Structure of off-shell covariant higher gauge theory

Satoshi Watamura
Tohoku Univ. Sendai

collaboration with
Ursula Carow-Watamura,
Marc A. Heller,
Noriaki Ikeda,
Yukio Kaneko

The paper is now available as arXiv:1606.03861, JHEP07(2016)125.

INTRODUCTION

- I would like to discuss a problem in a non-abelian theory of 2-form gauge fields appearing in the effective theory of multiple M2/M5-branes.

- This theory is considered to be a 6-dimensional N=(2,0) supersymmetric theory. And there are two major problems:

  1. Selfdual 2-form gauge field
  2. Interacting non-abelian 2-form gauge field theory

- Here we want to focus on the second problem.
  Our aim is

  1. List a wider class of higher gauge theories based on Lie 2-algebras.
     We find a new class of theory in 5 dimensions.
  2. Search for off-shell covariant theories
     and analyze the properties.

PHYSICAL BACKGROUND: MULTIPLE M2 AND M5

- Effective theory of multiple (i.e. n) M2-branes is now known as ABJM theory

  3-dimensional field theory on M2 world volume:
  N=6(8) Supersymmetric Chern-Simons matter theory

- Effective theory of n M5-branes is not known yet

WE EXPECT

- 6-dimensional field theory on single M5-brane world volume should be a theory of the following supermultiplet:
  - 5 scalar fields for transversal coordinates in 11-dim. spacetime
  - 1 fermion field as superpartner: 4-component Weyl spinor (8 d.o.f.)
- We need 3 more bosonic DOF on the world volume
  - ❌ Vector field in 6 dim.: massless 4=6-2, massive 5=6-1
  - ❌ 2-form field in 6 dim.: massless $\begin{pmatrix} 4 \\ 2 \end{pmatrix} = 6$, massive $\begin{pmatrix} 5 \\ 2 \end{pmatrix} = 10$
  - ⭕ Self-dual 2-form field in 6 dim.: 6/2=3

HIGHER GAUGE THEORY

- Single M5-brane effective theory: $X^i, B = \frac{1}{2} B_{\mu\nu} dx^\mu dx^\nu, \psi$

  with self-duality condition $dB = *dB$

- 2 MAJOR PROBLEMS

  ① Covariant action problem of self-dual 2-form gauge field:

  NO ACTION?

  $F^{(3)} = dB = *dB = *F^{(3)}$

  $$S \propto \int F^{(3)} \wedge *F^{(3)} = \int F^{(3)} \wedge F^{(3)} = 0$$

  ② Interacting (non-abelian) gauge theory of 2-form field:

  HIGHER GAUGE THEORY

  - Theory based on crossed module: Strict Lie 2-algebra [Baez-Crans, 2004]
  - Semi-strict Lie 2-algebra [Roytenberg, 2007]

- TRIALS: ① Discard Lorentz covariance [Chu-Ko 2005,Ho-Huang-Matsuo 2007]

  ② Use of deformed higher gauge theory [Ho-Matsuo 2009]

  NO solution yet

STARTING POINT OF HGT

- Gauge theory based on strict Lie 2-algebra [Baez 2002]

  Easy way: Define the Lie 2-algebra by the differential crossed module:

  - two Lie algebras: $\mathfrak{g}$ and $\mathfrak{h}$ and the following maps between them:

$$\underline{t} : \mathfrak{h} \longrightarrow \mathfrak{g} \qquad \underline{\alpha} : \mathfrak{g} \rightarrow Der(\mathfrak{h})$$

  s.t. for $g \in \mathfrak{g}$, $h, h' \in \mathfrak{h}$

  - $\mathfrak{g}$-invariance: $\underline{t}(\underline{\alpha}(g) \triangleright h) = [g, \underline{t}(h)]$
  - Peiffer identity: $\underline{\alpha}(\underline{t}(h)) \triangleright h' = [h, h']$

- Correspondingly, we introduce two gauge fields:
  - $\mathfrak{g}$-valued 1-form: A^a
  - $\mathfrak{h}$-valued 2-form: B^I

- Field strengths are given by
  - $F = dA + \frac{1}{2} A \wedge A - \underline{t}(B)$
  - $H = dB + \underline{\alpha}(A) \wedge B$

THEN Bianchi identity and closure of gauge transformations are analyzed

Crossed Module:

- Since the (differential) Crossed Module is a basic object in our construction, we introduce a little more detailed expressions:

- Crossed module: A pair of Lie algebras, $(\mathfrak{g}, \mathfrak{h})$ with homomorphisms t, α

$$t : \mathfrak{h} \ \rightarrow \ \mathfrak{g} \qquad\qquad \alpha : \mathfrak{g} \ \rightarrow \ Der(\mathfrak{h})$$

- To write down the field theory, we introduce a local basis:

$$g_a \in \mathfrak{g} \ , \ h_A \in \mathfrak{h} \ , \quad [g_a, g_b] = f^c_{ab} g_c \ , \ [h_A, h_B] = \tilde{f}^C_{AB} h_C$$

then the maps are

$$\alpha(g_a) \triangleright h_A = \alpha^B_{aA} h_B \ , \qquad\qquad t(h_A) = t^a_A g_a$$

- There are now 4 structure constants

$$\alpha^A_{aB} \qquad t^a_A \qquad f^c_{ab} \qquad \tilde{f}^C_{AB}$$

Among them there are some relations required by consistency

- To construct gauge field theories, we use a so-called QP-manifold. For our purpose, we take the graded manifold with a pair of vector spaces V and W

$W \sim \mathfrak{g}^*, V \sim \mathfrak{h}^*$

$$\mathcal{M}_n = T^*[n](W[1] \oplus V[2])$$

where $[n]$ shifts the degree n of the coordinates, thus the local coordinates are for $W \oplus V \oplus W^* \oplus V^*$

$$(q^a, Q^A, p_a, P_A) \quad \text{with degree} \quad (1, 2, n-1, n-2)$$

- QP-structure
  - P-structure $\omega = (-1)^n dq^a \wedge dp_a + dQ^A \wedge dP_A$

    then we have a corresponding graded Poisson bracket of degree -n, $\{-, -\}$
  - Q-structure is given by a Hamiltonian Θ :
    - Hamiltonian in the local coordinates is a polynomial of degree n+1
    - Master equation $\{\Theta, \Theta\} = 0$
    - Homological vector field of degree 1: $Q = \{\Theta, -\}$ $\quad Q^2 = 0$

- For example, a Hamiltonian: Each term contains a SINGLE momentum variable.

  On our supermanifold $\mathcal{M}_n = T^*[n](W[1] \oplus V[2])$

$$\Theta^{(1)} = t^a_A Q^A p_a - \frac{1}{2} f^a_{bc} q^b q^c p_a - \alpha^a_{aB} q^a Q^B P_A + \frac{1}{6} T^A_{abc} q^a q^b q^c P_A$$

- Here, the structure constants of crossed module appears.

  The master eq. $\{\Theta^{(1)}, \Theta^{(1)}\} = 0$ gives the relations among the structure constants. Extra structure constants

$T^A_{abc} = 0 \longrightarrow$ Strict Lie 2-algebra

$T^A_{abc} \neq 0 \longrightarrow$ Semi-strict Lie 2-algebra

Gauge Field and Field Strength

- To construct the gauge tr. rules and field strength, we follow AKSZ-Strobl:
  - Taking a d-dimensional spacetime Σ with coordinates: $(\sigma^\mu, \theta^\mu) \in T[1]\Sigma$
  - Fields are defined in the mapping space
    - Consider the map $a : T[1]\Sigma \to T^*[n](W[1] \oplus V[2])$
    - (super)gauge fields are defined by pullback: $a^*(q^a) = \mathbf{A}^a, \quad a^*(Q^A) = \mathbf{B}^A$
    - (super)Field strength: $\mathbf{F}_z = \mathbf{d}a^*(z) - a^*(Qz)$
  - Usual field corresponding to the coordinate z is the degree |z| component the of superfield $a^*(z)$
  - We define a degree preserving map with identification of the degree 1 coordinate of $T[1]\Sigma$ with a form $d\sigma^\mu$

$$\tilde{a} \; : \; T[1]\Sigma \; \to \; \mathcal{M}_n \qquad \tilde{a}^*(q^a) = A^a_\mu d\sigma^\mu \quad \tilde{a}^*(Q^A) = \frac{1}{2} B^A_{\mu\nu} d\sigma^\mu d\sigma^\nu$$

  - From degree |z|+1 part of $\mathbf{F}_z$ we get $F_z = d\tilde{a}^*(z) - \tilde{a}^*(Qz)$
  - From degree |z| part, we get the gauge transformation

$$\delta \tilde{a}^*(z) = d\tilde{a}^*_{-1}(z) - \tilde{a}^*_{-1}(Qz)$$

Classification of Hamiltonians

Now we classify the possible Hamiltonians for spacetime dimensions d=n+1

- We expand the Hamiltonian in the number of conjugate momenta (p_a, P_A)

$$\Theta = \sum_k \Theta^{(k)}$$

- We find that there is a limited number of types available, depending on the spacetime dimension d=n+1

1) dimension larger than 6 $n \geq 6$ $\Theta = \Theta^{(0)} + \Theta^{(1)}$

2) dimension 5 and 6 $n = 4, 5$ $\Theta = \Theta^{(0)} + \Theta^{(1)} + \Theta^{(2)}$

3) dimension 4 and less

- We see that case 1) is essentially the same as the semi-strict case:

$$\Theta^{(0)} = \frac{1}{d!} m_{ab\cdots de} q^a q^b \cdots q^d q^e + \frac{1}{(d-2)!} m_{a\cdots cA} q^a q^b \cdots Q^A + \cdots ,$$

$\{\Theta^{(1)}, \Theta^{(1)}\} = 0$ ⟵ Same relations as semi-strict Lie 2-algebra

$\{\Theta^{(0)}, \Theta^{(1)}\} + \{\Theta^{(1)}, \Theta^{(0)}\} = 0$

$\{\Theta^{(0)}, \Theta^{(0)}\} = 0$ ⟵ Automatic, no new relations

- Dimensions 5 and 6 provide interesting possibilities. We consider the case d=5.
  - The Hamiltonian is $\Theta = \Theta^{(0)} + \Theta^{(1)} + \Theta^{(2)}$

$$\Theta^{(0)} = \frac{1}{5!} m_{abcde} q^a q^b q^c q^d q^e + \frac{1}{3!} m_{abcA} q^a q^b q^c Q^A + \frac{1}{2} m_{aAB} q^a Q^A Q^B,$$

$$\Theta^{(1)} = \frac{1}{2} f_{ab}^c q^a q^b p_c + t_A^a Q^A p_a + \alpha_{aA}^B q^a Q^A P_B + \frac{1}{3!} T_{abc}^A q^a q^b q^c P_A,$$

$$\Theta^{(2)} = s^{aA} p_a P_A + \frac{1}{2} n_a^{AB} q^a P_A P_B$$

  - The master equation is decomposed by

$$\{\Theta^{(0)}, \Theta^{(0)}\} = 0,$$
$$\{\Theta^{(0)}, \Theta^{(1)}\} + \{\Theta^{(1)}, \Theta^{(0)}\} = 0$$
$$\{\Theta^{(1)}, \Theta^{(1)}\} + \{\Theta^{(0)}, \Theta^{(2)}\} + \{\Theta^{(2)}, \Theta^{(0)}\} = 0,$$
$$\{\Theta^{(1)}, \Theta^{(2)}\} + \{\Theta^{(2)}, \Theta^{(1)}\} = 0,$$
$$\{\Theta^{(2)}, \Theta^{(2)}\} = 0.$$

  Now we have some possibilities to extend the semi-strict Lie 2-algebra

NOTE: WHY 5 DIM

5-dimensional theory is also interesting since one can think of it as a KK compactification of the 6-dimensional theory, and the 6-dimensional theory itself is believed not to exhibit a covariant action.

Now we have some possibilities to extend the semi-strict Lie 2-algebra

1) $\Theta^{(0)} \neq 0$ and $\Theta^{(2)} = 0$

- It does not change the gauge transformation and field strength, since

$$\{\Theta^{(0)}, q^a\} = 0 \qquad \{\Theta^{(0)}, Q^A\} = 0$$

- It does not change the semi-strict Lie 2-algebra structure since $\{\Theta^{(1)}, \Theta^{(1)}\} = 0$

2) $\Theta^{(0)} = 0$ and $\Theta^{(2)} \neq 0$

- It changes the gauge transformation and field strength.
- It does not change the semi-strict Lie 2-algebra structure.

3) Both $\Theta^{(0)}, \Theta^{(2)} \neq 0$

- It changes the gauge transformation and field strength, and it also changes the semi-strict Lie 2-algebra structure, since $\{\Theta^{(0)}, \Theta^{(2)}\}$ term changes the relations given by $\{\Theta^{(1)}, \Theta^{(1)}\}$
- This deforms the constraints on the structure constants: $t^a_A \quad \alpha^A_{aB} \quad f^c_{ab} \quad T^A_{abc}$

and gives a new type of 2-form gauge field theory.

We focus on case 2), which already exhibits a very interesting structure from the covariantization point of view.

$\Theta^{(0)} = 0$ and $\Theta^{(2)} \neq 0$

$$\Theta^{(1)} = \frac{1}{2} f_{ab}^c q^a q^b p_c + t_A^a Q^A p_a + \alpha_{aA}^B q^a Q^A P_B + \frac{1}{3!} T_{abc}^A q^a q^b q^c P_A,$$

$$\Theta^{(2)} = s^{aA} p_a P_A + \frac{1}{2} n_a^{AB} q^a P_A P_B$$

$$\frac{1}{2} f_{e[a}^d f_{bc]}^e - \frac{1}{3!} t_A^d T_{abc}^A = 0,$$

$$t_A^c f_{cb}^a - t_B^a \alpha_{bA}^B = 0,$$

$$\frac{1}{2} \alpha_{cA}^B f_{ab}^c + \alpha_{[a|C|}^B \alpha_{b]A}^C + \frac{1}{2} t_A^c T_{cab}^B = 0,$$

$$\frac{3}{2} f_{[ab}^e T_{cd]e}^A + \alpha_{[a|B|}^A T_{bcd]}^B = 0,$$

$$\alpha_{a(A}^C t_{B)}^a = 0.$$

$$s^{a(A} n_a^{BC)} = 0,$$

$$s^{cA} f_{ca}^b + \alpha_{aB}^A s^{bB} - t_B^b n_a^{AB} = 0,$$

$$\frac{1}{2} s^{c(A} T_{abc}^{B)} + \frac{1}{4} n_c^{AB} f_{ab}^c + \alpha_{[a|C|}^{(A} n_{b]}^{B)C} = 0,$$

$$s^{a(A} \alpha_{aC}^{B)} + \frac{1}{2} t_C^a n_a^{AB} = 0,$$

$$t_A^{[a} s^{b]A} = 0.$$

2') set $T_{abc}^A = 0$ $\longrightarrow$ strict Lie 2-algebra

We focus on case 2), which already exhibits a very interesting structure from the covariantization point of view.

$\Theta^{(0)} = 0$ and $\Theta^{(2)} \neq 0$

$$\Theta^{(1)} = \frac{1}{2} f^c_{ab} q^a q^b p_c + t^a_A Q^A p_a + \alpha^B_{aA} q^a Q^A P_B + \frac{1}{3!} T^A_{abc} q^a q^b q^c P_A,$$

$$\Theta^{(2)} = s^{aA} p_a P_A + \frac{1}{2} n_a^{AB} q^a P_A P_B$$

$$\frac{1}{2} f^d_{e[a} f^e_{bc]} - \frac{1}{3!} t^d_A T^A_{abc} = 0,$$

$$t^c_A f^a_{cb} - t^a_B \alpha^B_{bA} = 0,$$

$$\frac{1}{2} \alpha^B_{cA} f^c_{ab} + \alpha^B_{[a|C|} \alpha^C_{b]A} + \frac{1}{2} t^c_A T^B_{cab} = 0,$$

$$\frac{3}{2} f^e_{[ab} T^A_{cd]e} + \alpha^A_{[a|B|} T^B_{bcd]} = 0,$$

$$\alpha^C_{a(A} t^a_{B)} = 0.$$

$$s^{a(A} n_a^{BC)} = 0,$$

$$s^{cA} f^b_{ca} + \alpha^A_{aB} s^{bB} - t^b_B n_a^{AB} = 0,$$

$$\frac{1}{2} s^{c(A} T^{B)}_{abc} + \frac{1}{4} n_c^{AB} f^c_{ab} + \alpha^{(A}_{[a|C|} n_{b]}^{B)C} = 0,$$

$$s^{a(A} \alpha^{B)}_{aC} + \frac{1}{2} t^a_C n_a^{AB} = 0,$$

$$t^{[a}_A s^{b]A} = 0.$$

2') set $T^A_{abc} = 0$ $\longrightarrow$ strict Lie 2-algebra

Now we have two new structure constants: s^{aA}, n_a^{AB}

- Structure of the maps,

$$
\begin{array}{rcccl}
[-,-] & : & W \times W & \longrightarrow & W \\
\underline{t} & : & V & \longrightarrow & W \\
\underline{\alpha} & : & W \times V & \longrightarrow & V \\
[-,-,-] & : & W \times W \times W & \longrightarrow & W \\
\underline{s} & : & W^* & \longrightarrow & V \\
\underline{n} & : & W \times V^* & \longrightarrow & W
\end{array}
$$

$$
\left.\begin{aligned}
[p_a, p_b] &= f^c_{ab} p_c, \\
\underline{t}(P_A) &= t^a_A p_a, \\
\underline{\alpha}(p_a) P_A &= \alpha^B_{aA} P_B,
\end{aligned}\right\} \text{differential crossed module}
$$

$$
\begin{aligned}
[p_a, p_b, p_c] &= T^A_{abc} P_A, \\
\underline{s}(q^a) &= s^{aA} P_A, \\
\underline{n}(p_a)(Q^A) &= n^{AB}_a P_B.
\end{aligned}
$$

- Other important maps
  - $\underline{\alpha} \circ \underline{t} : V \times V \longrightarrow V$, since $\alpha^C_{a(A} t^a_{B)} = 0$ this map is a bracket $[-,-]_V$

    with structure constant $\tilde{f}^C_{AB} = \alpha^C_{a[A} t^a_{B]}$
  - $\underline{s}' : W \to V$, since $t^{[a}_A s^{b]A} = 0$, $G^{ab} = t^a_A s^{bA}$ is a symmetric tensor.

    We can write $G^{ab} = \mathcal{P}^a_c g^{cb}$ where g is an invertible metric on W. Then $s^A_a = g_{ab} s^{bA}$ defines the map $\underline{s}'$ (g_{ab} has some ambiguity)

Field strength and gauge transformation

$$F^a = dA^a - \frac{1}{2} f^a_{bc} A^b \wedge A^c - t^a_A B^A - s^{aA} D_A,$$

$$H^A = dB^A + \alpha^A_{aB} A^a \wedge B^B + s^{bA} C_b + n^{AB}_a A^a \wedge D_B + \frac{1}{3!} T^A_{abc} A^a \wedge A^b \wedge A^c.$$

$$F^{(C)}_a = dC_a - f^c_{ab} A^b \wedge C_c - \alpha^A_{aB} B^B \wedge D_A - \frac{1}{2} n^{AB}_a D_A \wedge D_B - \frac{1}{2} T^A_{abc} A^b \wedge A^c \wedge D_A,$$

$$F^{(D)}_A = dD_A - t^a_A C_a - \alpha^A_{aB} A^a \wedge D_B.$$

$$\delta A^a = d\epsilon^a - f^a_{bc} A^b \epsilon^c - t^a_A \mu^A - s^{aA} \mu'_A,$$

$$\delta B^A = d\mu^A + \alpha^A_{aB}(A^a \wedge \mu^B + \epsilon^a B^B) + s^{bA} \epsilon'_b + n^{AB}_a (A^a \wedge \mu'_B + \epsilon^a \wedge D_B)$$
$$+ \frac{1}{2} T^A_{abc} A^a \wedge A^b \epsilon^c$$

$$\delta C_a = d\epsilon'_a - f^c_{ab}(A^b \wedge \epsilon'_c + \epsilon^b \wedge C_c) - \alpha^A_{aB}(B^B \wedge \mu'_A + \mu^B \wedge D_A) - n^{AB}_a D_A \wedge \mu'_B$$
$$- \frac{1}{2} T^A_{abc}(2A^b \wedge D_A \epsilon^c + A^b \wedge A^c \wedge \mu'_A),$$

$$\delta D_A = d\mu'_A - t^a_A \epsilon'_a - \alpha^B_{aA}(A^a \wedge \mu'_B + \epsilon^a D_B).$$

$$\delta F^a = f^a_{bc} F^b \epsilon^c,$$

$$\delta H^A = \alpha^A_{aB} H^B \epsilon^a - \alpha^A_{aB} F^a \wedge \mu^B - n^{AB}_a F^a \wedge \mu'_B . + n^{AB}_a F^{(D)}_B \epsilon^a + T^A_{abc} A^a \wedge F^c \epsilon^b$$

Field strength and gauge transformation, T=0

$$F^a = dA^a - \frac{1}{2} f^a_{bc} A^b \wedge A^c - t^a_A B^A - s^{aA} D_A,$$

$$H^A = dB^A + \alpha^A_{aB} A^a \wedge B^B + s^{bA} C_b + n^{AB}_a A^a \wedge D_B + \frac{1}{3!} T^A_{abc} A^a \wedge A^b \wedge A^c.$$

$$F^{(C)}_a = dC_a - f^c_{ab} A^b \wedge C_c - \alpha^A_{aB} B^B \wedge D_A - \frac{1}{2} n^{AB}_a D_A \wedge D_B - \frac{1}{2} T^A_{abc} A^b \wedge A^c \wedge D_A,$$

$$F^{(D)}_A = dD_A - t^a_A C_a - \alpha^A_{aB} A^a \wedge D_B.$$

$$\delta A^a = d\epsilon^a - f^a_{bc} A^b \epsilon^c - t^a_A \mu^A - s^{aA} \mu'_A,$$

$$\delta B^A = d\mu^A + \alpha^A_{aB}(A^a \wedge \mu^B + \epsilon^a B^B) + s^{bA} \epsilon'_b + n^{AB}_a (A^a \wedge \mu'_B + \epsilon^a \wedge D_B)$$

$$+ \frac{1}{2} T^A_{abc} A^a \wedge A^b \epsilon^c$$

$$\delta C_a = d\epsilon'_a - f^c_{ab}(A^b \wedge \epsilon'_c + \epsilon^b \wedge C_c) - \alpha^A_{aB}(B^B \wedge \mu'_A + \mu^B \wedge D_A) - n^{AB}_a D_A \wedge \mu'_B$$

$$- \frac{1}{2} T^A_{abc}(2A^b \wedge D_A \epsilon^c + A^b \wedge A^c \wedge \mu'_A),$$

$$\delta D_A = d\mu'_A - t^a_A \epsilon'_a - \alpha^B_{aA}(A^a \wedge \mu'_B + \epsilon^a D_B).$$

$$\delta F^a = f^a_{bc} F^b \epsilon^c,$$

$$\delta H^A = \alpha^A_{aB} H^B \epsilon^a - \alpha^A_{aB} F^a \wedge \mu^B - n^{AB}_a F^a \wedge \mu'_B . + n^{AB}_a F^{(D)}_B \epsilon^a + T^A_{abc} A^a \wedge F^c \epsilon^b$$

F=0 is necessary for covariance: fake curvature condition

Reduction to 2-form gauge theory

- The theory constructed so far contains auxiliary gauge fields C, D

  They are the auxiliary gauge fields of BF theory.
  Since we are interested in the theory only with A, B, which is non-topological,
  we drop the auxiliary gauge fields by imposing constraint conditions:

  - Trivial example: C=0, D=0
  - We look for a case in which the gauge transformation of the field strengths H, F is deformed/canceled in δH,

    For this, we shift δB by a term F

    We do this by adjusting the gauge freedom of the auxiliary gauge fields C, D

In general, we also analyze canonical transformations on the QP-manifold to identify equivalent theories.
One possibility is to take constraints:

$$s^{aA} C_a = \Gamma^A_{ab} A^a \wedge F^b \qquad\qquad D_A = 0$$

Reduction to 2-form gauge theory

- With constraints on the auxiliary gauge fields, a smaller class of gauge symmetries remains.
  - Identify the residual gauge symmetry, and the transformation of the field strengths (F, H) under this residual symmetry.
  - Identify also the conditions for the transformation of the 3-form field strength, δH, to be covariant.

The results are summarized in the end, with residual gauge transformation $\hat{\delta}$ and parameters $\hat{\mu}, \hat{\epsilon}$ as:

$$
\begin{aligned}
F^a &= dA^a - \frac{1}{2} f^a_{bc} A^b \wedge A^c - t^a_A B^A, \\
H^A &= dB^A + \alpha^A_{aB} A^a \wedge B^B - \alpha^A_{aB} s^B_c F^a \wedge A^c, \\
\hat{\delta} A^a &= d\hat{\epsilon}^a - f^a_{bc} A^b \hat{\epsilon}^c - t^a_A \hat{\mu}^A, \\
\hat{\delta} B^A &= d\hat{\mu}^A + \alpha^A_{jB} (A^j \wedge \hat{\mu}^B + \hat{\epsilon}^j B^B) - \alpha^A_{jB} s^B_c \hat{\epsilon}^c F^j, \\
\hat{\delta} F^a &= f^a_{bc} F^b (\hat{\epsilon}^c - (\mathcal{P}\hat{\epsilon})^c), \\
\hat{\delta} H^A &= \alpha^A_{aB} H^B (\hat{\epsilon}^a - (\mathcal{P}\hat{\epsilon})^a),
\end{aligned}
$$

Discussion

- Systematic reduction from Lie n-algebra gauge theory to Lie 2-algebra gauge theory is proposed
- 5-dim. case is analyzed carefully, and an example of a covariantized theory is constructed.
- The algebroid version can be a natural generalization

  We have anyway scalar field in M5 effective theory.
- Other possibility: inclusion of $\Theta^{(0)}$

  In this case, we modify the (semi)strict Lie 2-algebra structure.
- 4-dimensional case: n=3