{\text{DFT}} = \int d^{2D} X e^{-2d} \mathcal{R}(\mathcal{H}, d) \xrightarrow{\tilde{\partial}^i = 0} S_{\text{NS}}|_{\alpha'=0}$$

generalized curvature scalar

$$\begin{aligned} \mathcal{R} \equiv & 4 \mathcal{H}^{MN} \partial_M \partial_N d - \partial_M \partial_N \mathcal{H}^{MN} - 4 \mathcal{H}^{MN} \partial_M d \partial_N d + 4 \partial_M \mathcal{H}^{MN} \partial_N d \\ & + \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_K \mathcal{H}_{NL} \end{aligned}$$

Gauge transformations and generalized Lie derivatives

Recall g.c.t. from GR:

$$g'_{ij}(x') = \frac{\partial x^k}{\partial x'^i} \frac{\partial x^l}{\partial x'^j} g_{kl}(x)$$

Infinitesimally, $x'^i = x^i - \xi^i(x)$, governed by Lie derivatives

$$\delta_\xi g_{ij} = \mathcal{L}_\xi g_{ij} \equiv \xi^k \partial_k g_{ij} + \partial_i \xi^k g_{kj} + \partial_j \xi^k g_{ik}$$

In DFT gauge invariance governed by generalized Lie derivatives

$$\hat{\mathcal{L}}_\xi \mathcal{H}_{MN} = \xi^P \partial_P \mathcal{H}_{MN} + (\partial_M \xi^P - \partial^P \xi_M) \mathcal{H}_{PN} + (\partial_N \xi^P - \partial^P \xi_N) \mathcal{H}_{MP}$$

$$\hat{\mathcal{L}}_\xi (e^{-2d}) = \partial_M (\xi^M e^{-2d})$$

Invariance and closure, $[\hat{\mathcal{L}}_{\xi_1}, \hat{\mathcal{L}}_{\xi_2}] = \hat{\mathcal{L}}_{[\xi_1, \xi_2]_C}$, modulo strong constraint

$$\eta^{MN} \partial_M \partial_N = 2 \tilde{\partial}^i \partial_i = 0 \quad \eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Generalized coordinate transformations

Generalized g.c.t. that reproduce this infinitesimally:

$$S'(X') = S(X) \quad A'_M(X') = \mathcal{F}_M{}^N A_N(X)$$

and analogously on higher tensors, where [O.H., Zwiebach, 1207.4198]

$$\mathcal{F}_M{}^N \equiv \frac{1}{2} \left(\frac{\partial X^P}{\partial X'^M} \frac{\partial X'_P}{\partial X_N} + \frac{\partial X'_M}{\partial X_P} \frac{\partial X^N}{\partial X'^P} \right)$$

Setting $X'^M = X^M - \xi^M(X)$ we get $\delta_\xi = \hat{\mathcal{L}}_\xi$.

- $x^{i\prime} = x^{i\prime}(x)$, $\tilde{x}'_i = \tilde{x}_i$ leads to usual g.c.t.,
 $\tilde{x}'_i = \tilde{x}_i - \tilde{\xi}_i(x)$, $x^{i\prime} = x^i$ leads to $b_{ij} \rightarrow b_{ij} + \partial_i \tilde{\xi}_j - \partial_j \tilde{\xi}_i$
- composition according to BCH of C-bracket
- consistent patching for genuinely non-geometric backgrounds

Supersymmetric and Heterotic Extensions

(Generalized) vielbein formalism required [Siegel (1993), O.H. & Ki Kwak (2010)]

$$\mathcal{H}^{MN} = \hat{\eta}^{AB} E_A{}^M E_B{}^N , \quad \hat{\eta}_{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & \eta_{\bar{a}\bar{b}} \end{pmatrix}$$

local $SO(1, 9+n)_L \times SO(1, 9)_R$ Lorentz symmetry

Gauge fixing to diagonal subgroup

$$E_A{}^M = \begin{pmatrix} E_{ai} & E_a{}^i \\ E_{\bar{a}i} & E_{\bar{a}}{}^i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} e_{ia} + b_{ij} e_a{}^j & e_a{}^i \\ -e_{i\bar{a}} + b_{ij} e_{\bar{a}}{}^j & e_{\bar{a}}{}^i \end{pmatrix}$$

Fermions: singlets under $O(10+n, 10)$ and $\hat{\mathcal{L}}_\xi$

[O.H., S. Ki Kwak, 1111.7293]

- Ψ_a : vector of $SO(1, 9+n)_L$, spinor of $SO(1, 9)_R$
- ρ : spinor of $SO(1, 9)_R$,
- ϵ : spinor of $SO(1, 9)_R$

$\mathcal{N} = 1$ supersymmetric Lagrangian (including n vector multiplets)

$$\mathcal{L} = e^{-2d} \left(\mathcal{R}(E, d) - \bar{\Psi}^a \gamma^{\bar{b}} \nabla_{\bar{b}} \Psi_a + \bar{\rho} \gamma^{\bar{a}} \nabla_{\bar{a}} \rho + 2 \bar{\Psi}^a \nabla_a \rho \right)$$

$\mathcal{N} = 1$ supersymmetry transformations

$$E_{\bar{b}}{}^M \delta_\epsilon E_{aM} = \frac{1}{2} \bar{\epsilon} \gamma_{\bar{b}} \Psi_a \quad \delta_\epsilon d = -\frac{1}{4} \bar{\epsilon} \rho \quad \delta_\epsilon \Psi_a = \nabla_a \epsilon \quad \delta_\epsilon \rho = \gamma^{\bar{a}} \nabla_{\bar{a}} \epsilon$$

Proof of supersymmetric invariance: variation of bosonic term

$$e^{2d} \delta_\epsilon \mathcal{L}_B = \frac{1}{2} \bar{\epsilon} \rho \mathcal{R} + \bar{\epsilon} \gamma^{\bar{b}} \Psi^a \mathcal{R}_{a\bar{b}}$$

variation of fermionic terms

$$\begin{aligned} e^{2d} \delta_\epsilon \mathcal{L}_F &= -2 \bar{\Psi}^a \gamma^{\bar{b}} \nabla_{\bar{b}} \nabla_a \epsilon + 2 \bar{\rho} \gamma^{\bar{a}} \nabla_{\bar{a}} (\gamma^{\bar{b}} \nabla_{\bar{b}} \epsilon) + 2 \nabla^a \bar{\epsilon} \nabla_a \rho + 2 \bar{\Psi}^a \nabla_a (\gamma^{\bar{b}} \nabla_{\bar{b}} \epsilon) \\ &= -2 \bar{\Psi}^a [\gamma^{\bar{b}} \nabla_{\bar{b}}, \nabla_a] \epsilon + 2 \bar{\rho} (\gamma^{\bar{a}} \nabla_{\bar{a}} \gamma^{\bar{b}} \nabla_{\bar{b}} - \nabla^a \nabla_a) \epsilon \\ &= \bar{\Psi}^a \gamma^{\bar{b}} \mathcal{R}_{a\bar{b}} \epsilon - \frac{1}{2} \bar{\rho} \mathcal{R} \epsilon = -\frac{1}{2} \bar{\epsilon} \rho \mathcal{R} - \bar{\epsilon} \gamma^{\bar{b}} \Psi^a \mathcal{R}_{a\bar{b}} \end{aligned}$$

Thus: $\delta_\epsilon (S_B + S_F) = 0$

Comparison: standard $\mathcal{N} = 1$ supergravity action

$$\begin{aligned}
S = & \int d^{10}x e e^{-2\phi} \left[\left(R + 4\partial^i \phi \partial_i \phi - \frac{1}{12} \hat{H}^{ijk} \hat{H}_{ijk} - \frac{1}{4} F_{ij} F^{ij} \right) \right. \\
& - \bar{\psi}_i \gamma^{ijk} D_j \psi_k - 2\bar{\lambda} \gamma^i D_i \lambda - \frac{1}{2} \bar{\chi}^\alpha \not{D} \chi_\alpha \\
& + 2\bar{\psi}^i (\partial_i \phi) \gamma^j \psi_j - \bar{\psi}_i (\not{\partial} \phi) \gamma^i \lambda - \frac{1}{4} \bar{\chi}_\alpha \gamma^i \gamma^{jk} F_{jk}{}^\alpha \left(\psi_i + \frac{1}{6} \gamma_i \lambda \right) \\
& + \frac{1}{24} \hat{H}_{ijk} \left(\bar{\psi}_m \gamma^{mijkn} \psi_n + 6\bar{\psi}^i \gamma^j \psi^k - 2\bar{\psi}_m \gamma^{ijk} \gamma^m \lambda + \frac{1}{2} \bar{\chi}^\alpha \gamma^{ijk} \chi_\alpha \right) \\
& \left. + \text{quartic fermions} \right]
\end{aligned}$$

where

$$\hat{H}_{ijk} = 3 \left(\partial_{[i} b_{jk]} - A_{[i}{}^\alpha \partial_j A_{k]\alpha} \right)$$

Comparison: standard $\mathcal{N} = 1$ supersymmetry rules

$$\delta_\epsilon e_i{}^a = \frac{1}{2}\bar{\epsilon} \gamma^a \psi_i - \frac{1}{4}\bar{\epsilon} \lambda e_i{}^a ,$$

$$\delta_\epsilon \phi = -\bar{\epsilon} \lambda \quad , \quad \delta_\epsilon A_i{}^\alpha = \frac{1}{2}\bar{\epsilon} \gamma_i \chi^\alpha \quad ,$$

$$\delta_\epsilon \chi^\alpha = -\frac{1}{4}\gamma^{ij} F_{ij}{}^\alpha \epsilon$$

$$\delta_\epsilon \psi_i = D_i \epsilon - \frac{1}{8}\gamma_i(\not{\partial}\phi)\epsilon + \frac{1}{96}(\gamma_i{}^{klm} - 9\delta_i{}^k \gamma^{lm})\hat{H}_{klm}\epsilon ,$$

$$\delta_\epsilon \lambda = -\frac{1}{4}(\not{\partial}\phi)\epsilon + \frac{1}{48}\gamma^{ijk}\hat{H}_{ijk}\epsilon ,$$

$$\delta_\epsilon b_{ij} = \frac{1}{2}(\bar{\epsilon} \gamma_i \psi_j - \bar{\epsilon} \gamma_j \psi_i) - \frac{1}{2}\bar{\epsilon} \gamma_{ij} \lambda + \frac{1}{2}\bar{\epsilon} \gamma_{[i} \chi^\alpha A_{j]\alpha} .$$

Part II: Higher-derivative deformations

Geometrical structures for generalized vector $\Xi \equiv (\xi^M)$ in $\alpha' = 0$ DFT:

$$\langle \Xi_1 | \Xi_2 \rangle = \xi_1^M \xi_2^N \eta_{MN}, \quad [\Xi_1, \Xi_2]_C^M = \xi_{[1}^N \partial_N \xi_{2]}^M - \frac{1}{2} \xi_1^K \overset{\leftrightarrow}{\partial}^M \xi_{2K}$$

$$\hat{\mathcal{L}}_\Xi V^M = \xi^P \partial_P V^M + (\partial^M \xi_P - \partial_P \xi^M) V^P$$

All receive non-trivial higher-derivative corrections:

$$\langle \Xi_1 | \Xi_2 \rangle = \xi_1^M \xi_2^N \eta_{MN} - (\partial_N \xi_1^M)(\partial_M \xi_2^N)$$

$$[\Xi_1, \Xi_2]_C^M = \xi_{[1}^N \partial_N \xi_{2]}^M - \frac{1}{2} \xi_1^K \overset{\leftrightarrow}{\partial}^M \xi_{2K} + \frac{1}{2} (\partial_K \xi_1^L) \overset{\leftrightarrow}{\partial}^M (\partial_L \xi_2^K)$$

$$\mathcal{L}_\Xi V^M = \xi^P \partial_P V^M + (\partial^M \xi_P - \partial_P \xi^M) V^P - (\partial^M \partial_K \xi^L) \partial_L V^K$$

Closure and gauge invariance exact! ($\mathcal{L}_\Xi \langle V, W \rangle = \xi^N \partial_N \langle V, W \rangle$, etc.)

Not removable by $O(D, D)$ covariant redefinitions

Non-vanishing for $\tilde{x} = 0 \Rightarrow$ deformation of Courant bracket, etc.

DFT relations for $\mathcal{H} \in O(D, D)$

$$(\mathcal{H}^2)_{MN} \equiv \mathcal{H}_{MK}\mathcal{H}^K{}_N = \eta_{MN} \quad \text{Tr } \mathcal{H} \equiv \eta^{MN}\mathcal{H}_{MN} = 0$$

get deformed \Rightarrow dynamical equations!

$$(\mathcal{M} \star \mathcal{M})_{MN} \equiv 2(\mathcal{M}^2)_{MN} - \frac{1}{2}\partial_M\mathcal{M}^{PQ}\partial_N\mathcal{M}_{PQ} + \dots = 2\eta_{MN}$$

$$\text{tr } \mathcal{M} \equiv \eta^{MN}\mathcal{M}_{MN} - 3\partial_M\partial_N\mathcal{M}^{MN} + \dots = 0$$

In derivative expansion:

$$0^{\text{th}} \text{ order : } \mathcal{M}_{MN} = \mathcal{H}_{MN}, \quad \mathcal{H}^2 = \eta$$

$$1^{\text{st}} \text{ order : } \mathcal{M}_{MN} = \mathcal{H}_{MN} + \frac{1}{2}\{\mathcal{H}, \mathcal{V}^{(2)}\}_{MN}$$

Then

$$0 = \text{tr } \mathcal{M} = 3\mathcal{R}(\mathcal{H}, \phi) \quad [\text{dilaton eq.}] \quad \mathcal{V}^{(2)}\mathcal{H} - \mathcal{H}\mathcal{V}^{(2)} = 0 \quad [\text{gravity eq.}]$$

plus higher-derivative corrections!

CFT Derivation and Action

doubled world-sheet scalars $X^M(z)$, $M = 1, \dots, 2D$,
chirality condition: $P^M = X'^M \equiv Z^M$ $\left[' = \frac{\partial}{\partial z} \right]$

postulate the (two) Virasoro generators

$$\mathcal{S} \equiv \frac{1}{2}(Z^2 - \phi'') \quad \mathcal{T} \equiv \frac{1}{2}\mathcal{M}^{MN}Z_MZ_N - \frac{1}{2}(\widehat{\mathcal{M}}^M Z_M)'$$

OPE defines (various) ‘quantum products’. OPE yields Virasoro²

$$\mathcal{S}(z_1)\mathcal{S}(z_2) = \frac{D}{z_{12}^4} + \frac{2\mathcal{S}(z_2)}{z_{12}^2} + \frac{\mathcal{S}'(z_2)}{z_{12}} + \text{finite}, \quad \text{same for } \mathcal{T}$$

$$\mathcal{S}(z_1)\mathcal{T}(z_2) = \frac{2\mathcal{T}(z_2)}{z_{12}^2} + \frac{\mathcal{T}'(z_2)}{z_{12}} + \text{finite}$$

provided dilaton and gravity equations hold!

Gauge invariant action

$$S = \int e^\phi (\langle \mathcal{T}|\mathcal{S} \rangle - \frac{1}{6}\langle \mathcal{T}|\mathcal{T} \star \mathcal{T} \rangle) = \int e^\phi (\text{Tr} \mathcal{M} - \frac{1}{3}\mathcal{M}^3 + \dots)$$

Part III: Exceptional Field Theory

Cremmer-Julia [1979]: torus reduction of $D = 11$ SUGRA

$\rightarrow E_{6(6)} [D = 5], E_{7(7)} [D = 4], E_{8(8)} [D = 3]$

Larger mathematical framework that explains/makes it manifest?

Hillmann [2009]: truncation of $D = 11$ SUGRA in $4 + 7$ split,
keeping only 'internal' field components and coordinates,

$$G_{MN} = \begin{pmatrix} e^{2\Delta} \eta_{\mu\nu} & 0 \\ 0 & g_{mn}(y) \end{pmatrix}, \quad \text{etc.}$$

extending coordinates to fundamental 56 $\Rightarrow E_{7(7)}$ covariant action

more recently: other groups, geometry, covariant section constraints, etc.

[Berman & Perry (2010), Coimbra, Strickland-Constable & Waldram (2011), etc.]

Complete $D = 11$ SUGRA?? duality transformations in $D = 11$??

$E_{7(7)}$ Exceptional Field Theory

Coordinates (x^μ, Y^M) , $\mu = 0, \dots, 3$, M : fundamental **56** of $E_{7(7)}$

$$(t_\alpha)^{MN} \partial_M \otimes \partial_N = 0, \quad \Omega^{MN} \partial_M \otimes \partial_N = 0$$

with Ω^{MN} symplectic form of $E_{7(7)} \subset Sp(56)$

Generalized Lie derivative

$$\mathbb{L}_\Lambda V^M \equiv \Lambda^K \partial_K V^M - 12 \mathbb{P}^M{}_N{}^K{}_L \partial_K \Lambda^L V^N + \lambda \partial_P \Lambda^P V^M$$

with projector $\mathbb{P}^M{}_N{}^K{}_L$ onto adjoint; $\lambda(V)$ density weight.

$\rightarrow E_{7(7)}$ bracket

$$[\Lambda_1, \Lambda_2]_E^M = 2\Lambda_1^K \partial_K \Lambda_2^M + 12 (t_\alpha)^{MN} (t_\alpha)_{KL} \Lambda_1^K \partial_N \Lambda_2^L - \frac{1}{4} \Omega^{MN} \Omega_{KL} \partial_N (\Lambda_1^K \Lambda_2^L)$$

Jacobiator

$$J^M(\Lambda_1, \Lambda_2, \Lambda_3) = (t_\alpha)^{MN} \partial_N \chi_\alpha(\Lambda) + \Omega^{MN} \chi_N(\Lambda),$$

→ covariant curvature involves two 2-forms

$$\mathcal{F}_{\mu\nu}{}^M \equiv F_{\mu\nu}{}^M - 12 (t^\alpha)^{MN} \partial_N B_{\mu\nu\alpha} - \frac{1}{2} \Omega^{MN} B_{\mu\nu N}$$

where $B_{\mu\nu N}$ is *covariantly constrained* compensator field

Twisted (electric-magnetic) self-duality relations

$$\mathcal{F}_{\mu\nu}{}^M = -\frac{1}{2} e \varepsilon_{\mu\nu\rho\sigma} \Omega^{MN} \mathcal{M}_{NK} \mathcal{F}^{\rho\sigma K}$$

gauge vectors $A_\mu{}^M$ include 7 Kaluza-Klein vectors from $D = 11$ metric

→ also 7 dual gauge vectors → dual graviton in *non-linear* duality relation

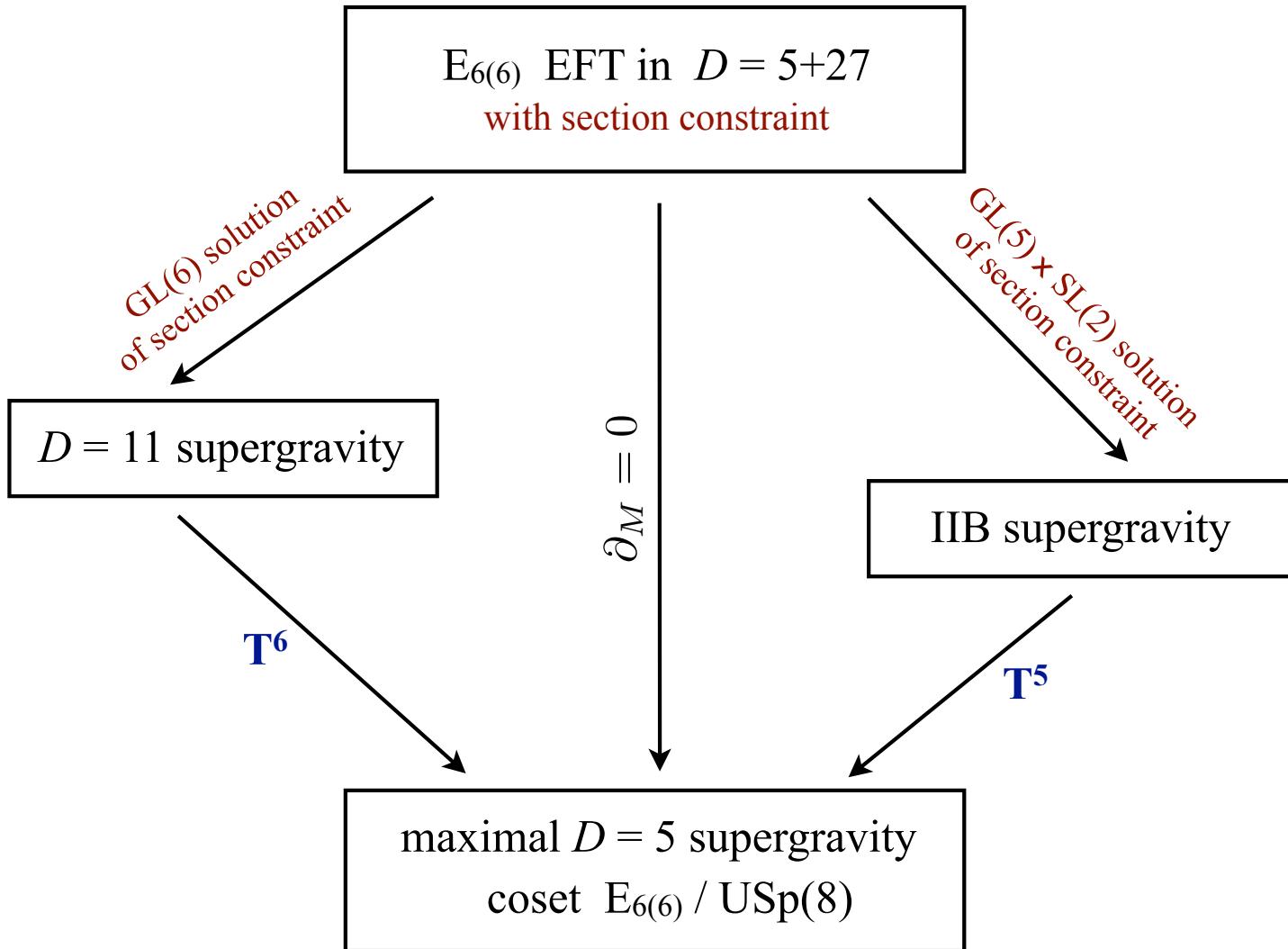
[no-go theorems: Bekaert, Boulanger, Henneaux (2002)]

‘Resolution’: dual graviton & compensating gauge field

[Boulanger, O.H. (2008)]

solving section constraint → only 7 $B_{\mu\nu N}$ survive

→ 7 ‘dual graviton’ fields pure gauge



Summary & Outlook

- DFT provides strikingly economic reformulation of supergravity
- strong evidence that it gives more than supergravity:
truly non-geometric backgrounds well-defined thanks to
DFT diffeomorphisms → compactification, generalized Scherk-Schwarz
[Aldazabal, Baron, Marques & Nunez, Geissbühler (2011)]
- intriguing new ‘quantum geometry’ beyond lowest order in α'
⇒ underlying gauge principle of string theory?
- Exceptional field theory generalization of DFT to M-theory/U-duality
⇒ same generalizations as for DFT: supersymmetry, higher derivative corrections, non-trivial backgrounds, etc.
- many obvious open questions/problems!