WG1COST Meeting

NC Geometry and deformation quantization approaches

Pointers for the discussion

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Let me give a few pointer for the discussion regarding the uses of deformation quantization in quantum gravity. I will say some probably obvious things, with the aim of setting the scenario.

Deformation quantization has its roots in the understanding quantum mechanics, and is an excellent example of fruitful collaborations between mathematics and physics.

Therefore most of the activity has been geared towards the problem of "quantization", with due interest to symmetries, structures etc.

A related and parallel approach led to first noncommutative topology (Gelfand Naijmark . . .) and later to noncommutative geometry (Connes)

The object which is deformed historically has been the phase space of a classical system (or variations thereof), so that for example the relevant structure was symplectic, or Poisson, and the problem are of a global nature

Quantum mechanics has a built-in cutoff in the infinitesimal

$\boxed{\hbar}$ was successfully introduced by Max Planck for this purpose.

At least in simple cases (a particle moving in empty space) we have a good theory, physically satisfactory.

Starting form Heisenberg, Snyder, Bronstein, the need was felt a quantization of Spacetime rather than phase space.

There are heuristic reasons (formation of micro black holes) which point to the fact that there should a physical scale under which a measurement of space is impossible

The difference is that, while we have a good theory of quantum mechanics, and even quantum field theory, we do not have anything comparable for quantum gravity.

Indeed there is some hope that a deformed spacetime could give an hand for some version of quantum gravity.

The explosion of interest for the application of deformed products which describe spacetime came form strings, with the work of Seiberg and Witten, which introduced the $*$ product to the masses.

The deformed theory now is not quantum mechanics, but a field theory.

A variety of deformed products have been used, apart form the original (Grönewold-Moyal) one. Note that for physical applications usually the framework of formal series is not suited. Convergence is normally required.

The main difference is however for symmetries. Symplectic structures are not relevant, while the Euclidean, Galiei, Poincaré or diffeos invariance become the requirement.

On one side this led to the study of possible detection of breaking of (typically Lorentz) invariance to detect non commutativity

Later, more in the spirit of deformation, to the study of deformed symmetries, Hopf algebra, Drinfeld twists etc.

This still goes on strongly, but is not clear how these deformed symmetries affect actual measurements.

In this line one can also see κ-Poincaré and κ-Minkowski

What more can deformations tell us?

For quantum mechanics the problem was in the form of global, large scale (often topological) limitations. For example given a Poisson manifold is it possible to find a deformed product? No matter how globally weird is it is (for example a constrained lagrangian submanifold of a nonorientable surface with fractional Betti numbers and a degenerate Poisson bracket)

The problems of spacetime are on the other side, more natural questions are like: given a space which at small scale is really weird (a network of fractal spin foams with noncommuting momenta valued in a deformed non associative exceptional group) can we come up with something so that we can describe fields on it, as a deformation of field theory?

Inspirations are already coming form various forms of noncommutative geometry, strings, loop quantum gravity, group field theory, microscopic black holes. . .

From the point of view of quantum gravity the frontier of deformation is in the small scale.

To go beyond the Planck length frontier a more radical thinking may be required. The space is open for the creation and understanding of more beautiful structures