Non-geometric fluxes and their back-reaction

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This talk is based on work with R. Blumenhagen and M. Fuchs ::

- *Partial SUSY breaking for asymmetric Gepner models and non-geometric flux vacua* [1608.00595]
- *The ACFT landscape in D=4,6,8 with extended supersymmetry* [1611.04617]

outline

1. introduction
2. partial susy breaking
3. gepner models
4. discussion

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    a) non-geometry
    b) fluxes
    c) back-reaction
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Dualities :: two different theories describe the same physics.

introduction :: dualities I

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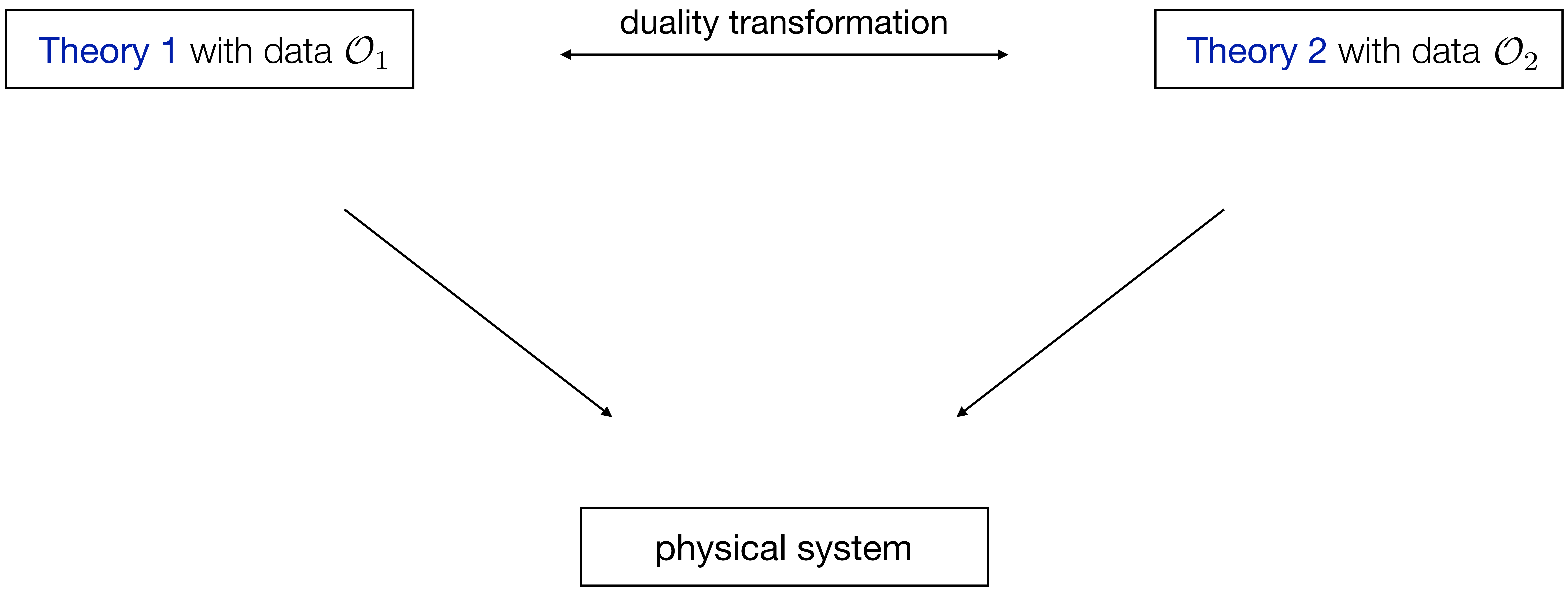
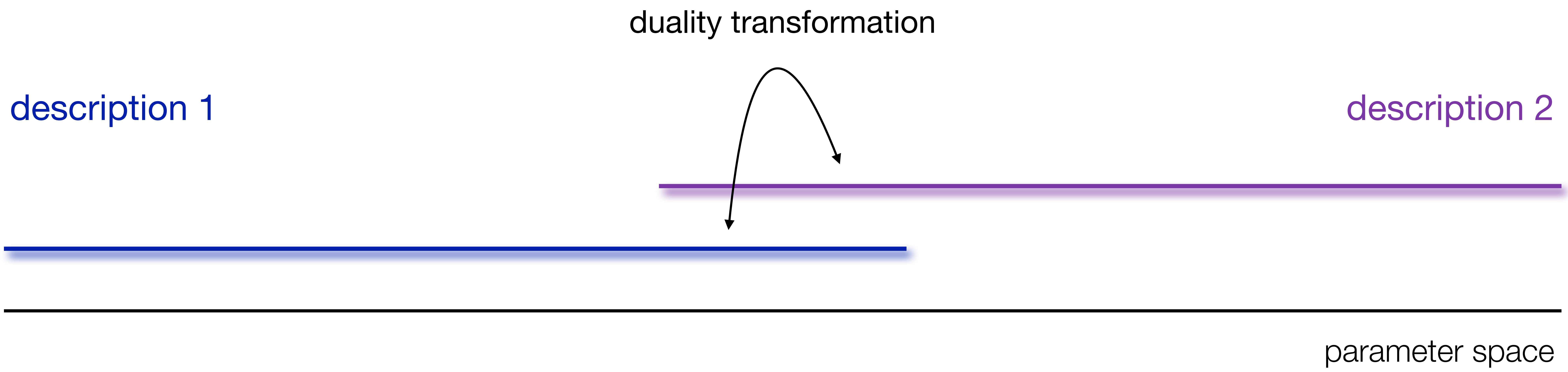


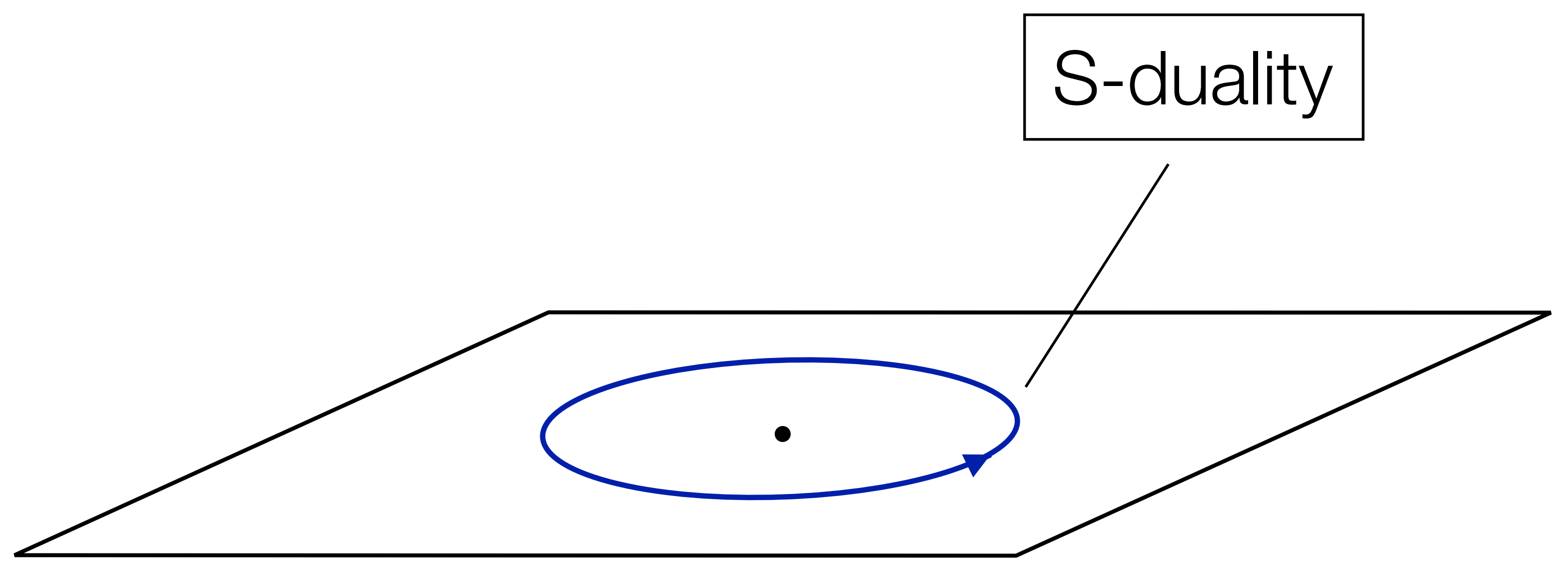
Regions in parameter space can have different descriptions.



String theory :: can use duality transformations as transition functions.

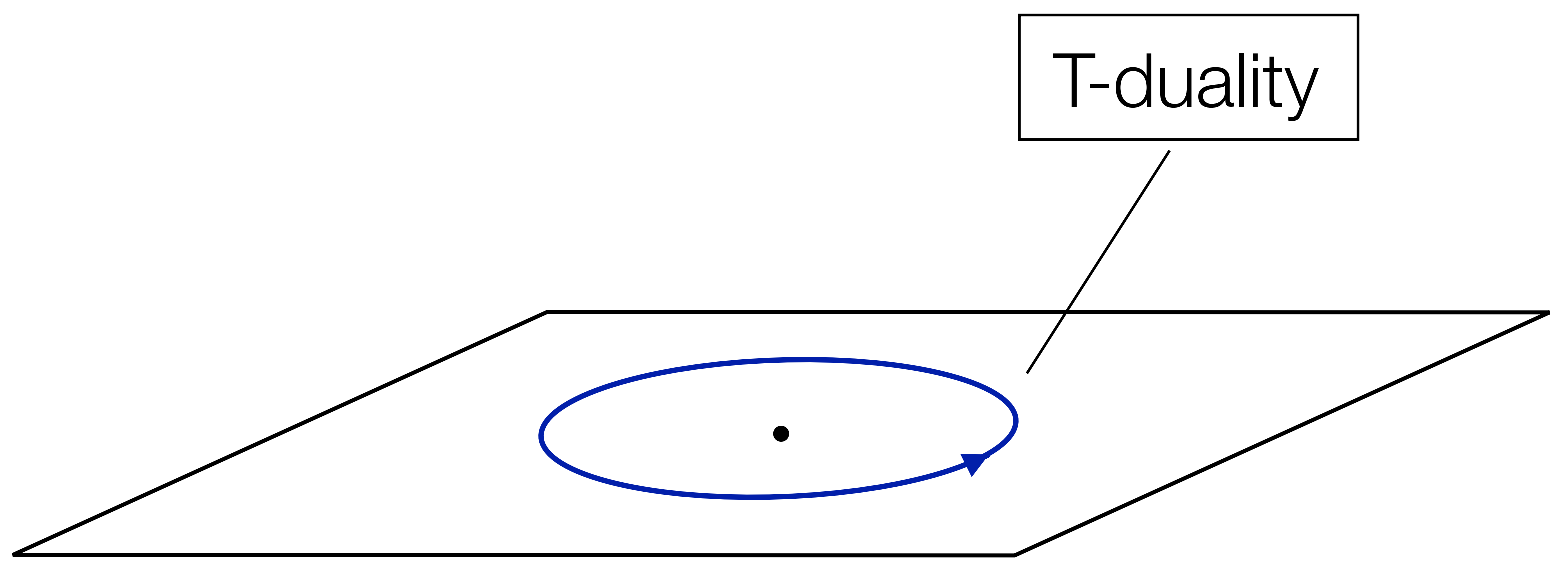
S-duality

- duality transformation $g_s \to 1/g_s$
- monodromy around *(p,q)*-branes contains S-duality



T-duality

- duality transformation $R \to 1/R$
- monodromy around defects may contain T-duality



Hellerman, McGreevy, Williams - 2003

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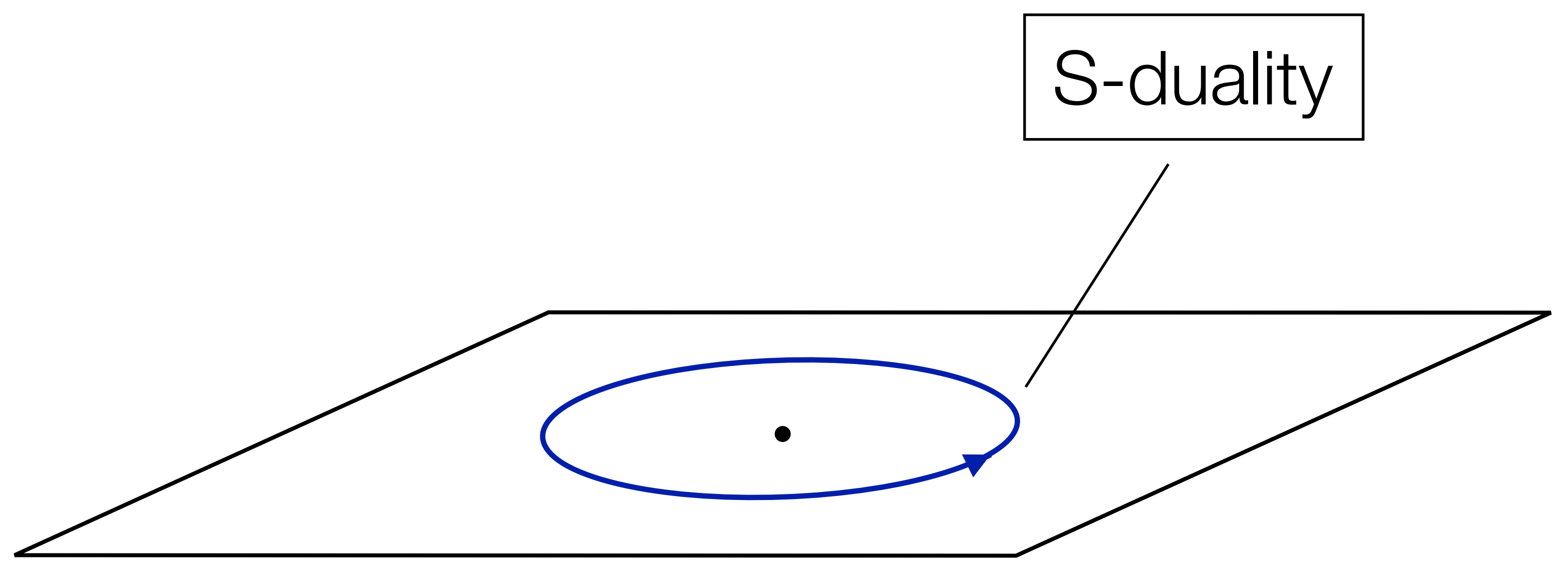

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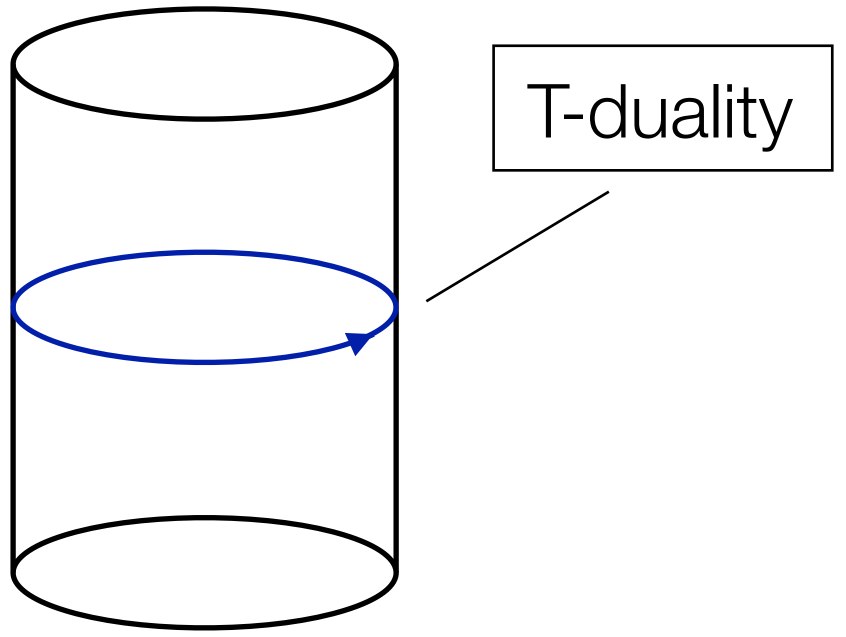


→ non-geometric background

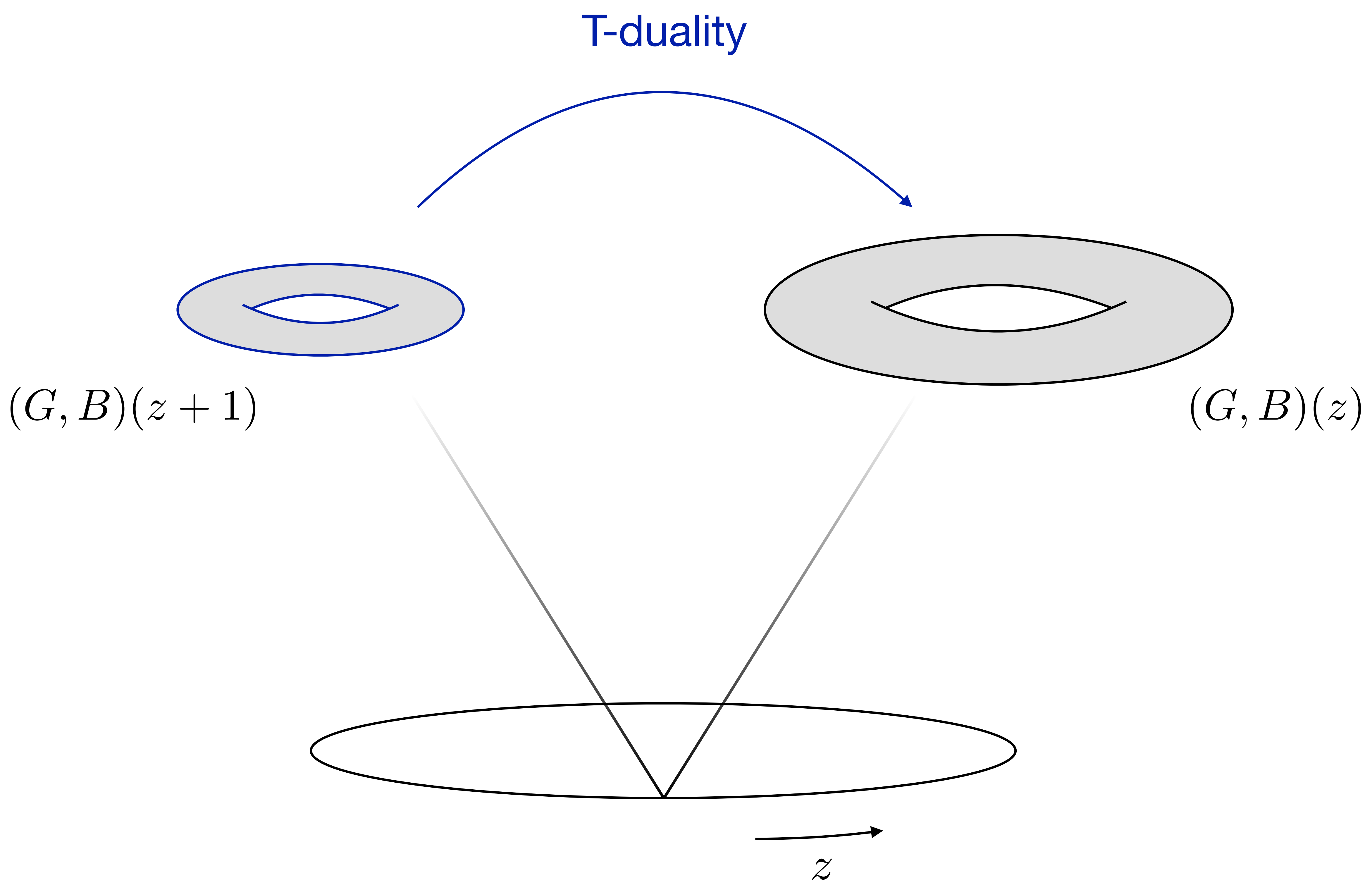
Hellerman, McGreevy, Williams - 2003
Hull - 2004

The prime example for a non-geometric background is the T-fold ::

$$ds^2 = \frac{1}{1+(Nz)^2}\,(dx^2+dy^2) + dz^2\,,$$

$$B_{xy} = \frac{Nz}{1+(Nz)^2}\,,$$

$$x \sim x+1\,, \quad y \sim y+1\,.$$



Hull - 2004

Question :: what measures the non-triviality of the fibration? (What is the parameter *N*?)

Consider the following family of T-dual backgrounds ::

name	metric	*B*-field	transformation	flux
$\mathbb{T}^3$ w/ *H*-flux	trivial	non-trivial	gauge transformation	H_{ijk}
twisted torus	non-trivial	trivial	diffeomorphism	${F_{ij}}^k$
T-fold	non-trivial	non-trivial	T-duality	${Q_i}^{jk}$
R-space	…	…	…	R^{ijk}

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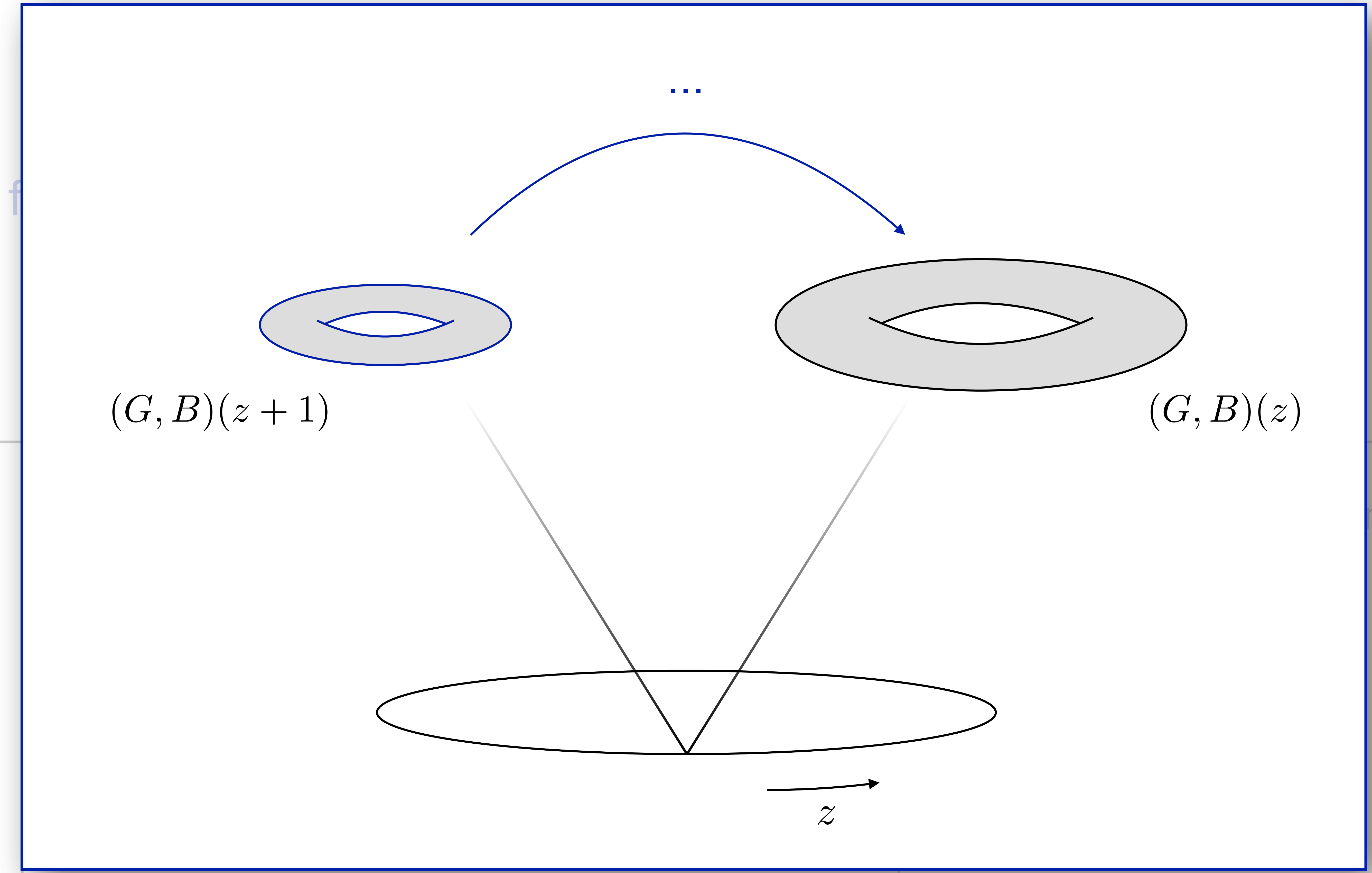

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R-space	…	…	…	R^{ijk}

Answer :: fluxes — geometric and non-geometric — measure the non-triviality of the fibration.

These fluxes are related via a T-duality chain ::

$$H_{ijk} \overset{T_k}{\longleftrightarrow} F_{ij}{}^{k} \overset{T_j}{\longleftrightarrow} Q_i{}^{jk} \overset{T_i}{\longleftrightarrow} R^{ijk} \,.$$

H_{ijk} ↑	$F_{ij}{}^{k}$ ↑	$Q_i{}^{jk}$ ↑	R^{ijk} ↑
torus with *H*-flux	twisted torus	T-fold	non-associative space

Shelton, Taylor, Wecht - 2005

The fluxes can be interpreted as operators and be combined into a twisted differential

$$\begin{aligned} H\wedge &: \quad p\text{-form} \quad \to \quad (p+3)\text{-form}\,, \\ F\circ &: \quad p\text{-form} \quad \to \quad (p+1)\text{-form}\,, \\ Q\bullet &: \quad p\text{-form} \quad \to \quad (p-1)\text{-form}\,, \\ R\llcorner &: \quad p\text{-form} \quad \to \quad (p-3)\text{-form}\,, \end{aligned}$$

$$\mathcal{D} = d - H\wedge\; - F\circ\; - Q\bullet\; - R\llcorner\,.$$

Aldazabal, Camara, Font, Ibanez - 2006
Villadoro, Zwirner - 2006
Shelton, Taylor, Wecht - 2006

When requiring nil-potency $\mathcal{D}^2 = 0$, Bianchi identities for the fluxes can be derived.

Shelton, Taylor, Wecht - 2006
Robins, Wrase - 2007

For Calabi-Yau three-folds, such fluxes describe *SU(3)xSU(3)* **structure** compactifications as

$$
\begin{aligned}
\mathcal{D}\alpha_\Lambda &= \; q_\Lambda{}^A\omega_A + f_{\Lambda\,A}\sigma^A\,, \\
\mathcal{D}\beta^\Lambda &= \; \tilde{q}^{\Lambda\,A}\omega_A + \tilde{f}^\Lambda{}_A\sigma^A\,, \\
\mathcal{D}\omega_A &= \; \tilde{f}^\Lambda{}_A\alpha_\Lambda - f_{\Lambda A}\beta^\Lambda\,, \\
\mathcal{D}\sigma^A &= -\tilde{q}^{\Lambda\,A}\alpha_\Lambda + q_\Lambda{}^A\beta^\Lambda\,,
\end{aligned}
$$

$$\{\alpha_\Lambda, \beta^\Lambda\} \in H^3(\mathcal{X})\,,$$

$$\{\omega_{\mathsf{A}}\} \in H^{1,1}(\mathcal{X})\,, \qquad \omega_0 = 1\,,$$

$$\{\sigma^{\mathsf{A}}\} \in H^{2,2}(\mathcal{X})\,, \qquad \sigma^0 = d\mathrm{vol}_6\,.$$

The *H*- and *R*-fluxes are contained in $f_{\Lambda\,0} = h_\Lambda\,,\ \ \tilde{f}^\Lambda{}_0 = \tilde{h}^\Lambda\,,\ \ q_\Lambda{}^0 = r_\Lambda\,,\ \ \tilde{q}^{\Lambda\,0} = \tilde{r}^\Lambda\,.$

Grana, Louis, Waldram - 2006

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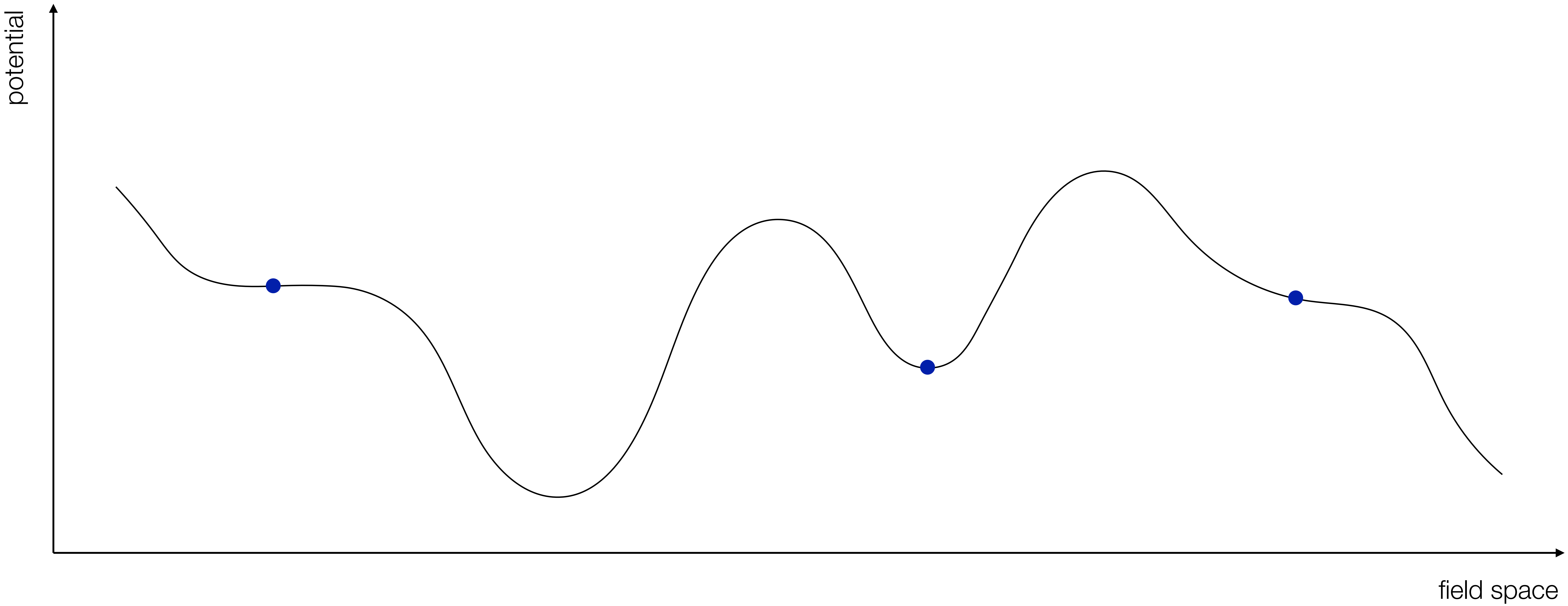
A string-theory background has to solve the equations of motion (vanishing β-functionals).



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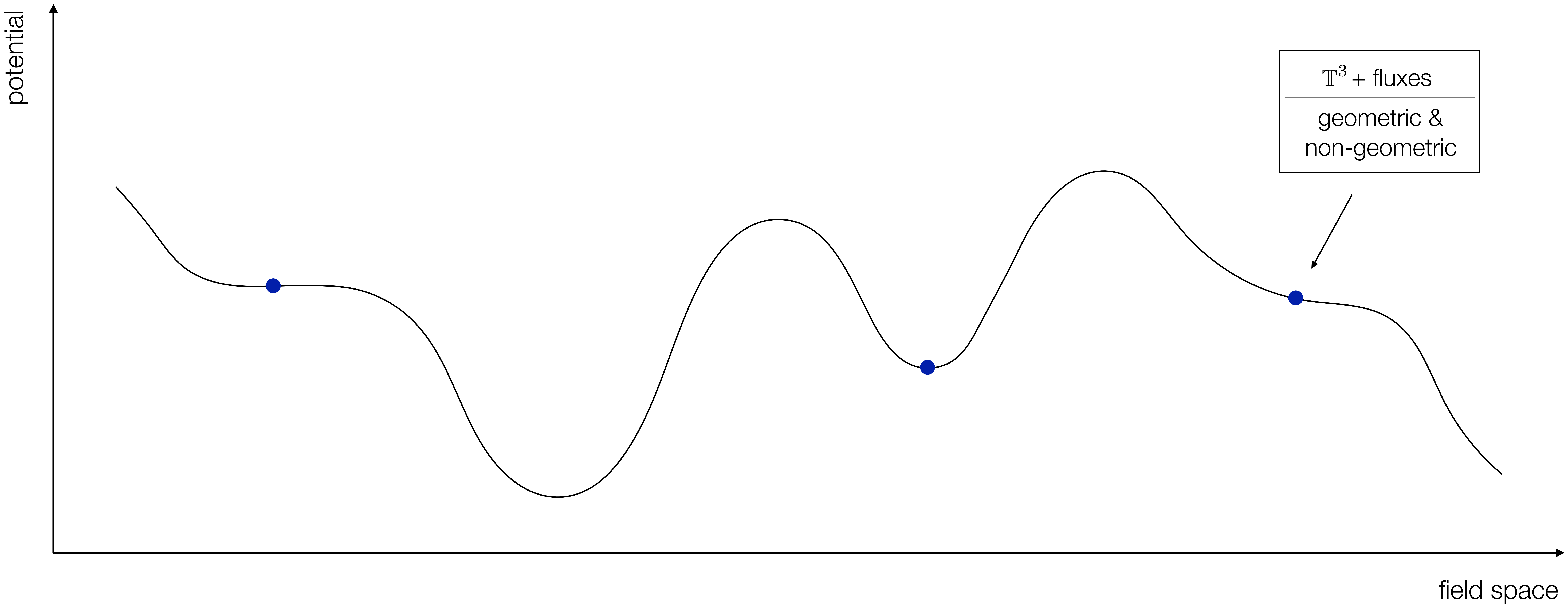

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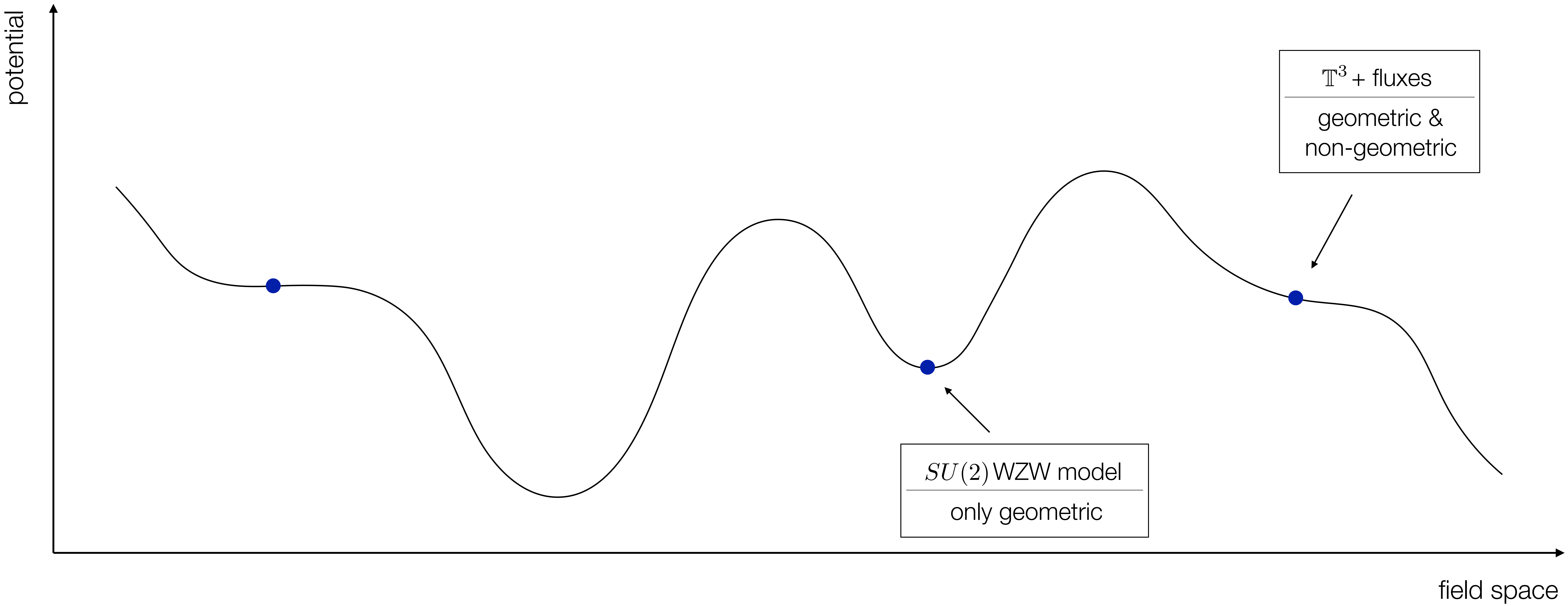

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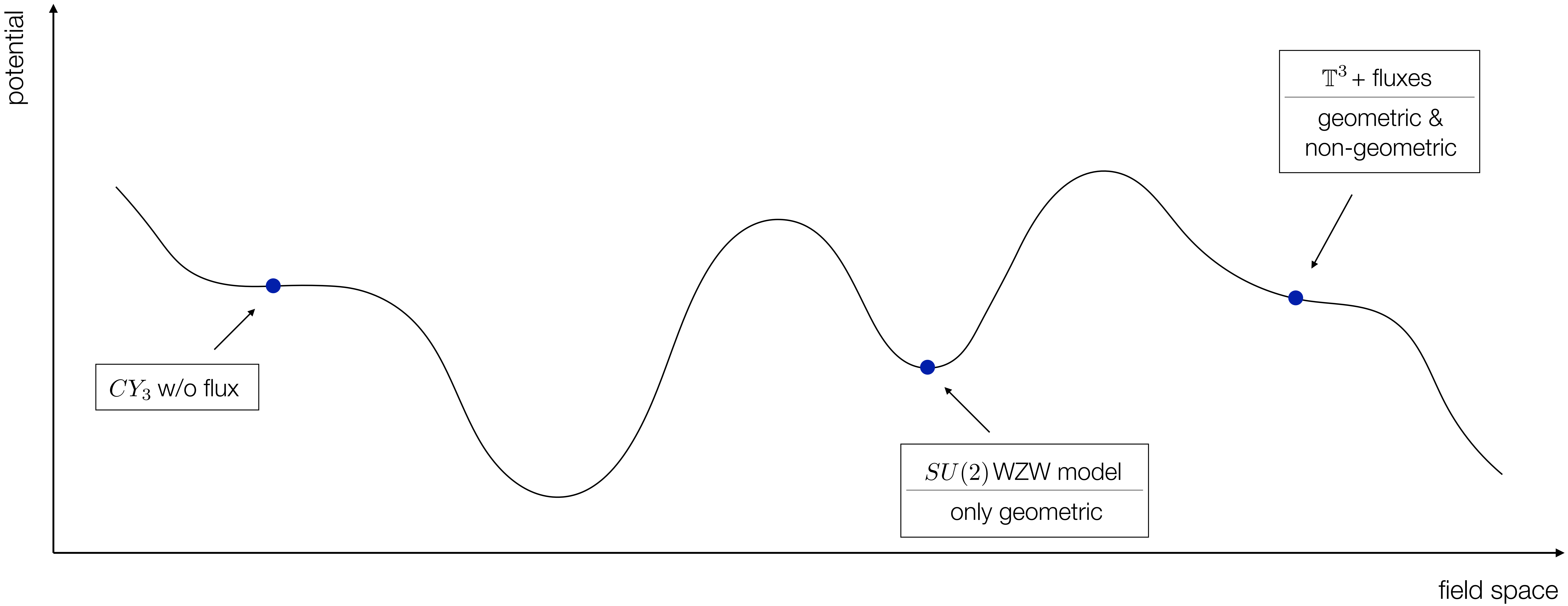

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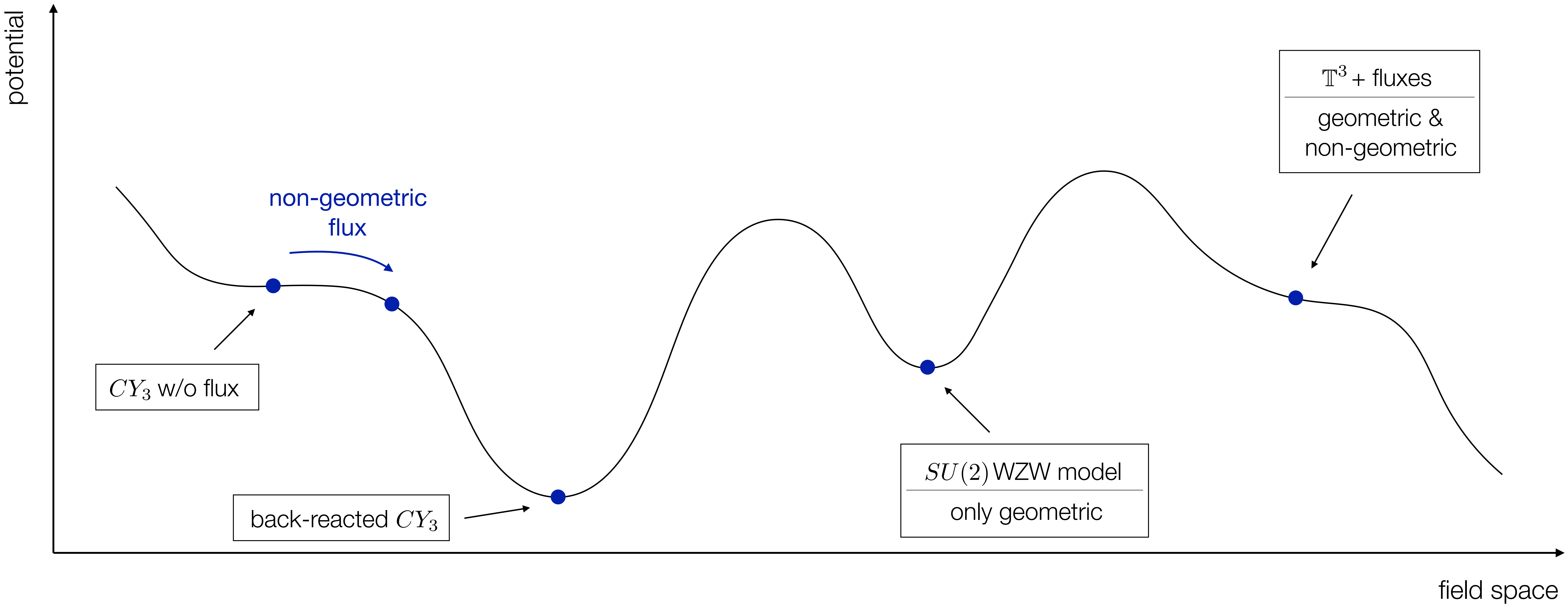

Question :: What is the back-reaction of non-geometric fluxes?

introduction :: back-reaction II

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Calabi-Yau three-fold N=2,1 susy in d=4	**Calabi-Yau three-fold** N=2,1 susy in d=4
↓ H-flux	↓ non-geometric flux
back-reacted manifold N=1,0 susy in d=4	…

Giddings, Kachru, Polchinski - 2001
...

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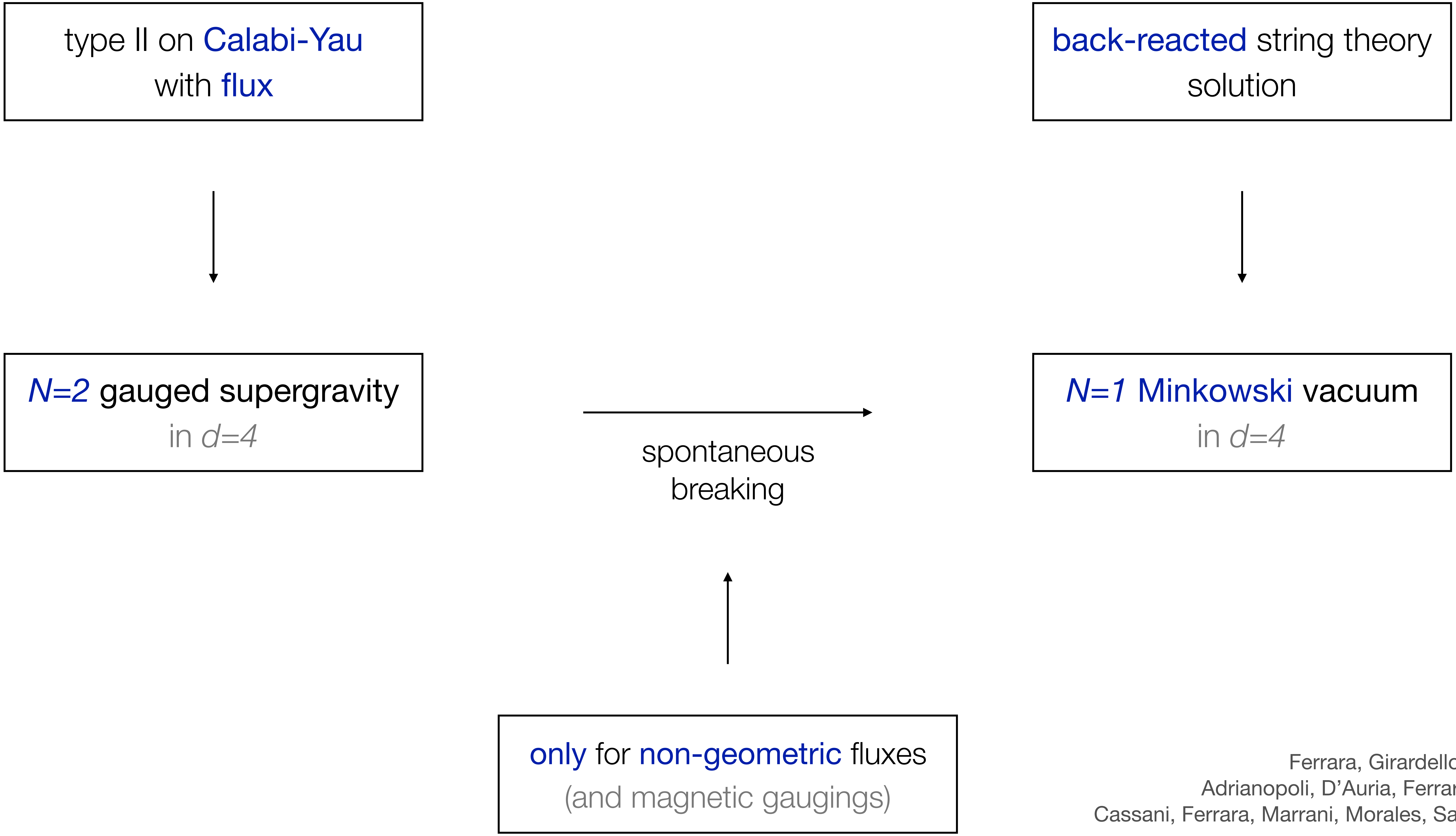
overview I

N=2 gauged supergravity in *d=4*	→ spontaneous breaking	*N=1* Minkowski vacuum in *d=4*

Ferrara, Girardello, Porrati - 1995
Adrianopoli, D'Auria, Ferrara, Lledo - 2002
Cassani, Ferrara, Marrani, Morales, Samtleben - 2009
Louis, Smyth, Triendl - 2009 & 2010

overview I



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The minimal process for spontaneous $\mathcal{N}=2 \to \mathcal{N}=1$ susy breaking (in the above context) is ::

$$\mathcal{G}_{(2)} + \mathcal{V}_{(2)} + \mathcal{H}_{(2)} \quad \to \quad G_{(1)} + \overline{S}_{(1)} + 2 \cdot C_{(1)} \,.$$

gravity	$\mathcal{G}_{(2)} = 1 \cdot [2] + 2 \cdot [\frac{3}{2}] + 1 \cdot [1]$		gravity	$G_{(1)} = 1 \cdot [2] + 1 \cdot [\frac{3}{2}]$
vector	$\mathcal{V}_{(2)} = 1 \cdot [1] + 2 \cdot [\frac{1}{2}] + 2 \cdot [0]$		chiral	$C_{(1)} = 1 \cdot [\frac{1}{2}] + 2 \cdot [0]$
hyper	$\mathcal{H}_{(2)} = 2 \cdot [\frac{1}{2}] + 4 \cdot [0]$		massive spin-$3/2$	$\overline{S}_{(1)} = 1 \cdot [\frac{3}{2}] + 2 \cdot [1] + 1 \cdot [\frac{1}{2}]$

Louis, Smyth, Triendl - 2009 & 2010

Requirements ::

- precisely one gravitino becomes massive,
- two gauge fields become massive $\longrightarrow$ Stückelberg mechanism,
- one fermion becomes massive.

Stückelberg mechanism — gauge field becomes massive by eating an axion.

For type IIB compactifications on Calabi-Yau three-folds ::

$h^{2,1}+1$	massless vector fields	$\xrightarrow{\mathcal{N}=2\to\mathcal{N}=1}$	$h^{2,1}-1$	massless vector fields
$2(h^{1,1}+1)+1$	massless axions		$2\,h^{1,1}+1$	massless axions

In general ::

- Fluxes gauge (axionic) shift symmetries $\longrightarrow$ vector fields become massive.
- Fluxes induce a scalar potential $\longrightarrow$ scalar fields become massive.

D'Auria, Ferrara, Trigiante - 2007
Cassani - 2008

This analysis can be generalized to multiple gaugings.

The following constraints — relating Calabi-Yau data to the *N=1* spectrum — have been derived ::

$$h^{2,1} - h^{1,1} - \Delta \leq N_V \leq h^{2,1} - 1\,,$$

$$N_V - N_{\text{ax}} \leq h^{2,1} - h^{1,1} - \Delta\,,$$

$$N_V - 2N_{\text{ax}} \geq h^{2,1} - 2h^{1,1} - \Delta\,,$$

$$N_0 \leq h^{2,1} + h^{1,1}\,,$$

N_V … massless vector fields,

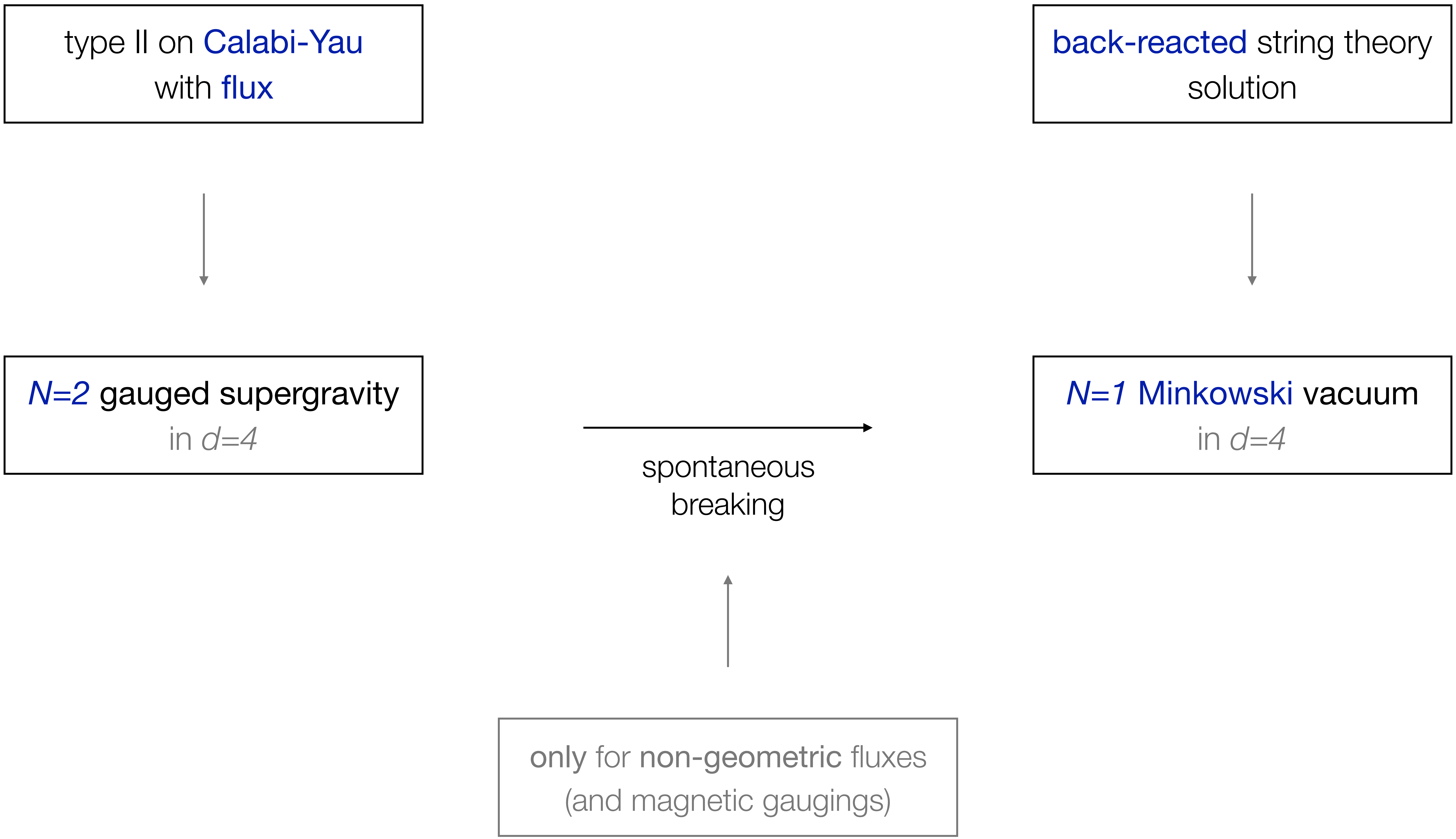
N_{ax} … massless complex R-R axions,

N_0 … massless complex NS-NS scalars,

$\Delta = 0, 1$ gauged NS-NS axion.

Blumenhagen, Fuchs, EP - 2016

overview II



type II on Calabi-Yau
with flux
back-reacted string theory
solution
N=2 gauged supergravity
in d=4
N=1 Minkowski vacuum
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spontaneous
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only for non-geometric fluxes
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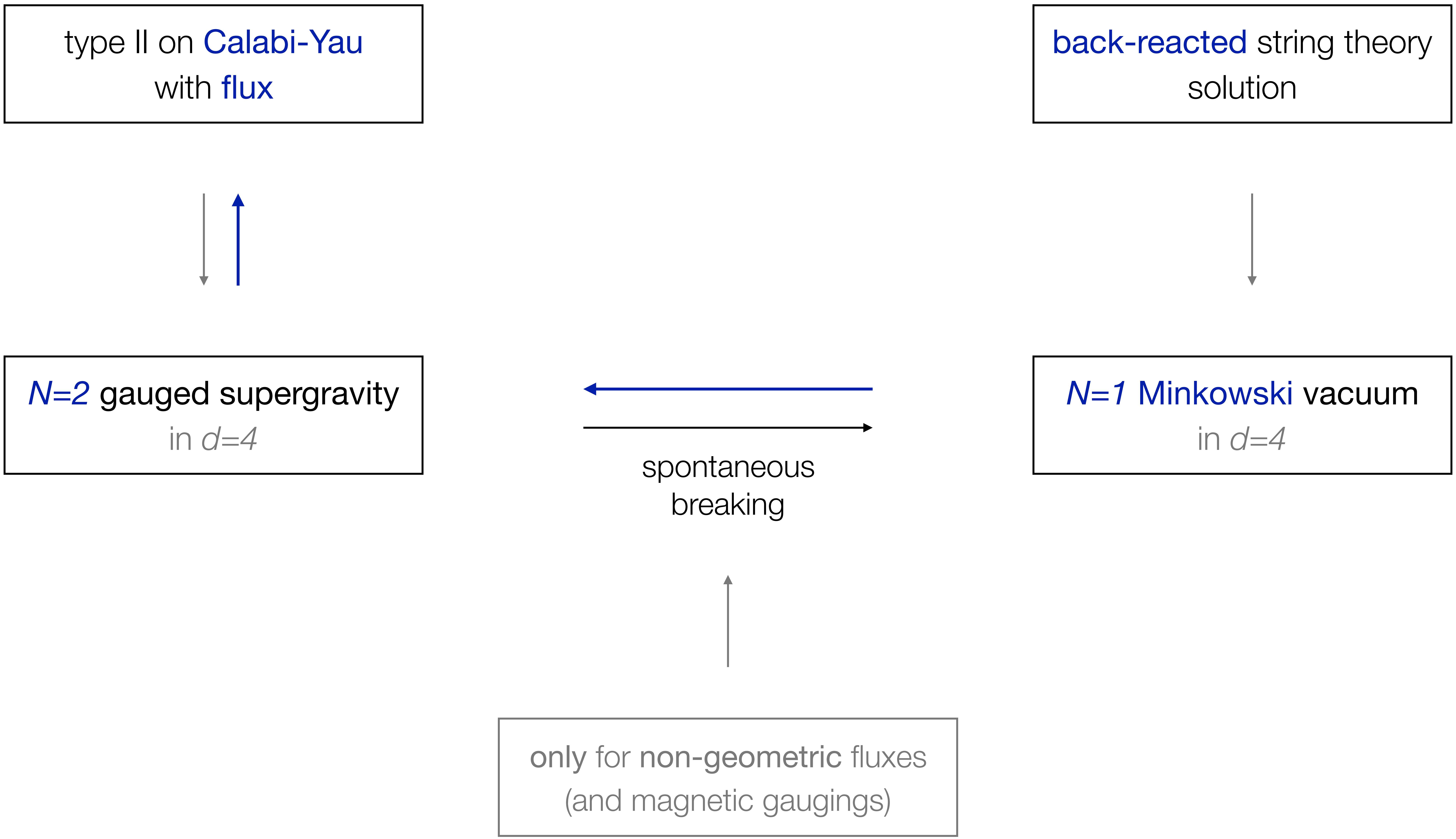
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Setting :: consider asymmetric simple-current extensions of Gepner models.

Gepner - 1987 & 1988
Schellekens, Yankielowicz - 1989 & 1990
Israël, Thiéry - 2013
Israël - 2015

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↓

product of minimal models

$$9 = c = \sum_i \frac{3\,k_i}{k_i + 2}$$

(N=2 Minkowski vacua)

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↓ certain primary fields (project to *N=1*)

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left sector ≠ right sector
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(*N=2* Minkowski vacua)

→ left-right asymmetric *N=1* Minkowski vacua in *d=4*.

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Geometry of Gepner models ::

massless fields	$\longrightarrow$	coordinates $x_i \in \mathbb{C}$
Yukawa couplings	$\longrightarrow$	constraints, e.g. $x_1^{k_1+2} + \ldots + x_5^{k_5+2} = 0$
U(1) charges	$\longrightarrow$	scaling weights of x_i

Example (where $d = \text{lcm}\{k_i + 2\}$) ::

$$(k_1,\, k_2,\, k_3,\, k_4,\, k_5) \longrightarrow \mathbb{P}_{\frac{d}{k_1+2}, \frac{d}{k_2+2}, \frac{d}{k_3+2}, \frac{d}{k_4+2}, \frac{d}{k_5+2}}[d]$$

Gepner - 1987
Witten - 1993

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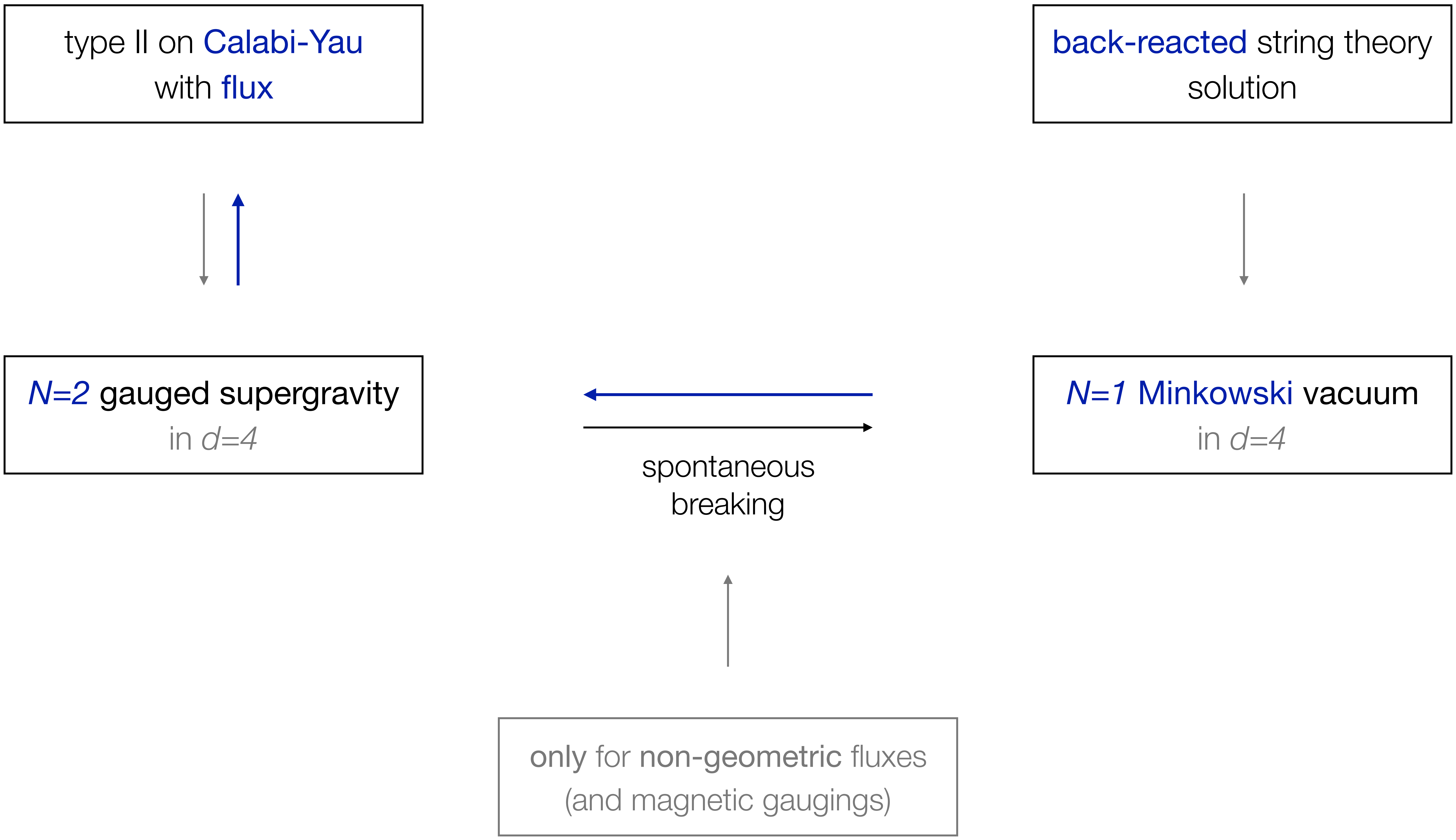
For **simple-current extensions** of Gepner models, this correspondence **does not work**.

Proposal :: a Calabi-Yau three-fold underlying the *N=1* theory can be constructed as

$$(2l-1,\ k_2,\ k_3,\ k_4,\ k_5) \text{ + simple current} \longrightarrow \mathbb{P}_{\frac{2d}{(2l+1)},\frac{ld}{(2l+1)},\frac{d}{k_2+2},\frac{d}{k_3+2},\frac{d}{k_4+2},\frac{d}{k_5+2}}\left[d \quad \frac{d(l+1)}{(2l+1)}\right]$$

Blumenhagen, Schimmrigk, Wißkirchen - 1995 & 1996
Blumenhagen, Fuchs, EP - 2016

overview III



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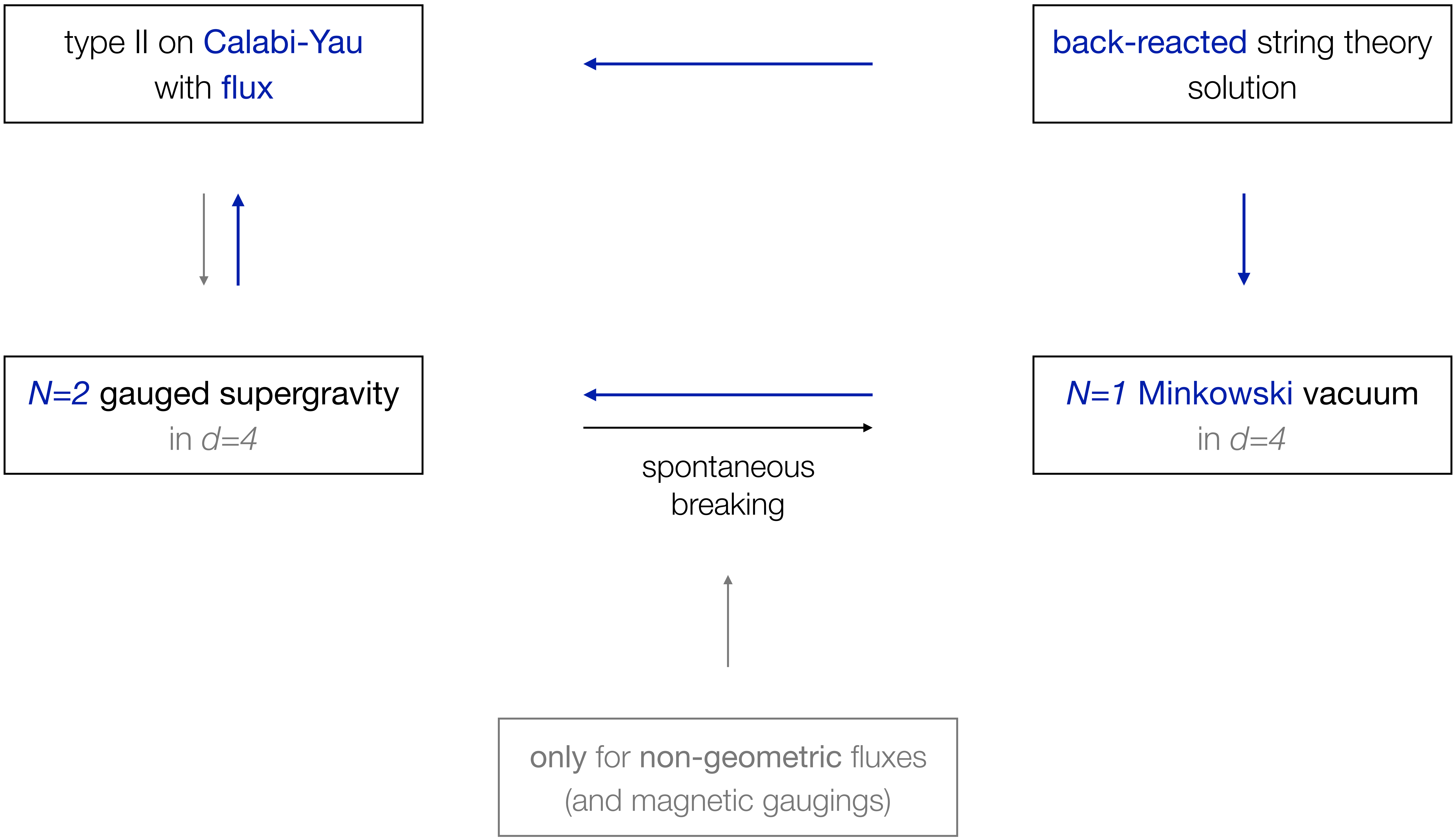

gepner models :: examples

Gepner model ACFT	$(N_V, N_{\mathsf{ax}}, N_0)$	conjectured *CY₃*	$(h^{2,1}, h^{1,1})$	susy-breaking constraints
$(3\,3\,3\,3\,3)$	$(80, 0, 74)$	$\mathbb{P}_{1,1,1,1,2,2}[4\ 4]$	$(83, 2)$	$N_V - N_{\mathsf{ax}} \leq 81 - \Delta$ $N_V - 2N_{\mathsf{ax}} \geq 79 - \Delta$
$(5\,5\,5\,12_D)$	$(86, 2, 80)$	$\mathbb{P}_{1,1,1,2,3,3}[7\ 4]$	$(89, 3)$	$N_V - N_{\mathsf{ax}} \leq 86 - \Delta$ $N_V - 2N_{\mathsf{ax}} \geq 83 - \Delta$
$(5\,5\,5\,12_A)$	$(86, 2, 80)$	$\mathbb{P}_{1,2,2,4,6,7}[14\ 8]$	$(88, 4)$	$N_V - N_{\mathsf{ax}} \leq 84 - \Delta$ $N_V - 2N_{\mathsf{ax}} \geq 80 - \Delta$
$(7\,7\,7\,1\,1)$	$(74, 2, 70)$	$\mathbb{P}_{1,1,2,3,3,4}[9\ 5]$	$(75, 6)$	$N_V - N_{\mathsf{ax}} \leq 69 - \Delta$ $N_V - 2N_{\mathsf{ax}} \geq 63 - \Delta$
$(6\,6\,6\,6_D)$	$(60, 4, 64)$	$\begin{matrix}\mathbb{P}_{1,1,2,3,4}\\ \mathbb{P}_{1,1}\end{matrix}\begin{bmatrix}7 & 4\\ 1 & 1\end{bmatrix}$	$(62, 6)$	$N_V - N_{\mathsf{ax}} \leq 56 - \Delta$ $N_V - 2N_{\mathsf{ax}} \geq 50 - \Delta$
$(10\,10\,4\,4)$	$(59, 5, 68)$	$\mathbb{P}_{1,2,2,3,4,9}[11\ 10]$	$(66, 8)$	$N_V - N_{\mathsf{ax}} \leq 58 - \Delta$ $N_V - 2N_{\mathsf{ax}} \geq 50 - \Delta$

Blumenhagen, Fuchs, EP - 2016

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Review :: Dualities, non-geometric backgrounds & non-geometric fluxes.

Question :: What is the back-reaction of non-geometric fluxes on Calabi-Yau three-folds?

Proposal :: Asymmetric simple-current extensions of Gepner models.

→ Concrete examples for CY_3 and back-reacted background.

→ Consistency with partial susy breaking has been checked.

discussion :: supergravity regime

In order to employ a supergravity framework, flux densities have to be small.

In order to employ a supergravity framework, flux densities have to be small ::

$$\text{Tr}\left[\mathcal{M}_1 \cdot \mathcal{O} \cdot \mathcal{M}_2^{-1} \cdot \mathcal{O}^T\right] \overset{!}{\ll} 1\,, \qquad \mathcal{O} = \begin{pmatrix} \tilde{f}^{\Lambda}{}_{A} & \tilde{q}^{\Lambda\, A} \\ f_{\Lambda\, A} & q_{\Lambda}{}^{A} \end{pmatrix},$$

$$\mathcal{M}_1 = \int_{\mathcal{X}} \begin{pmatrix} \alpha_\Lambda \wedge \star\alpha_\Sigma & \alpha_\Lambda \wedge \star\beta^\Sigma \\ \beta^\Lambda \wedge \star\alpha_\Sigma & \beta^\Lambda \wedge \star\beta^\Sigma \end{pmatrix},$$

$$\mathcal{M}_2 = \int_{\mathcal{X}} \begin{pmatrix} \langle \omega_A, \star_B\, \omega_B \rangle & \langle \omega_A, \star_B\, \sigma^B \rangle \\ \langle \sigma^A, \star_B\, \omega_B \rangle & \langle \sigma^A, \star_B\, \sigma^B \rangle \end{pmatrix}.$$

For the example of $\mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ and e.g. $\tilde{q}^{\,11} \neq 0$, this condition becomes

$$\left|\tilde{q}^{\,11}\right|^2 = \left(\tilde{q}^{\,11}\right)^2 \left[\int_{\mathcal{X}} \alpha_1 \wedge \star\alpha_1\right] \left[\int_{\mathcal{X}} \sigma^1 \wedge \star\sigma^1\right]^{-1} = \left(\tilde{q}^{\,11}\, \frac{R_5 R_6}{R_1}\right)^2 \overset{!}{\ll} 1\,.$$

Different types of fluxes on the "same cycle" are forbidden due to Bianchi identities.