BRST symmetry of doubled membrane sigma-models

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Outline

Based on: [1802.07003] and [1904.04857] by Chatzistavrakidis, Jonke, Khoo and Szabo

- Courant sigma-model
  - Gauge symmetry
  - BV-BRST symmetry and master action
- DFT
  - Projection to DFT
  - Projection of BRST
  - Gauge symmetry from projected BRST

Courant sigma-model

Courant sigma-model

[hep-th/0608150] Roytenberg

- Action functional:

$$S_C[\mathbb{X},\mathbb{A},F]=\int_{\Sigma_3}\Big(F_I\wedge \mathrm{d}\mathbb{X}^I+\tfrac{1}{2}\hat\eta_{\hat I\hat J}\,\mathbb{A}^{\hat I}\wedge \mathrm{d}\mathbb{A}^{\hat J}-\rho^I{}_{\hat J}(\mathbb{X})\,\mathbb{A}^{\hat J}\wedge F_I+\tfrac{1}{6}T_{\hat I\hat J\hat K}(\mathbb{X})\,\mathbb{A}^{\hat I}\wedge\mathbb{A}^{\hat J}\wedge\mathbb{A}^{\hat K}\Big)$$

where indices run: $I=1,\dots,2d$ and $\hat I=1,\dots,4d$ and $\mathbb{E}=T\mathcal{M}\oplus T^*\mathcal{M}$ over a doubled space $\mathcal{M}$, with objects:

$$\mathbb{X}=(\mathbb{X}^I):\Sigma_3\to\mathcal{M},\qquad \mathbb{A}\in\Omega^1(\Sigma_3,\mathbb{X}^*\mathbb{E}),\qquad F\in\Omega^2(\Sigma_3,\mathbb{X}^*T^*\mathcal{M})$$

$$\hat\eta=\begin{pmatrix}0 & 1_{2d}\\ 1_{2d} & 0\end{pmatrix},\qquad \rho:\mathbb{E}\to T\mathcal{M}$$

Courant sigma-model

[hep-th/0203043] Ikeda

- Gauge transformations:

$$\delta_{(\epsilon,t)}\mathbb{X}^I = \rho^I_{\hat{J}}\,\epsilon^{\hat{J}}$$

$$\delta_{(\epsilon,t)}\mathbb{A}^{\hat{I}} = \mathrm{d}\epsilon^{\hat{I}} + \hat{\eta}^{\hat{I}\hat{N}}\,T_{\hat{N}\hat{J}\hat{K}}\,\mathbb{A}^{\hat{J}}\epsilon^{\hat{K}} - \hat{\eta}^{\hat{I}\hat{J}}\rho^I{}_{\hat{J}}\;t_I$$

$$\delta_{(\epsilon,t)}F_I = -\,\mathrm{d}t_I - \partial_I\rho^J{}_{\hat{J}}\,\mathbb{A}^{\hat{J}}\wedge t_J - \epsilon^{\hat{J}}\partial_I\rho^J{}_{\hat{J}}\,F_J + \tfrac{1}{2}\epsilon^{\hat{J}}\partial_I\,T_{\hat{I}\hat{L}\hat{J}}\,\mathbb{A}^{\hat{I}}\wedge\mathbb{A}^{\hat{L}}$$

- Equations of motion:

$$\mathcal{D}\mathbb{X}^I := \mathrm{d}\mathbb{X}^I - \rho^I{}_{\hat{J}}\,\mathbb{A}^{\hat{J}} = 0$$

$$\mathcal{D}\mathbb{A}^{\hat{I}} := \mathrm{d}\mathbb{A}^{\hat{I}} - \hat{\eta}^{\hat{I}\hat{K}}\rho^I{}_{\hat{K}}F_I + \tfrac{1}{2}\hat{\eta}^{\hat{I}\hat{K}}\,T_{\hat{K}\hat{J}\hat{L}}\mathbb{A}^{\hat{J}}\wedge\mathbb{A}^{\hat{L}} = 0$$

$$\mathcal{D}F_I := \mathrm{d}F_I + \partial_I\rho^J{}_{\hat{K}}\,\mathbb{A}^{\hat{K}}\wedge F_J - \tfrac{1}{6}\partial_I\,T_{\hat{J}\hat{K}\hat{L}}\,\mathbb{A}^{\hat{J}}\wedge\mathbb{A}^{\hat{K}}\wedge\mathbb{A}^{\hat{L}} = 0$$

Gauge covariance

- Gauge transformations of equations of motion

$$\begin{aligned}\delta_{(\epsilon,t)}\mathcal{D}\mathbb{X}^I &= \epsilon^{\hat{J}}\partial_M\rho^I{}_{\hat{J}}\mathcal{D}\mathbb{X}^M - \hat{\eta}^{\hat{J}\hat{K}}\rho^I{}_{\hat{J}}\rho^L{}_{\hat{K}}\,t_L + \epsilon^{\hat{J}}\mathbb{A}^{\hat{K}}(2\rho^M{}_{[\hat{K}}\partial_{\underline{M}}\rho^I{}_{\hat{J}]} - \rho^I{}_{\hat{N}}\hat{\eta}^{\hat{N}\hat{M}}T_{\hat{M}\hat{K}\hat{J}})\\
\delta_{(\epsilon,t)}\mathcal{D}\mathbb{A}^{\hat{I}} &= -\hat{\eta}^{\hat{I}\hat{N}}(\partial_M T_{\hat{N}\hat{J}\hat{K}}\epsilon^{\hat{K}}\mathbb{A}^{\hat{J}} - \partial_M\rho^I{}_{\hat{N}}\,t_I)\wedge\mathcal{D}\mathbb{X}^M + \hat{\eta}^{\hat{I}\hat{N}}T_{\hat{N}\hat{J}\hat{K}}\epsilon^{\hat{K}}\mathcal{D}\mathbb{A}^{\hat{J}} +\\
&\quad + \tfrac{1}{2}\hat{\eta}^{\hat{I}\hat{K}}(3\rho^I{}_{[\hat{N}}\partial_I T_{\hat{J}\hat{L}]\hat{K}} - \rho^I{}_{\hat{K}}\partial_I T_{\hat{N}\hat{J}\hat{L}} - 3T_{\hat{K}\hat{R}[\hat{N}}\hat{\eta}^{\hat{R}\hat{P}}T_{\hat{J}\hat{L}]\hat{P}})\epsilon^{\hat{N}}\mathbb{A}^{\hat{J}}\wedge\mathbb{A}^{\hat{L}}\end{aligned}$$

- The requirement that the equations of motion be gauge covariant leads to the following conditions:

$$\begin{aligned}\hat{\eta}^{\hat{J}\hat{K}}\rho^I{}_{\hat{J}}\rho^L{}_{\hat{K}} &= 0\\
2\rho^M{}_{[\hat{K}}\partial_{\underline{M}}\rho^I{}_{\hat{J}]} - \rho^I{}_{\hat{N}}\hat{\eta}^{\hat{N}\hat{M}}T_{\hat{M}\hat{K}\hat{J}} &= 0\\
3\rho^I{}_{[\hat{N}}\partial_I T_{\hat{J}\hat{L}]\hat{K}} - \rho^I{}_{\hat{K}}\partial_I T_{\hat{N}\hat{J}\hat{L}} - 3T_{\hat{K}\hat{R}[\hat{N}}\hat{\eta}^{\hat{R}\hat{P}}T_{\hat{J}\hat{L}]\hat{P}} &= 0\end{aligned}$$

Closure of the gauge algebra

Closure of the gauge algebra on fields $\mathbb{X}$ and $\mathbb{A}$:

$$[\delta_{(\epsilon_1,t_1)},\delta_{(\epsilon_2,t_2)}]\mathbb{X}^I = \rho^I{}_{\hat{J}}\,\epsilon_{12}^{\hat{J}}$$
$$\epsilon_{12}^{\hat{I}} := \hat{\eta}^{\hat{I}\hat{J}}\,T_{\hat{J}\hat{K}\hat{L}}\epsilon_1^{\hat{K}}\epsilon_2^{\hat{L}}$$
$$[\delta_{(\epsilon_1,t_1)},\delta_{(\epsilon_2,t_2)}]\mathbb{A}^{\hat{I}} = \delta_{(\epsilon_{12},t_{12})}\mathbb{A}^{\hat{I}} - \hat{\eta}^{\hat{I}\hat{J}}\partial_M T_{\hat{J}\hat{K}\hat{L}}\epsilon_1^{\hat{K}}\epsilon_2^{\hat{L}}\mathcal{D}\mathbb{X}^M$$
$$t_{12I} := \partial_I T_{\hat{K}\hat{L}\hat{J}}\,\epsilon_1^{\hat{K}}\epsilon_2^{\hat{L}}\mathbb{A}^{\hat{J}} + 2\partial_I\rho^J{}_{\hat{K}}\,\epsilon_{[1}^{\hat{K}}\;t_{2]J}$$

Gauge algebra is closed only up to equations of motion.

Superfields

Introducing superfields:

$$\begin{aligned}
\mathbf{X}^I &= \mathbb{X}^I + F^{\dagger I} + t^{\dagger I} + v^{\dagger I} \\
\mathbf{A}^{\hat{I}} &= \epsilon^{\hat{I}} + \mathbb{A}^{\hat{I}} + \hat{\eta}^{\hat{I}\hat{J}}\mathbb{A}^{\dagger}_{\hat{J}} + \hat{\eta}^{\hat{I}\hat{J}}\epsilon^{\dagger}_{\hat{J}} \\
\mathbf{F}_I &= v_I + t_I + F_I + \mathbb{X}^{\dagger}_I
\end{aligned}$$

$\mathbf{X}^I$:	$\mathrm{gh}\,\mathbb{X}^I = 0,$	$\mathrm{gh}\,F^{\dagger I} = -1,$	$\mathrm{gh}\,t^{\dagger I} = -2,$	$\mathrm{gh}\,v^{\dagger I} = -3$
	$\deg\mathbb{X}^I = 0;$	$\deg F^{\dagger I} = 1;$	$\deg t^{\dagger I} = 2;$	$\deg v^{\dagger I} = 3;$
$\mathbf{A}^{\hat{I}}$:	$\mathrm{gh}\,\epsilon^{\hat{I}} = 1,$	$\mathrm{gh}\,\mathbb{A}^{\hat{I}} = 0,$	$\mathrm{gh}\,\mathbb{A}^{\dagger}_{\hat{I}} = -1,$	$\mathrm{gh}\,\epsilon^{\dagger}_{\hat{I}} = -2$
	$\deg\epsilon^{\hat{I}} = 0;$	$\deg\mathbb{A}^{\hat{I}} = 1;$	$\deg\mathbb{A}^{\dagger}_{\hat{I}} = 2;$	$\deg\epsilon^{\dagger}_{\hat{I}} = 3$
$\mathbf{F}_I$:	$\mathrm{gh}\,v_I = 2,$	$\mathrm{gh}\,t_I = 1,$	$\mathrm{gh}\,F_I = 0,$	$\mathrm{gh}\,\mathbb{X}^{\dagger}_I = -1$
	$\deg v_I = 0;$	$\deg t_I = 1;$	$\deg F_I = 2;$	$\deg\mathbb{X}^{\dagger}_I = 3$

Master action

[hep-th/9502010] Alexandrov, Kontshevich, Schwarz, Zaboronsky

Define the graded symplectic structure:

$$\omega = \mathrm{d}\mathbf{X}^I\, \mathrm{d}\mathbf{F}_I + \tfrac{1}{2}\hat{\eta}_{\hat{I}\hat{J}}\, \mathrm{d}\mathbf{A}^{\hat{I}}\, \mathrm{d}\mathbf{A}^{\hat{J}}$$

By the AKSZ procedure we can construct the BV master action:

$$\mathbf{S}_C[\mathbf{X},\mathbf{A},\mathbf{F}] = \int_{T[1]\Sigma_3} \mu \left(\mathbf{F}_I\, \mathrm{d}\mathbf{X}^I + \tfrac{1}{2}\hat{\eta}_{\hat{I}\hat{J}}\, \mathbf{A}^{\hat{I}}\, \mathrm{d}\mathbf{A}^{\hat{J}} - \rho^I{}_{\hat{J}}(\mathbf{X})\mathbf{A}^{\hat{I}}\, \mathbf{F}_J + \tfrac{1}{6} T_{\hat{I}\hat{J}\hat{K}}(\mathbf{X})\mathbf{A}^{\hat{I}}\mathbf{A}^{\hat{J}}\mathbf{A}^{\hat{K}} \right)$$

BRST transformations

The symplectic structure induces a graded Poisson bracket (the BV antibracket) that can be used to define the BRST transformations of the component fields:

$$\delta \mathbb{X}^I = \rho^I_{\hat{I}} \epsilon^{\hat{I}}$$

$$\delta \mathbb{A}^{\hat{I}} = \mathrm{d}\epsilon^{\hat{I}} - \hat{\eta}^{\hat{I}\hat{J}} \rho^I_{\hat{J}} t_I + \hat{\eta}^{\hat{I}\hat{J}} T_{\hat{J}\hat{K}\hat{L}} \mathbb{A}^{\hat{K}} \epsilon^{\hat{L}} - \hat{\eta}^{\hat{I}\hat{J}} \partial_J \rho^I_{\hat{J}} F^{\dagger J} v_I + \tfrac{1}{2} \hat{\eta}^{\hat{I}\hat{J}} \partial_J T_{\hat{J}\hat{K}\hat{L}} F^{\dagger J} \epsilon^{\hat{K}} \epsilon^{\hat{L}}$$

$$\begin{aligned}
\delta F_I = & -\mathrm{d}t_I - \partial_I \rho^J_{\hat{I}} \epsilon^{\hat{I}} F_J - \partial_I \rho^J_{\hat{I}} \mathbb{A}^{\hat{I}} t_J + \tfrac{1}{2} \partial_I T_{\hat{I}\hat{J}\hat{K}} \epsilon^{\hat{I}} \mathbb{A}^{\hat{J}} \mathbb{A}^{\hat{K}} + \\
& + \tfrac{1}{2} \partial_I T_{\hat{I}\hat{J}\hat{K}} \hat{\eta}^{\hat{K}\hat{L}} \epsilon^{\hat{I}} \epsilon^{\hat{J}} \mathbb{A}^\dagger_{\hat{L}} - \partial_I \rho^J_{\hat{I}} \hat{\eta}^{\hat{I}\hat{J}} \mathbb{A}^\dagger_{\hat{J}} v_J + \tfrac{1}{2} \partial_I \partial_J \partial_K \rho^L_{\hat{I}} F^{\dagger J} F^{\dagger K} \epsilon^{\hat{I}} v_L - \partial_I \partial_J \rho^K_{\hat{I}} t^{\dagger J} \epsilon^{\hat{I}} v_K - \\
& - \partial_I \partial_J \rho^K_{\hat{I}} F^{\dagger J} \epsilon^{\hat{I}} t_K + \partial_I \partial_J \rho^K_{\hat{I}} F^{\dagger J} \mathbb{A}^{\hat{I}} v_K - \tfrac{1}{12} \partial_I \partial_J \partial_K T_{\hat{I}\hat{J}\hat{K}} F^{\dagger J} F^{\dagger K} \epsilon^{\hat{I}} \epsilon^{\hat{J}} \epsilon^{\hat{K}} + \\
& + \tfrac{1}{6} \partial_I \partial_J T_{\hat{I}\hat{J}\hat{K}} t^{\dagger J} \epsilon^{\hat{I}} \epsilon^{\hat{J}} \epsilon^{\hat{K}} - \tfrac{1}{2} \partial_I \partial_J T_{\hat{I}\hat{J}\hat{K}} F^{\dagger J} \mathbb{A}^{\hat{I}} \epsilon^{\hat{J}} \epsilon^{\hat{K}}
\end{aligned}$$

$$\delta \epsilon^{\hat{I}} = \hat{\eta}^{\hat{I}\hat{J}} \rho^I{}_{\hat{J}} v_I - \tfrac{1}{2} \hat{\eta}^{\hat{I}\hat{J}} T_{\hat{J}\hat{K}\hat{L}} \epsilon^{\hat{K}} \epsilon^{\hat{L}}$$

$$\begin{aligned}
\delta t_I = & \, \mathrm{d}v_I - \partial_I \rho^J{}_{\hat{I}} \epsilon^{\hat{I}} t_J + \partial_I \rho^J{}_{\hat{I}} \mathbb{A}^{\hat{I}} v_J - \tfrac{1}{2} \partial_I T_{\hat{I}\hat{J}\hat{K}} \epsilon^{\hat{I}} \epsilon^{\hat{J}} \mathbb{A}^{\hat{K}} + \partial_I \partial_J \rho^K{}_{\hat{I}} F^{\dagger J} \epsilon^{\hat{I}} v_K \\
& - \tfrac{1}{6} \partial_I \partial_J T_{\hat{I}\hat{J}\hat{K}} F^{\dagger J} \epsilon^{\hat{I}} \epsilon^{\hat{J}} \epsilon^{\hat{K}}
\end{aligned}$$

$$\delta v_I = -\partial_I \rho^J{}_{\hat{I}} \epsilon^{\hat{I}} v_J + \tfrac{1}{6} \partial_I T_{\hat{I}\hat{J}\hat{K}} \epsilon^{\hat{I}} \epsilon^{\hat{J}} \epsilon^{\hat{K}}$$

DFT

DFT Projection

As has been shown one can obtain the DFT worldvolume theory by means of projection from a large Courant sigma-model. A splitting of the bundle is defined: $\mathbb{E} = \mathbb{T}\mathcal{M} = L_+ \oplus L_-$:

$$\mathbb{A} = \mathbb{A}_+^I e_I^+ + \mathbb{A}_-^I e_I^- \qquad \mathbb{A}_\pm^I = \tfrac{1}{2}(\mathbb{A}^I \pm \eta^{IJ}\widetilde{\mathbb{A}}_J)$$

$$e_I^\pm = \partial_I \pm \eta_{IJ}\, \mathrm{d}\mathbb{X}^J \qquad (\rho_\pm)^I{}_J = \rho^I{}_J \pm \eta_{JK}\widetilde{\rho}^{IK}$$

A projection is performed onto the L_+ subbundle:

$$\mathsf{p}_+ : \mathbb{E} \to L_+ \qquad \mathbb{A} \mapsto \mathbb{A}_+ = A$$

The projection obtains the C-bracket of DFT from the Courant bracket

$$[\![A, B]\!] = \mathsf{p}_+\left([\mathsf{p}_+(\mathbb{A}), \mathsf{p}_+(\mathbb{B})]_{\mathbb{E}}\right)$$

and gives the DFT membrane sigma model

$$S_{\mathrm{DFT}}[\mathbb{X}, A, F] = \int_{\Sigma_3} \left(F_I \wedge \mathrm{d}\mathbb{X}^I + \eta_{IJ} A^I \wedge \mathrm{d}A^J - (\rho_+)^I{}_J A^J \wedge F_I + \tfrac{1}{3}\hat{T}_{IJK} A^I \wedge A^J \wedge A^K\right)$$

with the choice of anchor and flux such that

$$\begin{aligned} \eta^{JK}(\rho_+)^I{}_J(\rho_+)^L{}_K &= \eta^{IL} \\ 2\rho^K{}_{[L}\partial_{\underline{K}}\rho^I{}_{M]} - \rho_{K[L}\partial^I\rho^K{}_{M]} &= \rho^I{}_J\eta^{JK}\hat{T}_{KLM} \end{aligned}$$

Projection of BRST transformations

The goal is to project on the BV-BRST level to obtain a consistent BRST description of DFT.

Projected master action:

$$\mathbf{S}_{\text{DFT}}[\mathbf{X},\mathbf{A},\mathbf{F}] = \int_{T[1]\Sigma_3} \mu \left(\mathbf{F}_I \, \mathrm{d}\mathbf{X}^I + \eta_{IJ} \mathbf{A}^I_+ \, \mathrm{d}\mathbf{A}^J_+ - (\rho_+)^J{}_I(\mathbf{X}) \mathbf{A}^I_+ \mathbf{F}_J + \tfrac{1}{3} \hat{T}_{IJK}(\mathbf{X}) \mathbf{A}^I_+ \mathbf{A}^J_+ \mathbf{A}^K_+ \right)$$

Projected BRST transformations:

$$\begin{aligned}
\delta \mathbb{A}^I_+ &= \mathrm{d}\epsilon^I_+ - \tfrac{1}{2}\eta^{IJ}\rho^K_{+J}\, t_K + \eta^{IL}\hat{T}_{LJK}\mathbb{A}^J_+\epsilon^K_+ - \tfrac{1}{2}\eta^{IJ}\partial_K\rho^L_{+J}F^{\dagger K}v_L + \tfrac{1}{2}\eta^{IJ}\partial_K\hat{T}_{JLM}F^{\dagger K}\epsilon^L_+\epsilon^M_+ \\
\delta \mathbb{A}^I_- &= \tfrac{1}{2}\eta^{IJ}\rho^K_{-J}\, t_K + \tfrac{1}{2}\eta^{IL}\theta_{JKL}\,\mathbb{A}^J_+\epsilon^K_+ + \tfrac{1}{2}\eta^{IJ}\partial_K\rho^L_{-J}F^{\dagger K}v_L + \tfrac{1}{4}\eta^{IJ}\partial_K\theta_{LMJ}F^{\dagger K}\epsilon^L_+\epsilon^M_+
\end{aligned}$$

$$\begin{aligned}
\delta \epsilon^I_+ &= \tfrac{1}{2}\eta^{IJ}\rho^K_{+J}v_K - \tfrac{1}{2}\eta^{IL}\hat{T}_{LJK}\epsilon^J_+\epsilon^K_+ \\
\delta \epsilon^I_- &= -\tfrac{1}{2}\eta^{IJ}\rho^K_{-J}v_K - \tfrac{1}{4}\eta^{IL}\theta_{JKL}\epsilon^J_+\epsilon^K_+
\end{aligned}$$

Since our projection equates to setting $\mathbb{A}_-$ and ϵ_- fields to zero they must remain vanishing after a BRST transformation.

Symmetry fixing

The requirement that we have a BRST compatible projection necessarily implies a reduction of our BRST symmetry by the fixing of ghost fields t and v:

$$\delta\epsilon_- = 0 \quad \Rightarrow \quad v_I = -\tfrac{1}{2}\eta_{IL}\eta^{NM}\rho^L_{-M}\theta_{JKN}\epsilon^J_+\epsilon^K_+ =: \tfrac{1}{2}\Theta_{IJK}(\mathbb{X})\epsilon^J_+\epsilon^K_+$$

$$\delta\mathbb{A}_- = 0 \quad \Rightarrow \quad t_I = \Theta_{IJK}(\mathbb{X})\mathbb{A}^J_+\epsilon^K_+ + \tfrac{1}{2}\partial_K\Theta_{ILM}F^{\dagger K}\epsilon^L_+\epsilon^M_+$$

However one must now make sure this fixing is BRST compatible in the sense that the BRST transformation coming from the fixing matches with the projected BRST transformation.

$$\begin{aligned}\delta t_I &= \mathrm{d}v_I - \epsilon^J_+\partial_I\rho^K_{+J}t_K + \mathbb{A}^J_+\partial_I\rho^K_{+J}v_K - \partial_I\hat{T}_{JKL}\epsilon^J_+\epsilon^K_+\mathbb{A}^L_+ - \partial_I\partial_J\rho^K_{+L}\epsilon^L_+F^{\dagger J}v_K - \\ &\quad - \tfrac{1}{3}\partial_I\partial_J\hat{T}_{KLM}F^{\dagger J}\epsilon^K_+\epsilon^L_+\epsilon^M_+ \\ \delta t_I &= \partial_L\Theta_{IJK}\delta\mathbb{X}^L\mathbb{A}^J_+\epsilon^K_+ + \Theta_{IJK}\delta\mathbb{A}^J_+\epsilon^K_+ + \Theta_{IJK}\mathbb{A}^J_+\delta\epsilon^K_+ + \tfrac{1}{2}\partial_A\partial_K\Theta_{ILM}\delta\mathbb{X}^AF^{\dagger K}\epsilon^L_+\epsilon^M_+ + \\ &\quad + \tfrac{1}{2}\partial_K\Theta_{ILM}\delta F^{\dagger K}\epsilon^L_+\epsilon^M_+ - \partial_K\Theta_{ILM}F^{\dagger K}\delta\epsilon^L_+\epsilon^M_+\end{aligned}$$

The same can be done for v.

Consistency condition

Define two objects:

$$S_{IJKL} := \partial_M \Theta_{I[JK}\rho^M_{+L]} - \Theta_{IM[J}\eta^{MN}\hat{T}_{\underline{N}KL]} + \tfrac{1}{2}\eta^{MN}\rho^P_{+N}\Theta_{IM[J}\Theta_{\underline{P}KL]} - \tfrac{2}{3}\partial_I \hat{T}_{JKL} + \partial_I \rho^M_{+[J}\Theta_{\underline{M}KL]}$$

$$R^I{}_{JK} := \eta^{AB}\rho^I_{+B}\Theta_{JAK} + 2\partial_J \rho^I_{+K}$$

The matching of the BRST transformations of fixed ghost fields t and v reduces to two conditions:

$$\begin{aligned} 3S_{IJKL}\mathbb{A}^J_+\epsilon^K_+\epsilon^L_+ + \partial_A S_{IBCD}F^{\dagger A}\epsilon^B_+\epsilon^C_+\epsilon^D_+ - \tfrac{1}{2}R^K{}_{IB}\partial_A\Theta_{KCD}F^{\dagger A}\epsilon^B_+\epsilon^C_+\epsilon^D_+ &= 0 \\ S_{IJKL}\epsilon^J_+\epsilon^K_+\epsilon^L_+ &= 0 \end{aligned}$$

However,

$$R^I{}_{JK} = 0 \qquad \Rightarrow \qquad S_{IJKL} = 0$$

in the case of ρ_+ and $\hat{T}$ being the DFT anchor and flux respectively.

Gauge transformations by projection

Going back to gauge symmetry one can obtain the gauge variations from the projected BRST transformations by setting antifields to zero:

$$\delta_\epsilon \mathbb{X}^I = \rho^I{}_J \epsilon^J$$

$$\delta_\epsilon A^I = \mathrm{d}\epsilon^I + \Phi^I{}_{JK} A^J \epsilon^K \qquad \Phi^I{}_{JK} := \eta^{IN}\big(\hat{T}_{NJK} - \tfrac{1}{2}\rho^M{}_N \Theta_{MJK}\big)$$

$$\delta_\epsilon F_I = -\,\mathrm{d}(\Theta_{IJK} A^J \epsilon^K) - \epsilon^J \partial_I \rho^K{}_J F_K + \epsilon^J A^K \wedge A^L (\partial_I \hat{T}_{KLJ} - \partial_I \rho^N{}_K \Theta_{NLJ})$$

The equations of motion coming from the projected worldvolume action:

$$\mathcal{D}\mathbb{X}^I := \mathrm{d}\mathbb{X}^I - \rho^I{}_J A^J = 0$$

$$\mathcal{D}A^I := \mathrm{d}A^I - \tfrac{1}{2}\eta^{IK}\rho^J{}_K F_J + \eta^{IK} \hat{T}_{KJL} A^J \wedge A^L = 0$$

$$\mathcal{D}F_I := \mathrm{d}F_I + \partial_I \rho^J{}_K A^K \wedge F_J - \tfrac{1}{3}\partial_I \hat{T}_{JKL} A^J \wedge A^K \wedge A^L = 0$$

Gauge covariance of equations of motion

And once again, now for the DFT case, gauge covariance of the equations of motion:

$$\delta_\epsilon \mathcal{D}\mathbb{X}^I = \epsilon^J \partial_M \rho^I{}_J \mathcal{D}\mathbb{X}^M + \epsilon^J A^K \overbrace{(2\rho^M{}_{[K}\partial_{\underline{M}}\rho^I{}_{J]} - \rho^I{}_N \Phi^N{}_{KJ})}^{\equiv R^I{}_{JK}=0}$$

$$\delta_\epsilon \mathcal{D}A^I = \eta^{IN}(\partial_M \hat{T}_{NJK} - \tfrac{1}{2}\partial_M \rho^L{}_N \Theta_{LJK})\epsilon^K \mathcal{D}\mathbb{X}^M \wedge A^J + \eta^{IN}\hat{T}_{NJK}\epsilon^K \mathcal{D}A^J +$$
$$+ \tfrac{1}{2}\eta^{IN}\epsilon^K A^M \wedge A^{M'} \underbrace{\left(3\rho^J{}_{[K}\partial_{\underline{J}}\hat{T}_{MM']N} - \rho^J{}_N \partial_J \hat{T}_{[KMM']} - 3\eta^{PJ}\hat{T}_{P[MM'}\hat{T}_{K]NJ}\right)}_{=0} +$$
$$+ \tfrac{1}{4}\eta^{IN}\rho^P{}_J \eta^{JL}\rho^S{}_L \epsilon^K \underbrace{\left(\Theta_{PKN}F_S + \Theta_{SMN}\Theta_{PM'K}A^M \wedge A^{M'}\right)}_{\text{s.c.}}$$

$$\eta^{PS}\Theta_{PKN}F_S = -2\rho_{M[K}\partial^S \rho^M{}_{N]}F_S$$

$$\Theta_{SMN}\eta^{PS}\Theta_{PM'K} = 4\eta_{SS'}\rho_{L[M}\partial^{S'}\rho^L{}_{N]}\rho_{J[M'}\partial^S \rho^J{}_{K]}$$

Closure of gauge algebra

Finally, taking a look at gauge closure:

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\mathbb{X}^I = \rho^I{}_J \epsilon_{12}^J$$

$$\epsilon_{12}^I := \Phi^I{}_{KL}\epsilon_1^K \epsilon_2^L$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]A^I = \delta_{\epsilon_{12}} A^I - \partial_L \Phi^I{}_{JK} \epsilon_1^J \epsilon_2^K \mathcal{D}\mathbb{X}^L + \\ + 3 \underbrace{\left(\Phi^I{}_{N[M} \Phi^N{}_{JK]} - \rho^N{}_{[M} \partial_{\underline{N}} \Phi^I{}_{JK]} \right)}_{=0} \epsilon_1^J \epsilon_2^K A^M$$

Gauge algebra is closed on-shell (as expected since it is an open algebra).

Conclusion

It has been shown:

- projection of a Courant sigma model over doubled space consistent on the BRST level
- algebra of gauge transformations closes on-shell
- equations of motion transform covariantly only up to the strong constraint

Open questions:

- Does there exist a modification of the DFT worldvolume action such that gauge invariance can be achieved without the use of the strong constraint?
- Is there a consistent way to define BRST transformations on the level of superfields?