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SUPERSPACE DUALITY

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DUALITY IN 4D $N=1$ SUPERSPACE

REAL LINEAR $\longleftrightarrow$ CHIRAL

$$D^2 G = 0 \qquad \bar{D}_{\dot{\alpha}} \phi = 0$$

$$\bar{D}^2 G = 0 \qquad D_\alpha \bar{\phi} = 0$$

COMPLEX LINEAR $\longleftrightarrow$ CHIRAL

$$\bar{D}^2 \Sigma = 0 \qquad \bar{D}_{\dot{\alpha}} \phi = 0$$

$$D^2 \bar{\Sigma} = 0 \qquad D_\alpha \bar{\phi} = 0$$

THERE IS A FUNDAMENTAL DIFFERENCE

BETWEEN THESE DUALITIES

REAL LINEAR

$$\int d^4\theta\, f(G) = \int d^4\theta \left(f(G) - \phi G - \bar{\phi} G \right)$$

G unconstrained
$\phi, \bar{\phi}$ chiral

$$\delta\phi : \bar{D}^2 G = 0 \qquad \delta\bar{\phi} : D^2 G = 0$$

BACK TO START

$$\delta G : \; f'(G) = \phi + \bar{\phi} \;\Rightarrow\; G = h(\phi + \bar{\phi})$$

$$\Rightarrow \int d^4\theta\, \tilde{f}(\phi + \bar{\phi})$$ Legendre transform

DUAL MODEL DEPENDS ON $\phi + \bar{\phi}$ ONLY

COMPLEX LINEAR

$$\int d^4\theta\, L(\Sigma,\bar{\Sigma}) = \int d^4\theta \left\{ L(\Sigma,\bar{\Sigma}) - \phi\Sigma - \bar{\phi}\bar{\Sigma} \right\}$$

$\Sigma, \bar{\Sigma}$ unconstrained

$\phi, \bar{\phi}$ chiral

$\delta\phi : \bar{D}^2\Sigma = 0 \qquad \delta\bar{\phi} : D^2\bar{\Sigma} = 0 \Rightarrow$ BACK TO START

$\delta\Sigma : \partial_\Sigma L(\Sigma,\bar{\Sigma}) = \phi \,, \quad \delta\bar{\Sigma} : \partial_{\bar{\Sigma}} L(\Sigma,\bar{\Sigma}) = \bar{\phi}$

$\Rightarrow \int d^4\theta\, K(\phi,\bar{\phi})$ ← Legendre transform

DUAL MODEL DOES NOT HAVE PREFERRED FORM OF $K(\phi,\bar{\phi})$

REAL LINEAR	COMPLEX LINEAR
DUAL MODEL OF SPECIAL FORM	DUAL MODEL OF GENERAL FORM
DUAL DESCRIPTIONS TRULY DIFFERENT	DUAL MODEL JUST A COORDINATE TRANS. OF ORIGINAL MODEL
Φ: φ (scalar), ψ_α (spinor)	Σ: φ (scalar), ψ_α (spinor)
G: B_{MN} (2-form), χ_α (spinor)	DESCRIBES KÄHLER GEOMETRY IN "WRONG" COORDINATES

LORE: $\Sigma \longleftrightarrow \Phi$ is always possible

$$\int d^4\theta \left\{ -\Sigma\bar{\Sigma} + \frac{1}{\Lambda^4} (\bar{D}\Sigma)^2 (D\bar{\Sigma})^2 \right\}$$

$$\int d^4\theta \left\{ -\Sigma\bar{\Sigma} + \frac{1}{\Lambda^4} (\bar{D}\Sigma)^2 (D\bar{\Sigma})^2 + \phi\Sigma + \bar{\phi}\bar{\Sigma} \right\}$$

$$\delta\bar{\Sigma}: \quad -\Sigma + \bar{\Phi} - \frac{1}{\Lambda^4} D^\alpha \left(D_\alpha \bar{\Sigma} (\bar{D}\Sigma)^2 \right) = 0$$

NEED Σ AND $\bar{D}_{\dot{\alpha}}\Sigma$ $\quad (\phi, \bar{\phi}, D\phi, \bar{D}\bar{\phi}, \ldots)$

START BY INVERTING

$$\bar{D}_{\dot{\alpha}}\bar{\Phi} = \bar{D}_{\dot{\alpha}}\Sigma + \frac{1}{\Lambda^4} \bar{D}_{\dot{\alpha}} D^\alpha \left(D_\alpha \bar{\Sigma} (\bar{D}\Sigma)^2 \right)$$

BUT WHAT ABOUT

$$\int d^4\theta \left\{ -\Sigma\bar{\Sigma} + \frac{1}{\Lambda^4}(D\Sigma)^2(\bar{D}\bar{\Sigma})^2 + \Phi\Sigma + \bar{\Phi}\bar{\Sigma} \right\}$$

$\delta\bar{\Sigma}$: $\quad -\Sigma + \bar{\Phi} - \frac{1}{\Lambda^4} D^\alpha\left(D_\alpha\Sigma(\bar{D}\bar{\Sigma})^2\right) = 0$

NEED $\Sigma, D_\alpha\Sigma \qquad (\Phi, \bar{\Phi}, D\Phi, \bar{D}\bar{\Phi}, \ldots)$

TRY SAME TRICK

$$\underbrace{D_\alpha\bar{\Phi}}_{=0} = D_\alpha\Sigma + \frac{1}{\Lambda^4} D_\alpha D^\beta\left(D_\beta\Sigma(\bar{D}\bar{\Sigma})^2\right)$$

NOT INVERTIBLE !

WHAT IS GOING ON?

PHYSICAL	AUXILLIARY
$\sigma = \Sigma\vert$	$F = D^2\Sigma\vert$
$\bar{\psi}_{\dot\alpha} = \bar{D}_{\dot\alpha}\Sigma\vert$	$\lambda_\alpha = D_\alpha\Sigma\vert$
	$P_{\alpha\dot\alpha} = \bar{D}_{\dot\alpha} D_\alpha \Sigma\vert$
	$\bar{\chi}_{\dot\alpha} = D^\alpha \bar{D}_{\dot\alpha} D_\alpha \Sigma\vert$

BOSONIC PART OF ACTION

$$\mathcal{L}_B = -F\bar{F} + \sigma\Box\bar{\sigma} + \frac{1}{2}P\cdot\bar{P} + \frac{1}{\Lambda^4}\left(F^2\bar{F}^2 + F\bar{F}\,P\cdot\bar{P} + P\cdot P\,\bar{P}\cdot\bar{P}\right)$$

SOLVE

$$\delta \bar{P}^M : \quad P_M + \frac{1}{\Lambda^4}\left(2 P\cdot P\, \bar{P}_M + P_M F\bar{F}\right) = 0$$

$$\delta \bar{F} : \quad -F + \frac{1}{\Lambda^4}\left(P\cdot\bar{P}\, F + 2 F F \bar{F} \right)$$

$P_M = 0$ ONLY LORENTZ INV. SOLUTION

TWO SOLUTIONS FOR F

$$F = 0 \qquad\qquad F\bar{F} = \frac{\Lambda^4}{2}$$

$F=0$

$$\mathcal{L} = \sigma \Box \bar{\sigma} + i\psi^{\alpha}\partial_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} +$$

$$+ \text{auxilliary}$$

SAME AS FREE CHIRAL THEORY

SUSY NOT BROKEN

$F\bar{F} = \frac{\Lambda^4}{2}$

$$\mathcal{L} = \sigma \Box \bar{\sigma} - i\psi^{\alpha}\partial_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} +$$

$$-2\frac{F\bar{F}}{\Lambda^4}\lambda^{\alpha} i\partial_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} + \dots$$

AUXILLIARY FERMION BECOMES PROPAGATING

SUSY:

$$\delta\lambda_{\alpha} = \varepsilon_{\alpha} F + \dots \rightarrow$$

$$\rightarrow \varepsilon_{\alpha}\frac{\Lambda^2}{\sqrt{2}} + \dots$$

~~SUSY~~ λ_{α} Goldstino

ON THE OTHER HAND

$$\int d^4\theta \left\{ -\Sigma\bar{\Sigma} + \frac{1}{\Lambda^4}\left(\bar{D}\Sigma\right)^2\left(D\bar{\Sigma}\right)^2 \right\}$$

BOSONIC ACTION

$$\mathcal{L}_B = -F\bar{F} + \sigma\Box\bar{\sigma} + \frac{1}{2}P\cdot\bar{P} + \frac{1}{\Lambda^4}\left(P\cdot P\,\bar{P}\cdot\bar{P}\right)$$

ONLY SOLUTION

$$F = 0 \qquad P_M = 0$$

NO SUSY BREAKING

IN 2D

MANY MORE REPRESENTATIONS

CHIRAL	TWISTED CH.	SEMICH.	LINEAR	TWISTED L.
$\phi, \bar\phi$	$\chi, \bar\chi$	$X_L\, X_R\, \bar X_L\, \bar X_R$	$\Sigma, \bar\Sigma$	$\Upsilon, \bar\Upsilon$
$\bar D_\pm \phi = 0$	$\bar D_+ \chi = 0$	$\bar D_+ X_L = 0$	$\bar D_+ \bar D_- \Sigma = 0$	$\bar D_+ D_- \Upsilon = 0$
$D_\pm \bar\phi = 0$	$D_+ \bar\chi = 0$	$D_+ \bar X_L = 0$	$D_+ D_- \bar\Sigma = 0$	$\bar D_- D_+ \bar\Upsilon = 0$
	$D_- \chi = 0$	$\bar D_- X_R = 0$		
	$\bar D_- \chi = 0$	$D_- \bar X_R = 0$		

REAL LINEAR

$$G \equiv \chi + \bar\chi$$

REAL TWISTED LINEAR

$$\tilde G = \phi + \bar\phi$$

MANY MORE DUALITIES

$$\int f(\phi+\bar{\phi}) \longleftrightarrow \int \tilde{f}(x+\bar{x})$$

$$\int f\left(\phi+\bar{\phi},\, x+\bar{x},\, i(\phi-\bar{\phi}+x-\bar{x})\right) \longleftrightarrow \int \tilde{f}\left(x_L+\bar{x}_L,\, x_R+\bar{x}_R,\, i(x_L-\bar{x}_L+x_R-\bar{x}_R)\right)$$

DUALITIES THAT CHANGE DESCRIPTION (T-DUALITIES)

$$\int K(\phi,\bar{\phi}) \longleftrightarrow \int \hat{K}(\Sigma,\bar{\Sigma})$$

$$\int K(x,\bar{x}) \longleftrightarrow \int \hat{K}(r,\bar{r})$$

DUALITIES THAT DON'T CHANGE DESCRIPTION

THERE IS A CORRESPONDING INCRESE IN
SCALAR AUXILLIARY FIELDS

$$P_{\alpha\dot\alpha} \longrightarrow P_{\#},\ P_{=},\ P_{+-} = K,\ P_{-+} = L$$

AND IN THE POSSIBLE HIGHER DERIVATIVE TERMS

$$\int d^4\theta\, D_+\Sigma\, D_-\Sigma\, \bar D_+\bar\Sigma\, \bar D_-\bar\Sigma\ ,\quad \int d^4\theta\, \bar D_+\Sigma\, \bar D_-\Sigma\, D_+\bar\Sigma\, D_-\bar\Sigma$$

$$\int d^4\theta\, D_+\Sigma\, \bar D_-\Sigma\, \bar D_+\bar\Sigma\, D_-\bar\Sigma\ ,\quad \int d^4\theta\, \bar D_+\Sigma\, D_-\Sigma\, D_+\bar\Sigma\, \bar D_-\bar\Sigma$$

$$\int d^4\theta\, D_+\Sigma\, \bar D_-\Sigma\, D_+\bar\Sigma\, \bar D_-\bar\Sigma + c.c. \quad \int d^4\theta\, D_+\Sigma\, \bar D_+\Sigma\, D_-\bar\Sigma\, \bar D_-\bar\Sigma + c.c.$$

ONLY CONSTRAINED BY LORENTZ + REALITY

ADDING ALL TERMS WITH ARBITRARY COEFFS. GIVES COMPLICATED 4'th ORDER POLYNOMIAL IN AUXILLIARY SCALAR FIELDS WITH MANY SOLN'S

SOME BASIC CASES:

$$-\Sigma\bar{\Sigma} + \frac{1}{\Lambda^4} D_+\Sigma D_-\Sigma \bar{D}_+\bar{\Sigma}\bar{D}_-\bar{\Sigma}$$

SAME AS 4D $\quad D_+D_-\Sigma = 0, \frac{\Lambda^2}{\sqrt{2}}$ ~~SUSY~~

IN SUPERSPACE

$$\Sigma = \bar{\Phi} + X$$

Prop $\qquad$ Goldstino

X CHIRAL SUCH THAT

$$X^2 = 0$$

$$\bar{X} D_+ D_- X = \frac{\Lambda^2}{\sqrt{2}} \bar{X}$$

$$X = \psi^2 + \theta^\alpha \psi_a + F$$

INSERT

$$D_+\Sigma D_-\Sigma \bar{D}_+\bar{\Sigma} \bar{D}_-\bar{\Sigma} \to D_+X D_-X \bar{D}_+\bar{X} \bar{D}_-\bar{X}$$

$$\left(0 = \frac{1}{2} D_+ D_- (X^2) = X D_+ D_- X + D_+ X D_- X \right)$$

$$\to X D_+ D_- X \bar{X} \bar{D}_+ \bar{D}_- \bar{X} = \frac{\Lambda^4}{2} X \bar{X}$$

$$D_\pm \Sigma = D_\pm X$$ AUXILLIARY FERM. → GOLDSTINO

$$\bar{D}_\pm \Sigma = \bar{D}_\pm \bar{\Phi}$$ PHYSICAL FERM.

$$D_+ D_- \Sigma = D_+ D_- X$$ VEV OF F

$$\bar{D}_+ D_- \Sigma = 0$$

$$D_+ \bar{D}_- \Sigma = 0$$

SAME SITUATION AS IN 4D

$$-\Sigma\bar{\Sigma}+\frac{1}{\Lambda^4}D_+\Sigma\bar{D}_-\Sigma D_+\bar{\Sigma}D_-\bar{\Sigma}$$

SOLUTION FOR $D_+\bar{D}_-\Sigma=\frac{\Lambda^2}{\sqrt{2}}$

NOW $\Sigma=\bar{\Phi}+\chi$

Goldstino $(\chi^2=0)$

FOR $\bar{\Phi}$ TO DROP OUT OF SECOND TERM

$$\left.\begin{aligned} D_\pm\bar{\Phi}&=0\\ \bar{D}_-\bar{\Phi}&=0\end{aligned}\right\} i\partial_=\bar{\Phi}$$

"LEFTON"

$$D_+ \Sigma = D_+ \chi \quad \text{Goldstino}$$

$$D_- \Sigma = 0$$

$$\bar{D}_+ \Sigma = \bar{D}_+ \bar{\Phi} \quad \text{lefton partner}$$

$$\bar{D}_- \Sigma = \bar{D}_- \chi \quad \text{Goldstino}$$

$$D_+ D_- \Sigma = 0$$

$$\bar{D}_+ D_- \Sigma = 0$$

$$D_+ \bar{D}_- \Sigma = D_+ \bar{D}_- \chi = L = \frac{\Lambda^2}{\sqrt{2}}$$

RESULTING THEORY

LEFTON + GOLDSTINO

- HIGHER DERIVATIVE TERMS SPONTANEOUSLY BREAK SUSY
- NEW MECHANISM. AUXILLIARY FERMION BECOMES PROPAGATING GOLDSTINO
- IN 2D MANY NEW EXOTIC THEORIES