Algebraic quantum field theories
Interpolating between non-commutativity and commutativity

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21 April 2018

Bayrischzell Workshop 2018

Joint work with Marco Benini and Alexander Schenkel

Outline

Algebraic quantum field theories

- **M**: bicomplete closed symmetric monoidal category, e.g. vector spaces or chain complexes.
- **Alg**: the category of algebras (actually: monoids) in **M**.
- **Loc**: category of oriented and time-oriented globally hyperbolic Lorentzian spacetimes with the distinguished class C of Cauchy morphisms.

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An *algebraic quantum field theory* is a functor $\mathfrak{A} : \mathbf{Loc} \to \mathbf{Alg}$ satisfying the following axioms:

1. *Einstein causality*: For morphisms $f_1 : M_1 \to M$ and $f_2 : M_2 \to M$ in **Loc** with causally disjoint images the diagram

$$\begin{tikzcd} \mathfrak{A}(M_1) \otimes \mathfrak{A}(M_2) \arrow[r, "\mathfrak{A}(f_1) \otimes \mathfrak{A}(f_2)"] \arrow[d, "\mathfrak{A}(f_1) \otimes \mathfrak{A}(f_2)"'] & \mathfrak{A}(M) \otimes \mathfrak{A}(M) \arrow[d, "\mu^{\mathrm{op}}"] \\ \mathfrak{A}(M) \otimes \mathfrak{A}(M) \arrow[r, "\mu"] & \mathfrak{A}(M) \end{tikzcd}$$

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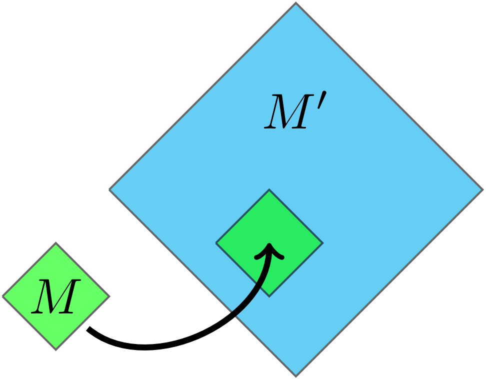
The time slice axiom can be hard-coded by defining $\mathfrak{A}$ on the localization $\mathbf{Loc}[C^{-1}]$.

Algebraic quantum field theories – a pictorial representation, part I

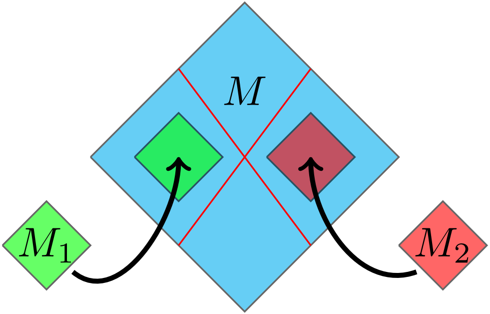
Functorial assignment of algebras to spacetime regions:



For any spacetime embedding $M \to M'$ we get an algebra morphism $\mathfrak{A}(M) \to \mathfrak{A}(M')$.

Algebraic quantum field theories – a pictorial representation, part II

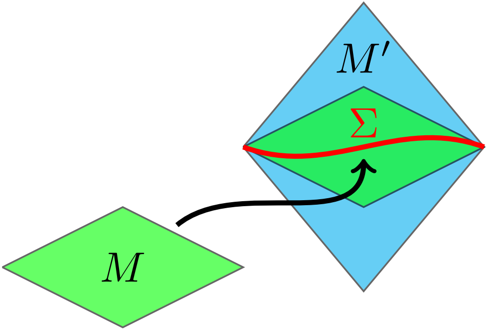
Algebras assigned to causally disjoint regions commute (in the algebra assigned to a bigger region):



We have $[\mathfrak{A}(M_1), \mathfrak{A}(M_2)] = 0$ in $\mathfrak{A}(M)$.

Algebraic quantum field theories – a pictorial representation, part III

Time slice axiom:



Σ is a Cauchy surface, i.e. intersected by every inextensible causal curve exactly once.
The embedding $M \to M'$ induces an isomorphism $\mathfrak{A}(M) \cong \mathfrak{A}(M')$.

Algebraic quantum field theories – a slight abstraction

Consider a category $\mathbf{C}$ equipped with an *orthogonality relation* $\perp$, i.e. a distinguished class of pairs of morphisms of $\mathbf{C}$ with common target:

$$(f_1 : c_1 \to t) \quad \perp \quad (f_2 : c_2 \to t).$$

Orthogonality is symmetric and stable under composition.
Denote by $\bar{\mathbf{C}} = (\mathbf{C}, \perp)$ the category $\mathbf{C}$ equipped with its orthogonality relation.

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A $\perp$*-commutative functor* $\mathfrak{A} : \bar{\mathbf{C}} \to \mathbf{Alg}$ is a functor $\mathbf{C} \to \mathbf{Alg}$ such that for orthogonal morphisms $f_1 : c_1 \to t \perp f_2 : c_2 \to t$ in $\mathbf{C}$ the diagram

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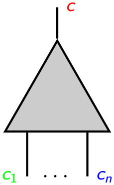
Other examples than algebraic quantum field theory

Euclidean field theories, chiral conformal field theory, ...

Colored operads

A (colored) operad encodes algebraic structures by giving objects (vector spaces, topological spaces, chain complexes) of operations with several inputs and one output.



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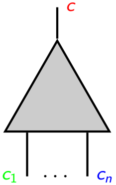

$$\in \mathbf{O}\big(\substack{c \\ (c_1,\dots,c_n)}\big)$$

The associative operad

The n-ary operations of the associative operad As are $\mathrm{As}(n) = \Sigma_n$, the permutation group on n letters.

Colored operads

An *algebra* A over a colored operad $\mathbf{O}$ is a concrete realization of the abstract operations in $\mathbf{O}$. It consists of colored objects A_c and morphisms

$$\alpha : \mathbf{O}\big(\substack{c \\ (c_1,\ldots,c_n)}\big) \otimes A_{c_1} \otimes \cdots \otimes A_{c_n} \longrightarrow A_c ,$$

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Can we build an operad such that its algebras are algebraic quantum field theories?

$\perp$-commutative functors as algebras over an operad

Theorem [Benini-Schenkel-W. '17]

Any orthogonal category $\bar{\mathbf{C}}$ gives rise to a $\mathbf{C}_0$-colored operad $\mathbf{O}_{\bar{\mathbf{C}}}$ such that the category $\mathbf{Alg}(\mathbf{O}_{\bar{\mathbf{C}}})$ of algebras over $\mathbf{O}_{\bar{\mathbf{C}}}$ is canonically isomorphic to the category $\mathbf{Fun}_\perp(\bar{\mathbf{C}}, \mathbf{Alg})$ of $\perp$-commutative functors $\bar{\mathbf{C}} \to \mathbf{Alg}$, i.e. to QFTs on $\bar{\mathbf{C}}$;

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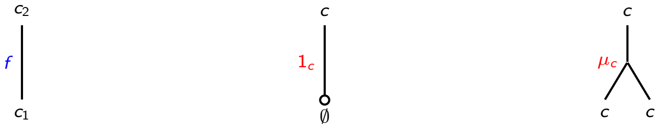
This can also be set up in a $*$-algebraic picture [Benini-Schenkel-W. '18].

The AQFT operad in terms of generators and relations

Generators:



Relations for functoriality:

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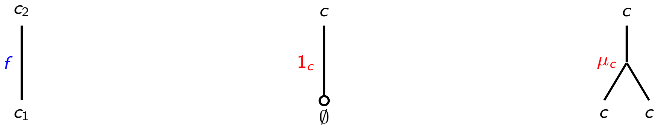
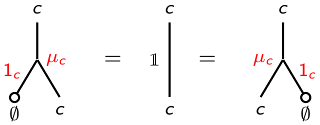
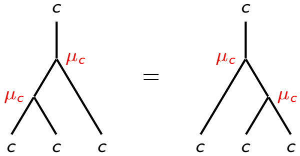


Relations for unital and associative multiplication:





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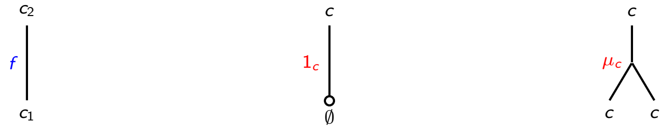
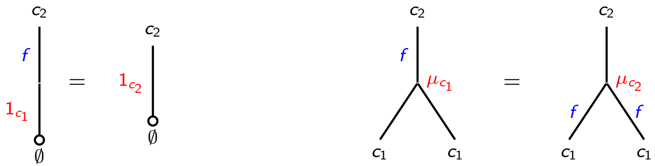


Relations for compatibility of functoriality and algebra structure:



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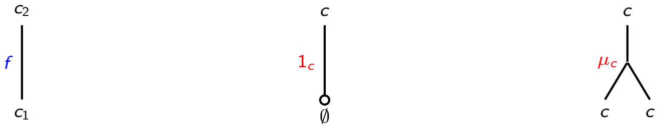
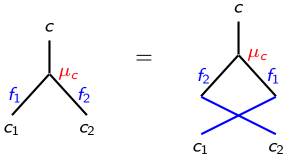


Relations for $\perp$-commutativity:



for $(f_1, f_2) \in \perp$

Change of color adjunctions

The operadic framework allows for an intrinsic treatment of $\perp$-commutativity (and hence of Einstein causality): Any orthogonal functor $F : \bar{\mathbf{C}} \to \bar{\mathbf{D}}$ gives rise to an adjoint pair of functors

$$\mathbf{Fun}_\perp(\bar{\mathbf{C}}, \mathbf{Alg}) = \mathbf{Alg}(\mathsf{O}_{\bar{\mathbf{C}}}) \underset{\mathsf{O}_F^*}{\overset{\mathsf{O}_{F\,!}}{\rightleftarrows}} \mathbf{Alg}(\mathsf{O}_{\bar{\mathbf{D}}}) \cong \mathbf{Fun}_\perp(\bar{\mathbf{D}}, \mathbf{Alg})$$

by the *operadic* left Kan extension.

An operadic version of Fredenhagen's universal algebra

Important special case related to Fredenhagen's universal algebra: Given an orthogonal embedding $J : \bar{\mathbf{C}} \to \bar{\mathbf{D}}$, we would like to extend $\mathfrak{A} \in \mathbf{Fun}_\perp(\bar{\mathbf{C}}, \mathbf{Alg}) = \mathbf{Alg}(\mathbf{O}_{\bar{\mathbf{C}}})$ to $\bar{\mathbf{D}}$. An extension of $\mathfrak{A}$, as a functor, is given by the left Kan extension

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Theorem [Benini-Schenkel-W. '17]

1. If $\mathrm{Lan}_J \mathfrak{A}$ is $\perp$-commutative again, then $\mathrm{Lan}_J \mathfrak{A} \cong \mathbf{O}_{J!}\mathfrak{A}$.
2. $\mathrm{Lan}_J \mathfrak{A}$ is $\perp$-commutative over $d \in \mathbf{D}$ if and only if for all orthogonal morphisms $f_1 : c_1 \to d$ and $f_2 : c_2 \to d$ in $\mathbf{D}$ with $c_1, c_2 \in \mathbf{C}$, there exists an object $c \in \mathbf{C}$ and a commutative diagram

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The assumption in 2. is violated for categories such as **Loc** for disconnected spacetimes. The general connected case is open.

Homotopical algebraic quantum field theory

Fact

In some cases, quantum gauge theory requires that our coefficient category **M** has a non-trivial notion of equivalence: **M** can be a model category, e.g. (co)chain complexes. Higher structures are needed. See e.g. Benini, Schenkel, Schreiber, Szabo.

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Reminder

A model category is a category equipped with three classes of morphisms: weak equivalences, fibrations and cofibrations. These are subject to various axioms. Model categories are an approach to the theory of ∞-categories.

Homotopical algebraic quantum field theory

The operadic framework allows for the needed homotopical relaxation of the axioms when our coefficients $\mathbf{M}$ are a sufficiently well-behaved model category like chain complexes or simplicial sets:

Definition [Benini-Schenkel '16, Benini-Schenkel-W. '17/work in progress]

A *homotopical (algebraic) quantum field theory* on an orthogonal category $\bar{\mathbf{C}}$ is an algebra over the operad $\mathbf{O}_{\bar{\mathbf{C}}}^{\infty}$, where $\mathbf{O}_{\bar{\mathbf{C}}}^{\infty} \xrightarrow{\simeq} \mathbf{O}_{\bar{\mathbf{C}}}$ is an admissible Σ-cofibrant resolution of $\mathbf{O}_{\bar{\mathbf{C}}}$.

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Example from quantum gauge theory [Benini-Schenkel-W., work in progress]

Given an ordinary AQFT $\mathfrak{A}$ defined on spacetimes together with a G-bundle, the homotopy orbifold of $\mathfrak{A}$ carries the structure of a homotopical AQFT that can be seen as cochain on bundle groupoids with AQFT coefficients. This implements the idea to see an AQFT as the coefficient system for a cohomology theory sensitive to causal structure.

Summary and outlook

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2. An important example of such a construction is the local-to-global construction. Hence, operads might allow to develop an notion of descent for AQFTs.
3. Operads are crucial to define AQFTs up to coherent homotopy.
4. Quantum gauge theory provides examples for homotopical AQFTs and suggests to see AQFTs as the coefficients of a (co)homology theory.