An extended standard model and its Higgs geometry from the matrix model

Jochen Zahn

Universität Wien

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joint work with Harold Steinacker

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Motivation

- The IKKT [Ishibashi, Kawai, Kitazawa, Tsuchiya 97] matrix model is a candidate for a nonperturbative definition of a fundamental theory of matter and gravity.
- Solutions can be interpreted as noncommutative branes, embedded in $\mathbb{R}^{10}$, giving rise to an emergent geometry [Steinacker 08].
- There are some promising hints:
  - The perturbations around the Moyal plane lead to Ricci-flat geometries in the absence of matter [Rivelles 03].
  - $3+1$ dimensions and an expanding universe seem to be dynamically generated [Kim, Nishimura, Tsuchiya 2012].

? How to embed the standard model in this framework?

- Generically, fermions are in the adjoint representation of $SU(N)$ and are not chiral. What about the Higgs?

Outline

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The IKKT model

The IKKT model is defined by the action

$$S = \Lambda_0^4 \operatorname{tr}\left([X^A, X^B][X_A, X_B] + \bar{\Psi}\Gamma^A[X_A, \Psi] \right),$$

where the $X^A \in \mathrm{Mat}(N \times N)$, $0 \leq A \leq 9$ are hermitean matrices, and the indices are raised and lowered with $\eta = \mathrm{diag}(1, -1, \dots, -1)$. The fermions Ψ are $\mathrm{Mat}(N \times N)$-valued Majorana-Weyl spinors of $SO(9,1)$, and the Γ's are the corresponding γ matrices. Λ_0 is some energy scale.

The action is invariant under 10-dimensional Poincaré transformations and unitary transformations of the X^A, Ψ. There is also an $\mathcal{N} = 2$ supersymmetry.

For $\Psi = 0$, the action leads to the equation of motion

$$[X^A, [X_A, X^B]] = 0.$$

A particular solution is thus the Moyal plane $\mathbb{R}^{2n}_\theta$, with commutation relations

$$[X^A, X^B] = i\lambda_{NC}^2 \theta^{AB}$$

for an antisymmetric matrix θ.

Noncommutative $\mathcal{N} = 4$ Super Yang-Mills

A particular solution is the 4-dimensional Moyal plane, i.e.,

$$\bar{X}^A = (\bar{X}^\mu, 0), \qquad \mu \in \{0, \dots, 3\}.$$

We denote the corresponding algebra by $\mathfrak{A}(\mathbb{R}^4_\theta)$, represented on $\mathcal{H}_\theta$. A stack of N coincident planes is described by $\mathfrak{A}(\mathbb{R}^4_\theta) \otimes \mathrm{Mat}(N \times N)$, represented on $\mathcal{H}_\theta \otimes \mathbb{C}^N$. For perturbations

$$Y^A = X^A - \bar{X}^A \otimes \mathrm{id} = (\theta^{\mu\nu} A_\nu, \Phi^i),$$

the IKKT action reduces to a NC $\mathcal{N} = 4$ super Yang-Mills $U(N)$ gauge theory:

$$S = \int \sqrt{G} \Big(- \tfrac{1}{4 g_{YM}^2} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2} D^\mu \phi^a D_\mu \phi_a + \tfrac{g_{YM}^2}{4} [\phi^a, \phi^b][\phi_a, \phi_b] + i \bar{\psi} \not{D} \psi + g \bar{\psi} \Gamma^a [\phi_a, \psi] \Big) \mathrm{d}^4 x.$$

Here

$$G^{\mu\nu} = \sqrt{|\theta^{-1}|} \theta^{\mu\mu'} \theta^{\nu\nu'} \eta_{\mu'\nu'}, \qquad D_\mu = \partial_\mu - i[A_\mu, \cdot], \qquad g_{YM} = \pi |\theta^{-1}|^{\frac{1}{4}} \Lambda_0^{-1},$$

and ϕ, ψ are rescaled versions of Φ, Ψ. All products are Moyal products. The tracial part of $U(N)$ corresponds to dynamical gravity [Steinacker 07] and will be ignored henceforth.

Chiral fermions at the intersection of quantum planes [Chatzistavrakidis, Steinacker, Zoupanos 11]:

$$\phi^{4,5} = \begin{pmatrix} X^{1,2} & 0 \\ 0 & 0 \end{pmatrix} \qquad \phi^{6,7} = \begin{pmatrix} 0 & 0 \\ 0 & Y^{1,2} \end{pmatrix} \qquad \psi = \begin{pmatrix} 0 & \psi_{(12)} \\ \psi_{(21)} & 0 \end{pmatrix}$$

The internal Dirac operator

$$\not{D}_{\mathrm{int}} \psi = \sum_{a=4}^{9} \Gamma^a [\phi_a, \psi]$$

then acts on $\psi_{(12)}$ as

$$\begin{aligned} \not{D}_{\mathrm{int}} \psi_{(12)} &= \Gamma^4 X^1 \psi_{(12)} + \Gamma^5 X^2 \psi_{(12)} + \Gamma^6 \psi_{(12)} Y^1 + \Gamma^7 \psi_{(12)} Y^2 \\ &= \alpha a^* \psi_{(12)} + \alpha^* a \psi_{(12)} + \beta \psi_{(12)} b^* + \beta^* \psi_{(12)} b \end{aligned}$$

where

$$a = X^1 - iX^2 \qquad b = Y^1 - iY^2$$
$$\alpha = \tfrac{1}{2}(\Gamma^4 - i\Gamma^5) \qquad \beta = \tfrac{1}{2}(\Gamma^6 - i\Gamma^7)$$

Hence, there is a zero mode localized at the intersection:

$$\psi_{(12)} = |0, \downarrow\rangle\langle 0, \uparrow|.$$

It has a definite chirality. This is stable under deformations [Steinacker, Z 13].

The fuzzy sphere

A particularly simple matrix geometry is the fuzzy sphere S^2_N [Hoppe; Madore]. Let J^i be the generators of the N-dimensional irreducible representation of $\mathfrak{su}(2)$. Then set

$$X^{i+3} = RJ^i.$$

It can be seen as a quantization with N quantum cells of the sphere of radius $R\frac{N-1}{2}$ with symplectic structure

$$\{x^i, x^j\} = \tfrac{2R}{N}\varepsilon^{ijk}x_k.$$

The fuzzy sphere is not a solution to the IKKT equation of motion:

$$[X^j, [X_j, X^i]] = 2R^2 X^i.$$

There are several possibilities to obtain a fuzzy sphere solution:

- Add a term $\operatorname{tr}\varepsilon^{ijk}_{456}X_iX_jX_k$ to the action.
- Add a term $\operatorname{tr}X_iX^i$ to the action.
- Let the sphere rotate, for example in the planes $4-7$, $5-8$, $6-9$.

By replacing R by R_i, one obtains a fuzzy ellipsoid. For $N=2$, consider

$$X^4 + iX^5 = \phi\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad X^6 = \frac{r}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

This can be seen as two quantum points at $(0, 0, \pm r/2)$, which connect to an ellipsoid upon switching on ϕ.

Chiral fermions on S^2_2 intersecting $\mathbb{R}^2_\Theta$

Consider the intersection of the fuzzy minimal ellipsoid with $\mathbb{R}^2_\Theta$:

$$\phi^i = \begin{pmatrix} \phi^i_{(1)} & 0 \\ 0 & \phi^i_{(2)} \end{pmatrix}, \quad \phi_{(1)} = \tfrac{1}{2}\begin{pmatrix} \phi\sigma_1 & \phi\sigma_2 & r\sigma_3 & 0 \end{pmatrix}, \quad \phi_{(2)} = \begin{pmatrix} 0 & 0 & X^1 & X^2 \end{pmatrix}.$$

Now consider again the fermions $\psi_{(12)}$ connecting the branes.

- For $\phi = 0$, there are in total 4 zero modes, located at $x^6 = \pm r/2$. At both locations, both chiralities occur.
- For $\phi > 0$, there are 2 zero modes $\psi_0^\pm$, located at $x^6 = \pm r/2$, of opposite chirality. There are mirror fermions $\psi_1^\pm$, also located at $x^6 = \pm r/2$, with opposite chirality of $\psi_0^\pm$, and
$$\not{D}_{\mathrm{int}} \psi_1^\pm = \pm \phi \psi_1^\mp.$$

On top of that, there are modes with masses of the order $\sqrt{\theta^{-1}}$.

Analogous results hold for $S^2_2 \times \mathbb{R}^2_\theta$ intersecting $\mathbb{R}^2_\theta$ if the supplementary plane spans the $8-9$ plane. We will replace the quantum planes by large fuzzy spheres, which locally look like the quantum plane. One then expects (and numerically confirms) would-be zero modes $\psi_0^\pm$, i.e.,

$$\not{D}_{\mathrm{int}} \psi_0^\pm = \pm \phi f_\psi \psi_0^\mp,$$

where $f_\psi \ll 1$ in the appropriate limit.

Outline

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- We consider branes of the form
$$\mathcal{D}_i = \mathbb{R}^4_\theta \times \mathcal{K}_i,$$
where $\mathcal{K}_i$ is a fuzzy matrix geometry, such as $S^2_{N_i}$.
- On the intersections, fermionic would-be zero modes form.
- Choosing $\mathcal{K}_i$ as a stack of n_i coincident branes, $\mathcal{K}_i = \tilde{\mathcal{K}}_i \otimes \mathrm{Mat}(n_i \times n_i)$, the brane $\mathcal{D}_i$ carries a gauge group $U(n_i)$.
- The fermions localized at the intersection of $\mathcal{D}_i$ and $\mathcal{D}_j$ are then charged in the bi-fundamental representation of $U(n_i) \times U(n_j)$.
- The goal is to choose the configuration such that the gauge group is broken to the standard model gauge group, with the fermionic would be zero modes appropriately charged.

The choice of branes

An arrangement of 6 branes $(\mathcal{D}_w, \mathcal{D}_x, \mathcal{D}_y, \mathcal{D}_z, \mathcal{D}_\ell, \mathcal{D}_B^3)$ to achieve fermions charged as in the standard model was proposed in [Chatzistavrakidis, Steinacker, Zoupanos 11]:

$$X^a = \begin{pmatrix} X_w^a & 0 & 0 & 0 & 0 & 0 \\ & X_x^a & 0 & 0 & 0 & 0 \\ & & X_y^a & 0 & 0 & 0 \\ & & & X_z^a & 0 & 0 \\ & & & & X_\ell^a & 0 \\ & & & & & X_B^a \end{pmatrix}, \qquad \Psi = \begin{pmatrix} 0 & 0 & 0 & 0 & \nu_L & u_L \\ & 0 & 0 & 0 & e_L & d_L \\ & & 0 & 0 & e_R & d_R \\ & & & 0 & \nu_R & u_R \\ & & & & 0 & 0 \\ & & & & & 0_3 \end{pmatrix},$$

The electric charge Q and the weak hypercharge Y are realized by the adjoint action of

$$t_Q = \tfrac{1}{2}\mathrm{diag}(1, -1, -1, 1, 1, -\tfrac{1}{3}), \qquad t_Y = \mathrm{diag}(0, 0, -1, 1, 1, -\tfrac{1}{3}).$$

For $X_w = X_x$, the gauge group is broken from $U(N)$ to $U(2) \times U(1)^3 \times U(3)$.

Problem: The intersections of compact branes always leads to fermions of both chiralities.

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Solution: Fuse $\mathcal{D}_w, \mathcal{D}_z$ and $\mathcal{D}_x, \mathcal{D}_y$ into $\mathcal{D}_u$ and $\mathcal{D}_d$, by turning on Higgs fields $\phi_{u/d}$. Concretely, $\mathcal{D}_{d,u} = S^2_{N_{d,u}} \times S^2_2$, such that e_L, ν_L are located at the south pole of S^2_2 and e_R, ν_R at the north pole, with the poles connected by $\phi_{u/d}$. With $X_w = X_x$ and $X_y = X_z$, we have a $SU(2)$ symmetry at both poles, spontaneously broken by $\phi_{u,d}$.

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Modified action

We want to find such a configuration of intersecting branes, which is a solution to a modified IKKT model. Quasi-massless fermionic modes should be present, which are quasi-localized on the correct pole $X_{w/x/y/z}$. There is a close analogy between branes in the IKKT model and in supergravity, so typically one expects an attractive interaction generated by quantum effects. We model this by adding to the IKKT action a term,

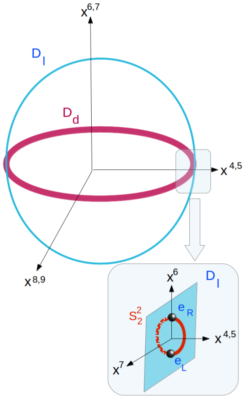
$$S = S_{IKKT} - f(\mathrm{tr} \sum_{i=4}^{9} X_i X^i),$$

where f should have a nontrivial minimum. Hence, the equation of motion becomes

$$[X^\mu, [X_\mu, X^j]] = -c f'(\mathrm{tr} \sum_{i=4}^{9} X_i X^i) X^j$$

Intersecting brane solutions

$$X_{d(u)} = \begin{pmatrix} R_d' L_3 \otimes \mathbb{1}_2 + \phi_d \mathbb{1}_{N_d} \otimes \sigma_1' \\ \phi_d \mathbb{1}_{N_d} \otimes \sigma_2' \\ r_d \mathbb{1}_{N_d} \otimes \sigma_3' \\ 0 \\ R_d L_1 \otimes \mathbb{1}_2 \\ R_d L_2 \otimes \mathbb{1}_2 \end{pmatrix}, \quad X_\ell = \begin{pmatrix} R_\ell' K_3 \\ 0 \\ R_\ell K_1 \\ R_\ell K_2 \\ 0 \\ 0 \end{pmatrix}.$$



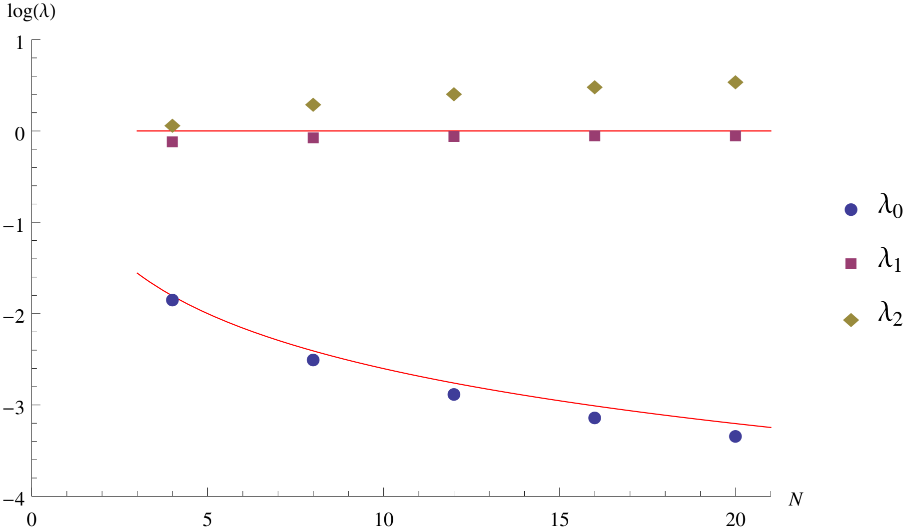
Here L_i, K_i are the generators of the $N_{d/\ell}$-dimensional irrep of $\mathfrak{su}(2)$, and $\sigma_i' = \frac{1}{2}\sigma_i$. This is a solution provided that

$$R_d^2 = {R_d'}^2 = R_\ell^2 = {R_\ell'}^2 = r_d^2 = \phi_d^2 = -c'f'.$$

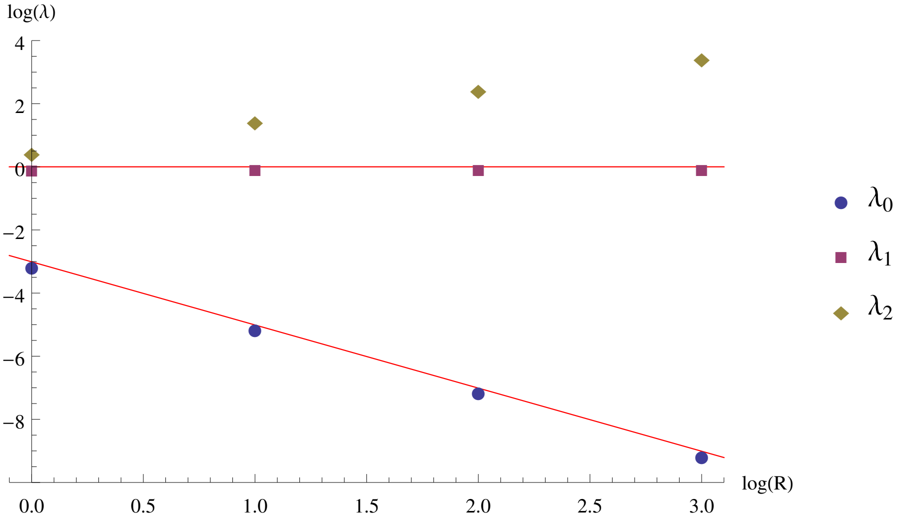
There are two intersection regions for $N_\ell \sim N_d \gg 1$, locally looking like the intersection of $\mathbb{R}^2_\Theta \times S^2_2$ and $\mathbb{R}^2_\Theta$. We expect (quasi-) massless fermionic modes of (quasi-) definite chirality, (quasi-) localized at the poles of S^2_2. The lowest eigenvalues can be estimated ($R = R_{d,\ell} = R'_{d,\ell}$, $N = N_{d,\ell}$):

$$\not{D}_{\rm int} \psi_i^\pm = \pm \lambda_i \psi_i^\mp, \qquad \lambda_0 \sim \frac{\phi r^2}{4N^2R^2}, \qquad \lambda_1 \sim \phi.$$

The lowest eigenvalues as a function of N, for $R = r = \phi = 1$:

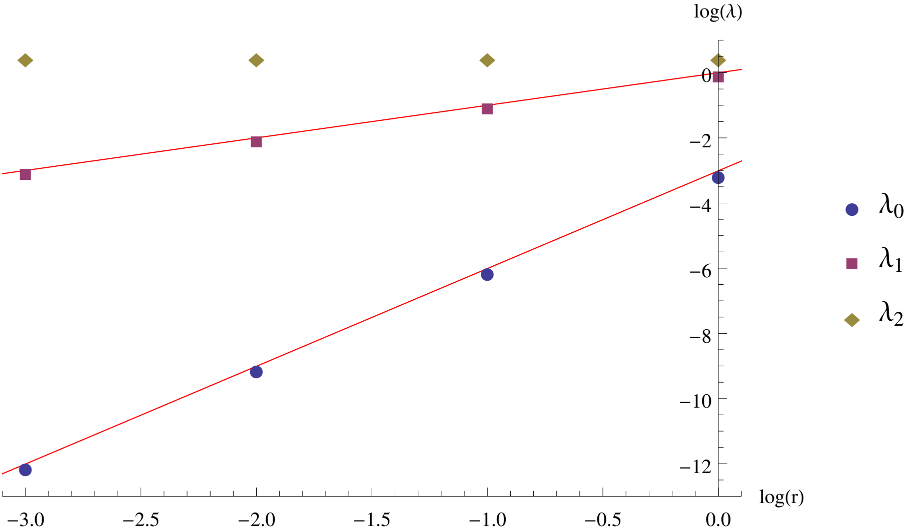


The lowest eigenvalues as a function of R, for $r = \phi = 1$, $N = 16$:



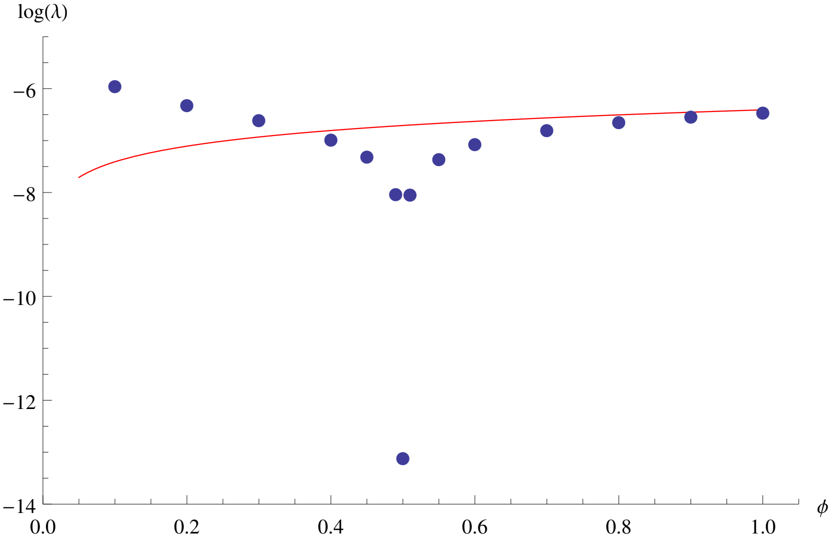
Numerical test of the expectations III

The lowest eigenvalues as a function of r, for $r = \phi$, $R = 1$, $N = 16$:



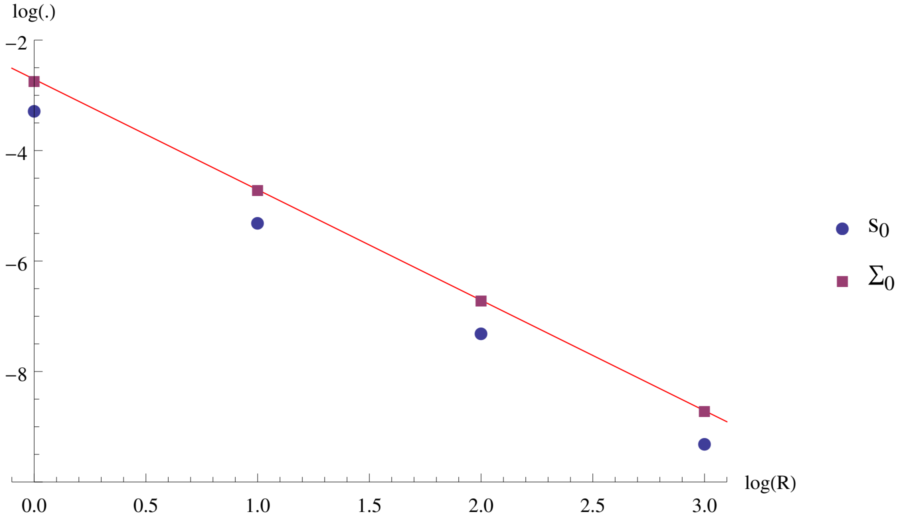
Numerical test of the expectations IV

The lowest eigenvalue as a function of ϕ, for $R = r = 1$, $N = 16$:



Numerical test of the expectations V

Expectation values of $s = 1 - \sigma_3$ and $\Sigma = 1 - \Sigma_{45}$ in the lowest eigenvalue as a function of R, for $r = \phi = 1$, $N = 16$. The estimate is $\Sigma \sim \frac{r^2}{2N^2R^2}$.



The singlet Higgs should link $\mathcal{D}_u$ and $\mathcal{D}_\ell$, so we use the ansatz

$$H_S^a = h^a S + \text{h.c.}, \qquad S = \sum_n |+, p_n\rangle_u \langle q_n|_\ell$$

where

$$|+, p\rangle_u = |+\rangle|p\rangle, \qquad L_3|+\rangle = \tfrac{N_u - 1}{2}|+\rangle.$$

For suitably chosen p_n, q_n, this becomes an eigenvector of the linearized wave operator if we choose

$$h^a = h(e^8 + ie^9).$$

One verifies numerically that $\sum_n |p_n\rangle\langle q_n|$ can be chosen to be (quasi-) localized at ν_R, as expected. The resulting mode of the linearized wave operator is instable. We assume that it is non-linearly stabilized, so that h acquires a non-zero value.

- For $h = 0$, we have one stack of 2 branes $\mathcal{D}_d, \mathcal{D}_u$, and one stack of 4 branes, $\mathcal{D}_\ell, \mathcal{D}_B$. Hence we have a $U(2) \times U(4)$ symmetry. Turning on h breaks it to

$$SU(3)_c \times U(1)_Q \times U(1)_B \times U(1)_{\text{tr}}.$$

- The singlet Higgs can induce a Majorana mass for ν_R,

$$\text{tr}_N(\nu_R^T \gamma^0 S^* \nu_R S^*).$$

The gauge bosons

The gauge bosons are obtained from fluctuation of the X^μ:

$$A = g(W_- t_+ + W_+ t_- + W_3 t_3) + \tfrac{1}{2} g' B t_Y + g_5 B_5 t_5 + g_S A_\alpha t_\alpha ,$$

where, in the basis $(\mathcal{D}_L, \mathcal{D}_y, \mathcal{D}_z, \mathcal{D}_\ell, \mathcal{D}_B^3)$, with $\mathcal{D}_L = (\mathcal{D}_w, \mathcal{D}_x)$,

$$t_{\pm,3} = \tfrac{1}{2}\mathrm{diag}(\mathbb{1}_{N_1} \otimes \sigma_{\pm,3}, 0, 0, 0, 0_3) \qquad t_Y = \mathrm{diag}(0_2, -\mathbb{1}_{N_1}, \mathbb{1}_{N_1}, \mathbb{1}_{N_2}, -\tfrac{1}{3}\mathbb{1}_{N_2} \otimes \mathbb{1}_2)$$

$$t_\alpha = \mathrm{diag}(0_2, 0, 0, 0, \mathbb{1}_{N_2} \otimes \lambda_\alpha) \qquad t_5 = \mathrm{diag}(\mathbb{1}_{N_1} \otimes \mathbb{1}_2, -\mathbb{1}_{N_1}, -\mathbb{1}_{N_1}, -\mathbb{1}_{N_2}, \tfrac{1}{3}\mathbb{1}_{N_2} \otimes \mathbb{1}_2)$$

with $N_u = N_d = N_1$, $N_B = N_\ell = N_2$, and

$$g = \frac{g_{YM}}{\sqrt{N_1}}, \qquad g' = \frac{g_{YM}}{\sqrt{N_1 + \frac{2}{3}N_2}}, \qquad g_5 = \frac{g_{YM}}{\sqrt{8(N_1 + \frac{1}{3}N_2)}}, \qquad g_S = \frac{g_{YM}}{\sqrt{N}}.$$

The λ_α are generators of the fundamental $\mathfrak{u}(3)$ representation. The identity generator gives $U(1)_B$, which is anomalous and expected to disappear from the low energy spectrum. Note that $[t_5, \cdot]$ acts as $B - L + \gamma_5$ on the chiral fermions, so $U(1)_5$ is also anomalous (it is also broken by the Higgs ϕ).

As usual, the electroweak Higgses $\phi_u = \phi_d = \phi$ induce mass terms for some gauge bosons,

$$\phi^2 N_1 \left(\tfrac{1}{2} g^2 (W_1^2 + W_2^2) + \tfrac{1}{2} g_Z^2 Z^2 + 2 g_5^2 B_5^2 \right),$$

where

$$g_Z Z = g W_3 - g' B.$$

In particular, we obtain the Weinberg angle

$$\sin^2 \theta_W = \frac{1}{1 + \frac{g^2}{g'^2}} = \frac{1}{2 + \frac{2}{3} \frac{N_2}{N_1}}.$$

For $N_1 = N_2$, we then obtain

$$\sin^2 \theta_W = \tfrac{3}{8}, \qquad g_S = g,$$

as in the $SU(5)$ GUT.

Fermion & Higgs masses

The Yukawa mass terms for the would-be zero modes can be made arbitrarily small by choosing N_i large enough (provided that our heuristic estimate is correct). The mirror fermions with opposite chirality have Yukawa masses

$$m \sim g_{YM}\phi = \sqrt{2} m_W$$

However, one has to keep in mind that these are tree level masses at some high energy scale. Due to the coupling to massive Kaluza-Klein modes, one may expect that quantum effects raise this gap.

For the fluctuation of the Higgs ϕ, one obtains a mass

$$m_\phi^2 = 4 m_W^2 \left(1 + 2\pi^2 f''\right) .$$

Outline

Summary & Outlook

Summary:

- Configurations in the IKKT model whose low-energy physics resembles the standard model.
- Predicts mirror fermions and Kaluza-Klein towers of gauge fields.

Issues:

- Introduced an ad-hoc supplementary term to the IKKT action, breaking 10-dimensional Poincaré and super-symmetry. Motivated by quantum effects, but can this be made precise?
- Non-linear stabilization of the scalar Higgs?
- With our configuration, only an even number of generations can be achieved.
- Low scale of the mirror fermions.

Despite these problems, one may find it remarkable that one can get standard model like low energy energy physics from an $\mathcal{N} = 4$ supersymmetric gauge theory.

Thank you for your attention!