One Loop Two Point Function in a Nonlocal U(1) Gauge Theory Model

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- Based on 1306.1239, see also 0807.4886, 1109.2485, 1111.4951 and 1402.6184.

Outline

Noncommutative/nonlocal gauge theories

Photon two point function

Including a neutral fermion

$D \to 2$ result

Outlook

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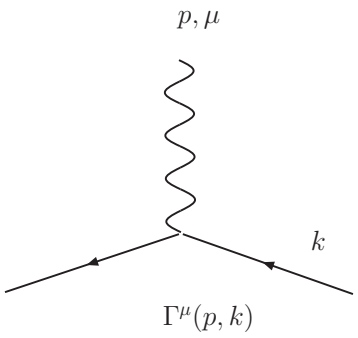
Noncommutative gauge theory

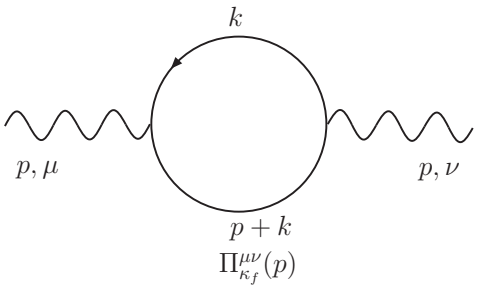
- NCGFT on Moyal space

$$S = -\frac{1}{2}\int F_{\mu\nu} \star F^{\mu\nu}, F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu \overset{\star}{,} A_\nu]$$
$$\delta_\Lambda A_\mu = \partial_\mu \Lambda + i[\Lambda \overset{\star}{,} A_\mu],\ \delta_\Lambda F_{\mu\nu} = i[\Lambda \overset{\star}{,} F_{\mu\nu}];$$

- Moyal product induces phase factor in loop integral.
- When phase factor is nontrivial, the the Schwinger parameterized loop integral integral are regularized by it and become a integral over modified bessel functions K_ν, which process an IR singularity. Such phenomenon is called *UV/IR mixing*.

NCQGFT, an example of neutral fermion





$$S = \int -\frac{1}{2}F^2 + i\bar{\Psi}\gamma^\mu(\partial_\mu\Psi - i[A_\mu \overset{\star}{,} \Psi]),\ \Gamma^\mu[p,k] = \gamma^\mu \sin\frac{p\theta k}{2}$$

$$\Pi^{\mu\nu} = -\mathrm{tr}\ \mu^{d-D}\int \frac{d^Dk}{(2\pi)^D}\sin^2\frac{k\theta p}{2}\gamma^\mu\frac{i(p+k)^\rho\gamma_\rho}{(p+k)^2}\gamma^\nu\frac{ik^\sigma\gamma_\sigma}{k^2}$$

$$= \mathrm{tr}\ \mu^{d-D}\int\frac{d^Dk}{(2\pi)^D}2^{-2}(\underbrace{2}_{\text{planar}}\ \underbrace{-e^{ik\theta p} - e^{-ik\theta p}}_{\text{nonplanar}})\frac{\gamma^\mu(p+k)^\rho\gamma_\rho\gamma^\nu k^\sigma\gamma_\sigma}{(p+k)^2k^2}$$

$$\Pi\mu\nu_{D\to 4} = \pi^{-2}\Bigg[\underbrace{\left(g^{\mu\nu}p^2 - p^\mu p^\nu\right)\left(\frac{1}{6}\left[\frac{2}{\epsilon} + \ln\pi e^{\gamma_E} - \ln\frac{p^2}{\mu^2}\right]\right.}_{\text{Planar UV divergence}}$$

$$\underbrace{\left.-2\int_0^1 dx\, x(1-x)K_0\left[(x(1-x)p^2(\theta p)^2)^{\frac{1}{2}}\right]\right) - 2(\theta p)^\mu(\theta p)^\nu\int_0^1 dx\, x(1-x)p^2(\theta p)^{-2}K_2\left[(x(1-x)p^2(\theta p)^2)^{\frac{1}{2}}\right]\Bigg]}_{\text{nonplanar Bessel K}-\text{function integrals}}.$$

Seiberg-Witten map from phenomenological viewpoint

- We would like to deform certain gauge theory model (of phenomenological importance), introduce additional interactions induced by $\theta^{\mu\nu}$, without changing the field content and gauge group representation of the original model.
- Take pure gauge theory as an example. Consider a composite operator $F_{\mu\nu}(a_\mu, \theta^{\mu\nu})$ which transforms in such a way that $\delta_\lambda F_{\mu\nu} = i[\Lambda(\lambda, a_\mu) \stackrel{\star}{,} F_{\mu\nu}]$ then

$$S \propto \mathrm{tr} \int F_{\mu\nu} \star F^{\mu\nu}$$

  is invariant under this gauge transformation.
- Seiberg-Witten map consistency conditions

$$\partial_\mu \Lambda + i[\Lambda, A_\mu] = \delta_\lambda A_\mu[a_\mu],$$
$$[\Lambda[\alpha, a_\mu] \stackrel{\star}{,} \Lambda[\beta, a_\mu]] + i\delta_\alpha \Lambda[\beta, a_\mu] - i\delta_\beta \Lambda[\alpha, a_\mu] = \Lambda[[\alpha, \beta], a_\mu].$$

  ensures that $F_{\mu\nu}$ is then the conventional noncommutative field strength of the composite operator (noncommutative gauge field) $A_\mu(a_\mu, \theta)$.

Seiberg-Witten map expansions

- Phenomenology applications require the commutative fields a_μ etc. to be considered as primary, thus the aforementioned action has to be expanded.
- Initially nonlocality was hidden due to an expansion over NC parameter θ

$$A_\mu = a_\mu - \frac{1}{2}\theta^{\nu\rho} a_\nu (\partial_\rho a_\mu + f_{\rho\mu}) + \mathcal{O}(\theta^2)\,,$$
$$\Psi = \psi - \theta^{\mu\nu} a_\mu \partial_\nu \psi + \mathcal{O}(\theta^2)\psi\,,$$
$$F_{\mu\nu} = f_{\mu\nu} + \theta^{\rho\tau}\Big(f_{\mu\rho} f_{\nu\tau} - a_\rho \partial_\tau f_{\mu\nu}\Big) + \mathcal{O}(\theta^2)\,.$$

- Remove nonlocality is on the cost of higher derivative terms (poor power-counting) .
- Expansion over a_μ recovers the nonlocality, for example for U(1) model

$$A_\mu = a_\mu - \frac{1}{2}\theta^{\nu\rho} a_\nu \star_2 (\partial_\rho a_\mu + f_{\rho\mu}) + \mathcal{O}(a^3)\,,$$
$$\Psi = \psi - \theta^{\mu\nu} a_\mu \star_2 \partial_\nu \psi + \mathcal{O}(a^2)\psi\,,$$
$$F_{\mu\nu} = f_{\mu\nu} + \theta^{\rho\tau}\Big(f_{\mu\rho} \star_2 f_{\nu\tau} - a_\rho \star_2 \partial_\tau f_{\mu\nu}\Big) + \mathcal{O}(a^3)\,.$$

From noncommutative to a nonlocal U(1) Action

- New generalized star product $\star_2$ is commutative and non-associative.

$$f \star_2 g = f(x_1) \frac{\sin \frac{\partial_1 \theta \partial_2}{2}}{\frac{\partial_1 \theta \partial_2}{2}} g(x_2) \Big|_{x_1 = x_2 = x} = g(x_2) \frac{\sin \frac{\partial_2 \theta \partial_1}{2}}{\frac{\partial_2 \theta \partial_1}{2}} f(x_1) \Big|_{x_1 = x_2 = x} = g \star_2 f \,,$$

$$\int f \star_2 g = \int f \cdot g, \; (f \star_2 g) \star_2 h \neq f \star_2 (g \star_2 h), \; \int (f \star_2 g) \star_2 h = \int f \star_2 (g \star_2 h) \,.$$

- A nonlocal $U(1)$ action with photon and a neutral fermion (photino).

$$S = \int -\frac{1}{2} f_{\mu\nu} f^{\mu\nu} + i \bar{\psi} \partial^\mu \gamma_\mu \psi$$
$$+ \theta^{ij} f^{\mu\nu} \left(\frac{1}{4} f_{ij} \star_2 f_{\mu\nu} - f_{\mu i} \star_2 f_{\nu j} \right) - i \theta^{ij} \bar{\psi} \gamma^\mu \left(\frac{1}{2} f_{ij} \star_2 \partial_\mu \psi - f_{\mu i} \star_2 \partial_j \psi \right) .$$

- This action is U(1) gauge invariant term by term.

- Seiberg-Witten map of $F_{\mu\nu}$ allows a further freedom

$$F_{\mu\nu}(\kappa_g) = f_{\mu\nu} + \theta^{\rho\tau}\left(\kappa_g f_{\mu\rho} \star_2 f_{\nu\tau} - a_\rho \star_2 \partial_\tau f_{\mu\nu}\right) + \mathcal{O}(a^3)\,.$$

- This motivates us to add two free parameters κ_g and κ_f

$$\begin{aligned} S = \int & -\frac{1}{2} f_{\mu\nu} f^{\mu\nu} + i\bar{\psi}\partial^\mu \gamma_\mu \psi \\ & + \theta^{ij} f^{\mu\nu}\left(\frac{1}{4} f_{ij} \star_2 f_{\mu\nu} - \kappa_g f_{\mu i} \star_2 f_{\nu j}\right) \\ & - i\theta^{ij}\bar{\psi}\gamma^\mu\left(\frac{1}{2} f_{ij} \star_2 \partial_\mu \psi - \kappa_f f_{\mu i} \star_2 \partial_j \psi\right). \end{aligned}$$

- By variating the linear combinations of the gauge invariant interaction terms, we obtain gauge invariant nonlocal theories without SW map correspondence.
- These parameters affect the loop behavior, as shown below.

Outline

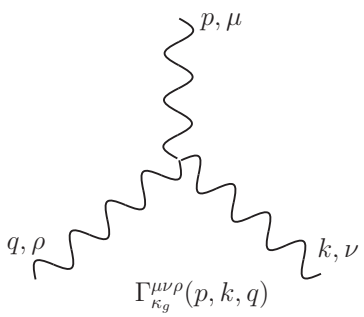
Action

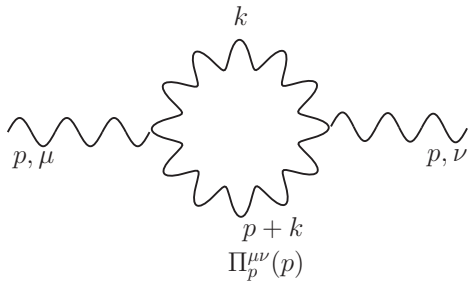
- A nonlocal U(1) gauge invariant action on Euclidean spacetime with a dimensionless free parameter κ_g

$$S_g = \int -\frac{1}{2} f^{\mu\nu} f_{\mu\nu} + \theta^{ij} f^{\mu\nu} \left(\frac{1}{4} f_{ij} \star_2 f_{\mu\nu} - \kappa_g f_{\mu i} \star_2 f_{\nu j} \right).$$

- No Seiberg-Witten map correspondence is available for this action, i.e. it is nonlocal but not really "noncommutative".
- Only a three photon self-interaction vertex exists in this model.

Vertex&diagram





$$\Gamma^{\mu\nu\rho}_{\kappa_g}(p,k,q) = F(k,q)V^{\mu\nu\rho}_{\kappa_g}(p,k,q); \quad F(k,q) = \frac{\sin\frac{k\theta q}{2}}{\frac{k\theta q}{2}},$$

$$\begin{aligned} V^{\mu\nu\rho}_{\kappa_g}(p,k,q) =& \kappa_g \Big\{ -(p\theta k)(p-k)^\rho g^{\mu\nu} - \theta^{\mu\nu}\Big[p^\rho(kq) - k^\rho(pq)\Big] + (\theta p)^\nu \Big[g^{\mu\rho}q^2 - q^\nu q^\rho\Big] \\ & + (\theta p)^\rho \Big[g^{\mu\nu}k^2 - k^\mu k^\nu\Big] + \theta^{\mu\sigma}(k+q+\kappa_g^{-1}p)_\sigma \Big[g^{\nu\rho}(kq) - q^\nu k^\rho\Big]\Big\} \\ & + \text{cyclic permutations.} \end{aligned}$$

$$\Pi^{\mu\nu}_p(p) = \frac{1}{2}\int \frac{d^Dk}{(2\pi)^D}\Gamma^{\mu\rho\sigma}_{\kappa_g}(p,k,-p-k)\frac{-ig_{\rho\rho'}}{k^2}\Gamma^{\nu\rho'\sigma'}_{\kappa_g}(-p,-k,k+p)\frac{-ig_{\sigma\sigma'}}{(p+k)^2}.$$

Parametrization

- $\star_2$ induces diffraction factor $F(p,k)$ instead of just phase factor, some additional measure has to be taken for it.
- Our current parametrization goes as follows

$$\frac{1}{k^2(p+k)^2}\frac{1}{k\theta p} = 2i\int_0^1 dx \int_0^\infty dy \int_0^\infty d\lambda\lambda^2$$
$$\exp\Big[-\lambda[(1-x)k^2 + x(p+k)^2 + iy(k\theta p)]\Big].$$

- When the phase part is multiplied to the denorminator

$$\frac{2-e^{ik\theta p}-e^{-ik\theta p}}{k^2(p+k)^2(k\theta p)}\cdot\{\text{numerator}\} = 2\mathrm{i}\int_0^1 \mathrm{dx}\int_0^{\frac{1}{\lambda}} \mathrm{dy}\int_0^\infty \mathrm{d}\lambda\lambda^2$$
$$\exp\left[-\lambda\big(l^2 + x(1-x)p^2 + \frac{y^2}{4}(\theta p)^2\big)\right]\{y \text{ odd terms of the numerator}\},$$
$$l = k + xp + \frac{i}{2}y(\theta p).$$

Parametrization, cont'd

- Two integral ends of parameter y loosely correspond to the planar and nonplanar parts of the amplitude.
- Higher negative powers of $k\theta p$ bring out new contributions

$$\frac{2 - e^{ik\theta p} - e^{-ik\theta p}}{k^2(p+k)^2(k\theta p)^2} \cdot \{\text{numerator}\}$$

$$= -2 \int_0^1 dx \int_0^{\frac{1}{\lambda}} dy \int_0^\infty d\lambda \lambda^3 \exp\left[-\lambda\left(l^2 + x(1-x)p^2 + \frac{y^2}{4}(\theta p)^2\right)\right]$$

$$\cdot \Big(\underbrace{y \cdot \{y \text{ even terms of the numerator}\}}_{\text{Planar and Bessel K}-\text{function integrals}}$$

$$\underbrace{-\lambda^{-1} \cdot \{y \text{ even terms of the numerator}\}}_{\text{hypergeometric integrals}} \Big).$$

Tensor structure

- In QED, vacuum polarization is associated with $p^\mu p^\nu - p^2 g^{\mu\nu}$ to satisfy Ward identity.
- NCQED brings additional noncommutative structure proportional to $(\theta p)^\mu (\theta p)^\nu$.
- The nonlocal model here has much more tensor structures in general

$$\Pi^{\mu\nu}_{\kappa_g}(p)_D = \frac{1}{(4\pi)^2}\Big\{\Big[g^{\mu\nu}p^2 - p^\mu p^\nu\Big]B_1^{\kappa_g}(p) + (\theta p)^\mu(\theta p)^\nu B_2^{\kappa_g}(p)$$
$$+\Big[g^{\mu\nu}(\theta p)^2 - (\theta\theta)^{\mu\nu}p^2 + p^{\{\mu}(\theta\theta p)^{\nu\}}\Big]B_3^{\kappa_g}(p)$$
$$+\Big[(\theta\theta)^{\mu\nu}(\theta p)^2 + (\theta\theta p)^\mu(\theta\theta p)^\nu\Big]B_4^{\kappa_g}(p) + (\theta p)^{\{\mu}(\theta\theta\theta p)^{\nu\}}B_5^{\kappa_g}(p)\Big\}.$$

- Each of the five tensor structures above satisfies the Ward identity by itself.

Lots of divergences arise when $D \to 4$

$$B_1^{\kappa_g}(p) \sim \left(\frac{2}{3}(1-3\kappa_g)^2 + \frac{2}{3}(1+2\kappa_g)^2 \, \frac{p^2(\mathrm{tr}\theta\theta)}{(\theta p)^2} + \frac{4}{3}(1+4\kappa_g+\kappa_g^2) \, \frac{p^2(\theta\theta p)^2}{(\theta p)^4} \right)$$
$$\cdot \left[\frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2) \right] \quad - \frac{16}{3}(1-\kappa_g)^2 \, \frac{1}{(\theta p)^6} \left((\mathrm{tr}\theta\theta)(\theta p)^2 + 4(\theta\theta p)^2 \right),$$

$$B_2^{\kappa_g}(p) \sim \left(\frac{8}{3}(1-\kappa_g)^2 \, \frac{p^4(\theta\theta p)^2}{(\theta p)^6} + \frac{2}{3}(1-2\kappa_g-5\kappa_g^2) \frac{p^4(\mathrm{tr}\theta\theta)}{(\theta p)^4} + \frac{2}{3}(25-86\kappa_g \right.$$
$$\left. + 73\kappa_g^2) \frac{p^2}{(\theta p)^2} \right) \left[\frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2) \right] - \frac{16}{3}(1-3\kappa_g)(3-\kappa_g) \frac{1}{(\theta p)^4}$$
$$+ \frac{32}{3}(1-\kappa_g)^2 \frac{1}{(\theta p)^8} \left((\mathrm{tr}\theta\theta)(\theta p)^2 + 6(\theta\theta p)^2 \right),$$

$$B_3^{\kappa_g}(p) \sim -\frac{1}{3}(1-2\kappa_g-11\kappa_g^2) \frac{p^2}{(\theta p)^2} \left[\frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2) \right] - \frac{8}{3(\theta p)^4}(1-10\kappa_g+17\kappa_g^2),$$

$$B_4^{\kappa_g}(p) \sim -2(1+\kappa_g)^2 \frac{p^4}{(\theta p)^4} \left[\frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2) \right] - \frac{32p^2}{3(\theta p)^6}(1-6\kappa_g+7\kappa_g^2),$$

$$B_5^{\kappa_g}(p) \sim \frac{4}{3}(1+\kappa_g+4\kappa_g^2) \frac{p^4}{(\theta p)^4} \left[\frac{2}{\epsilon} + \ln(\mu^2(\theta p)^2) \right] + \frac{64p^2}{3(\theta p)^6}(1-\kappa_g)(1-2\kappa_g).$$

Special $\theta^{\mu\nu}$

- A lot of divergences appear in our 1-loop amplitude, some simplification needed.
- Introducing a noncommutative parameter with unique arithmetic property

$$\theta_s^{\mu\nu} = \frac{1}{\Lambda_{\rm NC}^2}\begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}, \ (\theta\theta)^{\mu\nu} = -\frac{1}{\Lambda_{\rm NC}^4} g^{\mu\nu}.$$

- Five tensor structures reduce to two

$$\begin{aligned}
\Pi_{\kappa_g}^{\mu\nu}(p)_4\Big|^{\theta_s} &= \frac{1}{(4\pi)^2}\left\{\left[g^{\mu\nu}p^2 - p^\mu p^\nu\right] B_I^{\kappa_g}(p) + (\theta p)^\mu(\theta p)^\nu\, B_{II}^{\kappa_g}(p)\right\} \\
&= \frac{1}{(4\pi)^2}\left\{\left[g^{\mu\nu}p^2 - p^\mu p^\nu\right]\left(B_1^{\kappa_g} + 2\frac{B_3^{\kappa_g}}{\Lambda_{\rm NC}^4} - \frac{B_4^{\kappa_g}}{\Lambda_{\rm NC}^8}\right)\right. \\
&\left. + (\theta p)^\mu(\theta p)^\nu\left(B_2^{\kappa_g} - 2\frac{B_5^{\kappa_g}}{\Lambda_{\rm NC}^4}\right)\right\},
\end{aligned}$$

Divergence cancelation

- Summing over divergent terms according to the relations induced by θ_s

$$B_I^{\kappa_g}(p) \sim (1-3\kappa_g)\left\{\frac{4(1-3\kappa_g)}{3}\left(\frac{2}{\epsilon}+\ln\left(\mu^2(\theta p)^2\right)\right)+\frac{16}{3}\frac{(1+\kappa_g)}{p^2(\theta p)^2}\right\},$$

$$B_{II}^{\kappa_g}(p) \sim (1-3\kappa_g)\left\{2\,p^2\frac{(7-9\kappa_g)}{(\theta p)^2}\left(\frac{2}{\epsilon}+\ln\left(\mu^2(\theta p)^2\right)\right)-\frac{16}{3}\frac{(7-5\kappa_g)}{(\theta p)^4}\right\}.$$

- All divergences, UV and IR, vanish when we set $\kappa_g = 1/3$.....

Finite terms when $\kappa_g = 1/3$

- We then managed to express the full amplitude for $\theta = \theta_s$ and $\kappa_g = 1/3$.

$$B_I^{\kappa_g=1/3}(p) = \frac{112}{27} + \frac{2}{9}\mathcal{I}, B_{II}^{\kappa_g=1/3} = -\frac{p^2}{(\theta p)^2}\left[8 - \mathcal{I}\right],$$

- Some explicit computation shows that the special function integral $\mathcal{I} = 0$, thus

$$qqq \qquad \Pi^{\mu\nu}_{\kappa_g=1/3}(p)_4\Big|^{\theta_s} = \frac{p^2}{\pi^2}\left\{\frac{7}{27}\left[g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right] - \frac{1}{2}\frac{(\theta p)^\mu(\theta p)^\nu}{(\theta p)^2}\right\},$$

$$
\begin{aligned}
\mathcal{I} = & \int_0^1 dx \left(8 - 6x(1-x)\right) K_0[(x(1-x)p^2(\theta p)^2)^{\frac{1}{2}}] + \left(3 - 16x(1-x)\right) \lim_{\epsilon \to 0} (\theta p)^{2\epsilon} \\
& \cdot \left[(x(1-x)p^2(\theta p)^2)^{1-\epsilon} \Gamma\left[\epsilon - 1\right] {}_1F_2 \left(\frac{1}{2}; \frac{3}{2}, 2-\epsilon; \frac{x(1-x)p^2(\theta p)^2}{4} \right) \right. \\
& \left. - \frac{2^{2-2\epsilon}}{2\epsilon - 1} \Gamma\left[1-\epsilon\right] {}_1F_2 \left(\frac{2\epsilon - 1}{2}; \epsilon, \frac{1+2\epsilon}{2}; \frac{x(1-x)p^2(\theta p)^2}{4} \right) \right] \\
= & - \int_0^1 dx \left(8 - 6x(1-x)\right) \left(\sum_{k=0}^{\infty} \frac{1}{\Gamma\left[2k+2\right]} \left(\frac{p^2 \theta p^2}{4} \right)^k \left(\frac{1}{2} \ln \frac{p^2(\theta p)^2}{4} - \psi(2k+2) \right) \right) \\
& + 4\left(3 - 16x(1-x)\right) \left(1 - \sum_{k=0}^{\infty} \frac{2x^k(1-x)^k}{\Gamma[k+1]\Gamma[k+2](2k+1)} \left(\frac{p^2(\theta p)^2}{4} \right)^{k+1} \right. \\
& \left. \cdot \left(\frac{1}{2} \ln \frac{x(1-x)p^2(\theta p)^2}{4} + \frac{1}{2}\psi(k+1) + \frac{1}{2}\psi(k+2) + \frac{1}{2k+1} \right) \right) \\
= & - 8\left(\left(\frac{1}{2} \ln \frac{p^2(\theta p)^2}{4} - \psi(2) \right) - 6 \cdot \frac{1}{\Gamma[4]} \left(\frac{1}{2} \ln \frac{p^2(\theta p)^2}{4} + 1 - \psi(4) \right) \right) - \left(3 \cdot 4 \right. \\
& \left. - 16 \cdot \frac{2}{3} \right) + \sum_{k=0}^{\infty} \left(\frac{p^2(\theta p)^2}{4} \right)^{k+1} \left[\left(\ln \frac{p^2(\theta p)^2}{4} - 2\psi(2k+4) \right) \left(- \frac{4}{\Gamma\left[2k+4\right]} + \frac{24(k+2)^2}{\Gamma[2k+6]} \right. \right. \\
& \left. + \frac{12(k+1)}{\Gamma\left[2k+4\right](2k+1)} - \frac{64(k+1)(k+2)^2}{\Gamma\left[2k+6\right](2k+1)} \right) + \left(\frac{12(k+1)}{\Gamma[2k+4](2k+1)} \right. \\
& \cdot \left(\frac{1}{k+1} - \frac{2}{2k+1} \right) - \frac{64(k+1)(k+2)^2}{\Gamma\left[2k+6\right](2k+1)} \cdot \left(\frac{1}{k+1} + \frac{1}{k+2} - \frac{2}{2k+1} \right. \\
& \left. \left. \left. - \frac{2}{2k+5} \right) + \frac{48(k+2)^2}{\Gamma[2k+6]} \left(\frac{1}{2k+4} - \frac{1}{2k+5} \right) \right) \right] = 0.
\end{aligned}
$$

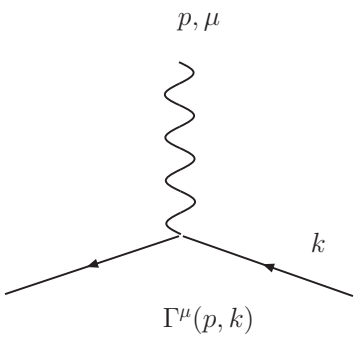
Outline

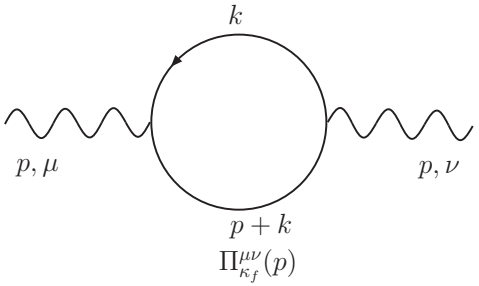
NC neutral fermion coupling

- A neutral fermion can couple to photon via a star commutator $i[A_\mu \stackrel{\star}{,} \Psi]$.
- Using nearly identical SW map trick for photon one could define a nonlocal U(1) model including neutral fermion

$$
\begin{aligned}
S =& S_g + S_f \\
=& S_g + i \int \bar{\psi}\gamma^\mu \partial_\mu \psi - \theta^{ij} \bar{\psi}\gamma^\mu \left(\frac{1}{2} f_{ij} \star_2 \partial_\mu \psi - \kappa_f f_{\mu i} \star_2 \partial_j \psi \right).
\end{aligned}
$$

Vertex and diagram





$$\Gamma^\mu_{\kappa_f}(k,p) = F(k,p)\Big[\kappa_f\Big(k^\rho\gamma_\rho(\theta p)^\mu - \gamma^\mu(k\theta p)\Big) - (\theta k)^\mu p^\rho\gamma_\rho\Big],$$

$$\Pi^{\mu\nu}_{\kappa_f}(p)_D = -\mathrm{tr}\ \mu^{d-D}\int\frac{d^Dk}{(2\pi)^D}\Gamma^\mu_{\kappa_f}(-p,p+k)\frac{i(p+k)^\rho\gamma_\rho}{(p+k)^2}\Gamma^\nu_{\kappa_f}(p,k)\frac{ik^\sigma\gamma_\sigma}{k^2}$$

Amplitudes

- Standard evaluation yields following results

$$\Pi^{\mu\nu}_{\kappa_f}(p)_D = \frac{1}{(4\pi)^2}\left[\left(g^{\mu\nu}p^2 - p^\mu p^\nu\right)F_1^{\kappa_f}(p) + (\theta p)^\mu(\theta p)^\nu F_2^{\kappa_f}(p)\right].$$

- No new tensor structure arises in $\Pi^{\mu\nu}_{\kappa_f}(p)_D$ when comparing with NCQED w/o SW map.

Amplitudes when $D \to 4$

$$
\begin{aligned}
F_1^{\kappa_f}(p) = &- \kappa_f^2 \, \frac{8}{3} \left[\frac{2}{\epsilon} + \ln \pi e^{\gamma_E} + \ln \left(\mu^2 (\theta p)^2 \right) \right] \\
&+ 4\kappa_f^2 p^2 (\theta p)^2 \sum_{k=0}^{\infty} \frac{(k+2)(p^2(\theta p)^2)^k}{4^k \Gamma[2k+6]} \\
&\cdot \left[(k+2) \Big(\ln \left(p^2 (\theta p)^2 \right) - \psi(2k+6) - \ln 4 \Big) + 2 \right],
\end{aligned}
$$

$$
\begin{aligned}
F_2^{\kappa_f}(p) = &\kappa_f \, \frac{8}{3} \frac{p^2}{(\theta p)^2} \left[\kappa_f - 8 \big(\kappa_f + 2 \big) \frac{1}{p^2 (\theta p)^2} \right] \\
&- 4\kappa_f^2 p^4 \sum_{k=0}^{\infty} \frac{(p^2(\theta p)^2)^k}{4^k \Gamma[2k+6]} \\
&\cdot \left[(k+1)(k+2) \Big(\ln \left(p^2 (\theta p)^2 \right) - 2\psi(2k+6) - \ln 4 \Big) + 2k + 3 \right].
\end{aligned}
$$

- One can verify that F_is remain the same as NC U(1) w/o SW map when $\kappa_f = 1$, vanish when $\kappa_f = 0$.

Outline

Two dimensional $\theta^{\mu\nu}$

- $\theta^{\mu\nu} = \Lambda_{NC}^{-2} i\sigma_2$ is unique and rotation invariant in two dimension.
- The tensor structures reduce to a single term

$$\begin{aligned}\Pi^{\mu\nu}_{\kappa_g}(p)_2 &= \frac{1}{(4\pi)^2}\left[g^{\mu\nu}p^2 - p^\mu p^\nu\right]\left(B_1^{\kappa_g} + \frac{B_2^{\kappa_g} + 2B_3^{\kappa_g}}{\Lambda_{\mathrm{NC}}^4} - \frac{B_4^{\kappa_g} + 2B_5^{\kappa_g}}{\Lambda_{\mathrm{NC}}^8}\right)\\ &= \frac{1}{4\pi}\left[g^{\mu\nu}p^2 - p^\mu p^\nu\right]B^{\kappa_g}(p).\end{aligned}$$

$$\begin{aligned}\Pi^{\mu\nu}_{\kappa_f}(p)_2 =& \frac{1}{(4\pi)^2}\left(g^{\mu\nu}p^2 - p^\mu p^\nu\right)\left[F_1^{\kappa_f}(p) + \frac{(\theta p)^2}{p^2}F_2^{\kappa_f}(p)\right]\\ =& \frac{1}{4\pi}\left[g^{\mu\nu}p^2 - p^\mu p^\nu\right]F^{\kappa_f}(p).\end{aligned}$$

Form factors in 2D

- Explicit evaluation yields

$$B^{\kappa_g}(p) = \frac{16}{p^2}\left(1 - 7\kappa_g + 7\kappa_g^2\right)$$
$$F^{\kappa_f}(p) = \frac{8}{p^2}\kappa_f\left(\kappa_f - 2\right).$$

- $B^{\kappa_g}(p)$ vanishes when $\kappa_g = (7 \pm \sqrt{21})/14$, $F^{\kappa_f}(p)$ vanishes when $\kappa_f = 0, 2$.

Outline

Summary

- We derived closed-form result for one loop two point functions of a (series of) nonlocal U(1) gauge theory model(s).
- The evaluation method could be generalized to other similar problems.
- Away from pure noncommutative U(1) gauge theory, we found a nonlocal pure U(1) model processing finite one loop two point function.
- Adjoint (neutral) fermion correction behaves differently from photon in the model we consider.

Outlook

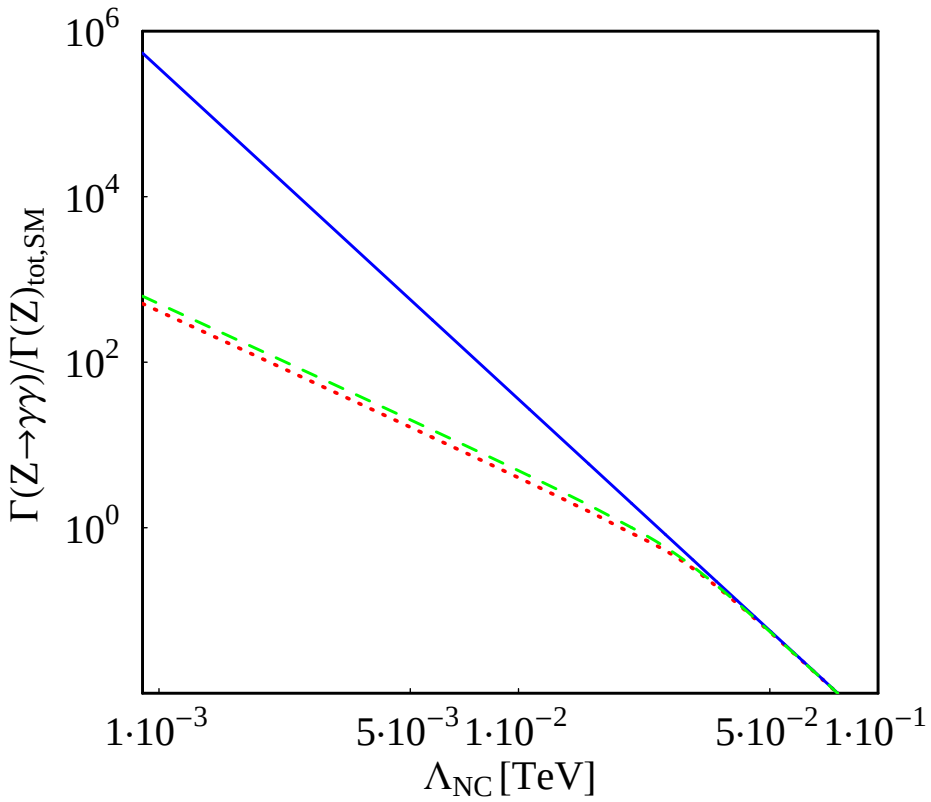
- Why fine-tune $\kappa_g = 1/3$? Stability, higher point functions etc..
- U(1) model with four photon self-coupling terms, moving back to the noncommutative U(1) theory.
- Possible extension to the Non-Abelian gauge theories.
- Exotic phenomenological consequences sensible by experiments like further CTA (Cherenkov Telescope Array) .
- .....

$Z\gamma\gamma$, 1204.6201

$$\mathcal{L}_{Z\gamma\gamma}(\kappa_g) = \frac{e}{4}\sin 2\vartheta_W \, \mathrm{K}_{Z\gamma\gamma} \, \theta^{\rho\tau} \cdot \Big[2Z^{\mu\nu}\left(2\kappa_g A_{\mu\rho} \star_2 A_{\nu\tau} - A_{\mu\nu} \star_2 A_{\rho\tau}\right) + A^{\mu\nu}\left(8\kappa_g Z_{\mu\rho} \star_2 A_{\nu\tau} - Z_{\rho\tau} \star_2 A_{\mu\nu}\right)\Big],$$

$$\begin{aligned}
\Gamma(Z \to \gamma\gamma) = {} & \frac{\alpha}{24}\sin^2 2\vartheta_W K^2_{Z\gamma\gamma} M_Z \\
& \cdot \Bigg[-8\left(\left(9 - 34\kappa_g + 35\kappa_g^2\right) + 2\frac{|\vec{B_\theta}|^2}{|\vec{E_\theta}|^2} + (1-\kappa_g)(1+3\kappa_g)\frac{(\vec{E_\theta}\vec{B_\theta})^2}{|\vec{E_\theta}|^4}\right) \\
& + \left(2\left(11\kappa_g - 42\kappa_g + 43\kappa_g^2\right) + \left(1 + 2\kappa_g + 5\kappa_g^2\right)\frac{|\vec{B_\theta}|^2}{|\vec{E_\theta}|^2} + (1-\kappa_g)(1+3\kappa_g)\frac{(\vec{E_\theta}\vec{B_\theta})^2}{|\vec{E_\theta}|^4}\right) \\
& \cdot \left(M_Z^2|\vec{E_\theta}|\,\mathrm{Si}\left(\frac{1}{2}M_Z^2|\vec{E_\theta}|\right) + 2\cos\left(\frac{1}{2}M_Z^2|\vec{E_\theta}|\right)\right) + 2\Bigg(-(1-\kappa_g)(1+3\kappa_g)\frac{|\vec{B_\theta}|^2}{|\vec{E_\theta}|^2}\left(1 - 3\frac{(\vec{E_\theta}\vec{B_\theta})^2}{|\vec{E_\theta}|^2|\vec{B_\theta}|^2}\right) \\
& + 2\left(7 - 26\kappa_g + 27\kappa_g^2\right)\Bigg)\frac{\sin\left(\frac{1}{2}M_Z^2|\vec{E_\theta}|\right)}{\left(\frac{1}{2}M_Z^2|\vec{E_\theta}|\right)}\Bigg],
\end{aligned}$$

$Z \to \gamma\gamma$ decay



$\Gamma(Z \to \gamma\gamma)/\Gamma(Z)_{\rm tot,SM}$ vs. $\Lambda_{\rm NC}$, for fixed coupling constant $|K_{Z\gamma\gamma}| = 0.33$. The black horizontal line is the experimental upper limit $\Gamma(Z \to \gamma\gamma)/\Gamma(Z)_{\rm tot,SM} < 5.2 \cdot 10^{-5}$ [64]. Dashed and solid curves correspond to the gauge deformation freedom parameter $\kappa_g = 1, 1/3$, respectively. Red corresponds to the light-like case $|\vec{E_\theta}| = |\vec{B_\theta}| = 1/\sqrt{2}\Lambda_{\rm NC}^2$ and $\vec{E_\theta}\vec{B_\theta} = 0$, (overlapped with $\vec{E_\theta}\vec{B_\theta} = 1/2\Lambda_{\rm NC}^4$). Black is: $|\vec{E_\theta}| = \vec{E_\theta}\vec{B_\theta} = 0$, and $|\vec{B_\theta}| = 1/\Lambda_{\rm NC}^2$.

Second order θ-exact Seiberg-Witten map

C.P. Martin 1206.2814

$$\Lambda^{(2)} = -\frac{1}{8}\theta^{ij}\theta^{kl}\big((\partial_i\lambda a_k(\partial_l a_j + f_{lj}))_{\star_{3alt}} - (a_i\partial_j(a_k\partial_l\lambda))_{\star_{3alt}}\big),$$

$$A^{(2)}_\mu = -\frac{1}{8}\theta^{ij}\theta^{kl}\big(((\partial_i a_\mu + f_{i\mu})a_k(\partial_l a_j + f_{lj}))_{\star_{3alt}} - (a_i\partial_j(a_k(\partial_l a_\mu + f_{l\mu})))_{\star_{3alt}} + 2(a_i(f_{jk}f_{\mu l} - a_k\partial_l f_{j\mu}))_{\star_{3alt}}\big),$$

$$[fgh]_{\star_{3alt}} = \cdot\left(\left(\frac{\cos\left[\left(\frac{\partial_f\theta\partial_g}{2} + \frac{\partial_f\theta\partial_h}{2} - \frac{\partial_g\theta\partial_h}{2}\right)\right] - 1}{\left(\frac{\partial_f\theta\partial_g}{2} + \frac{\partial_f\theta\partial_h}{2} - \frac{\partial_g\theta\partial_h}{2}\right)\left(\frac{\partial_g\theta\partial_h}{2}\right)} - \frac{\cos\left[t\left(\frac{\partial_f\theta\partial_g}{2} + \frac{\partial_f\theta\partial_h}{2} + \frac{\partial_g\theta\partial_h}{2}\right)\right] - 1}{\left(\frac{\partial_f\theta\partial_g}{2} + \frac{\partial_f\theta\partial_h}{2} + \frac{\partial_g\theta\partial_h}{2}\right)\left(\frac{\partial_g\theta\partial_h}{2}\right)}\right) f\otimes g\otimes h\right).$$

Second order θ-exact Seiberg-Witten map

Mehen & Wise hep-th/0010204

$$\Lambda = \lambda - \frac{1}{2}\theta^{ij} a_i \star_2 \partial_j \lambda + \frac{1}{2}\theta^{ij}\theta^{kl}[\frac{1}{2}(a_k \star_2 (\partial_l a_i + f_{li})) \star_2 \partial_j \lambda + \frac{1}{2} a_i \star_2 \partial_j (a_k \star_2 \partial_l \lambda)] - \frac{1}{2}\theta^{ij}\theta^{kl}[\partial_k \partial_i \lambda a_j a_l + \partial_k \lambda a_i \partial_l a_j]_{\star_3} + \mathcal{O}(a^3),$$

$$A_\mu = a_\mu - \frac{1}{2}\theta^{ij} a_i \star_2 (\partial_j a_\mu + f_{j\mu}) + \frac{1}{2}\theta^{ij}\theta^{kl}[\frac{1}{2}(a_k \star_2 (\partial_l a_i + f_{li})) \star_2 (\partial_j a_\mu + f_{j\mu}) + a_i \star_2 (\partial_j (a_k \star_2 (\partial_l a_\mu + f_{l\mu})) - \frac{1}{2}\partial_\mu (a_k \star_2 (\partial_l a_j + f_{lj})))] - \frac{1}{2}\theta^{ij}\theta^{kl} a_i \star_2 (\partial_k a_j \star_2 \partial_l a_\mu) - \frac{1}{2}\theta^{ij}\theta^{kl}[-a_i \partial_k a_\mu (\partial_j a_l + f_{jl}) + \partial_k \partial_i a_\mu a_j a_l + 2\partial_k a_i \partial_\mu a_j a_l]_{\star_3} + \mathcal{O}(A^4),$$

$$[f(x)g(x)h(x)]_{\star_3} = \int dp_1 e^{ip_1 x} \tilde{f}(p_1) \int dp_2 e^{ip_2 x} \tilde{g}(p_2) \int dp_3 e^{ip_3 x} \tilde{h}(p_3) \cdot \left[\frac{\sin(\frac{p_2 \wedge p_3}{2}) \sin(\frac{p_1 \wedge (p_2+p_3)}{2})}{\frac{(p_1+p_2)\wedge p_3}{2} \frac{p_1 \wedge (p_2+p_3)}{2}} + \frac{\sin(\frac{p_1 \wedge p_3}{2}) \sin(\frac{p_2 \wedge (p_1+p_3)}{2})}{\frac{(p_1+p_2)\wedge p_3}{2} \frac{p_2 \wedge (p_1+p_3)}{2}} \right].$$

Thanks!