

Quantum reference systems and quantum general covariance

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Bayrischzell
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based on: Vanrietvelde, PH, Giacomini, Castro-Ruiz, arXiv:1809.00556
Vanrietvelde, PH, Giacomini, arXiv:1809.05093
PH, Vanrietvelde, arXiv:1810.04153
PH, arXiv:1811.00611

General covariance

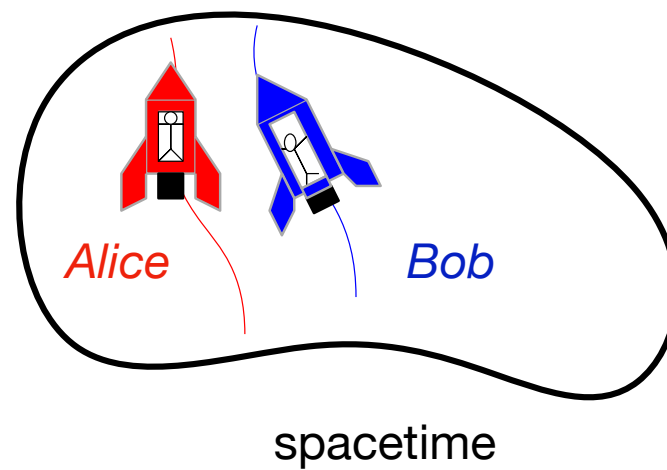
“All the laws of physics are the same in all reference frames.”

General covariance

“All the laws of physics are the same in all reference frames.”

frames usually

- idealized
- external



General covariance

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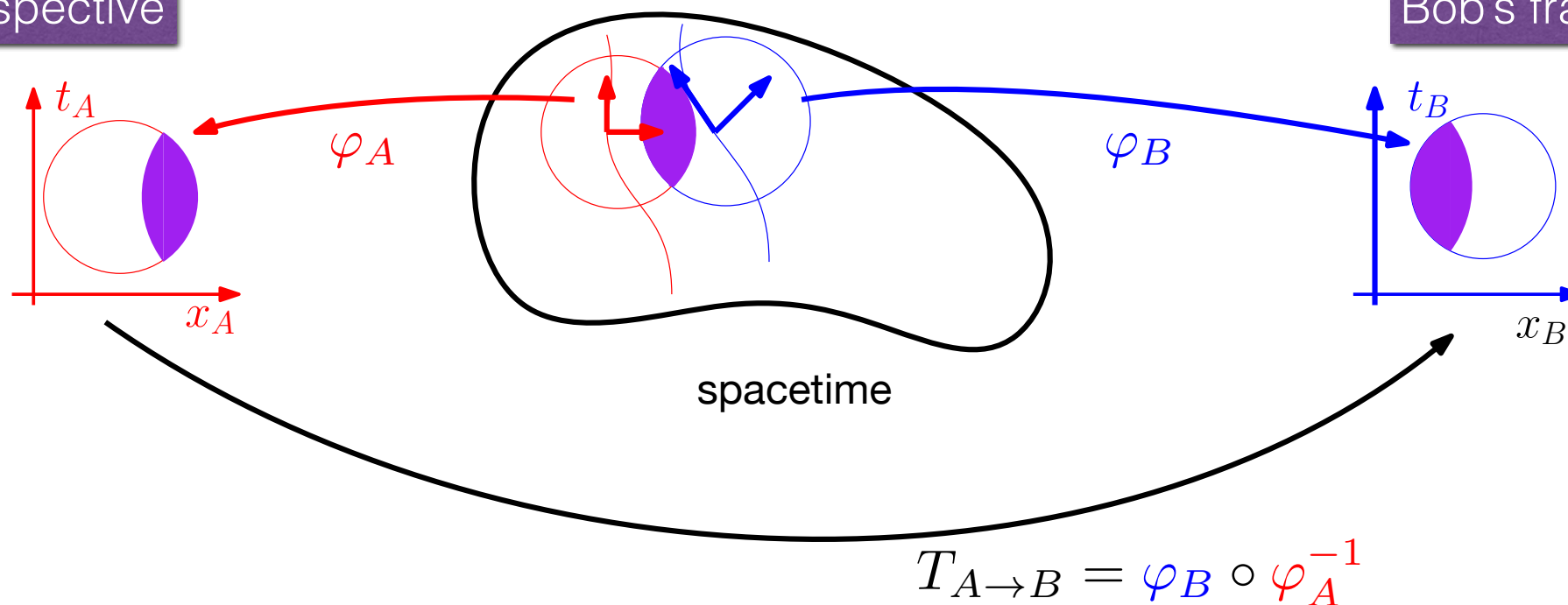
frames usually

- idealized
- external

frame-neutral

Alice's frame perspective

Bob's frame perspective

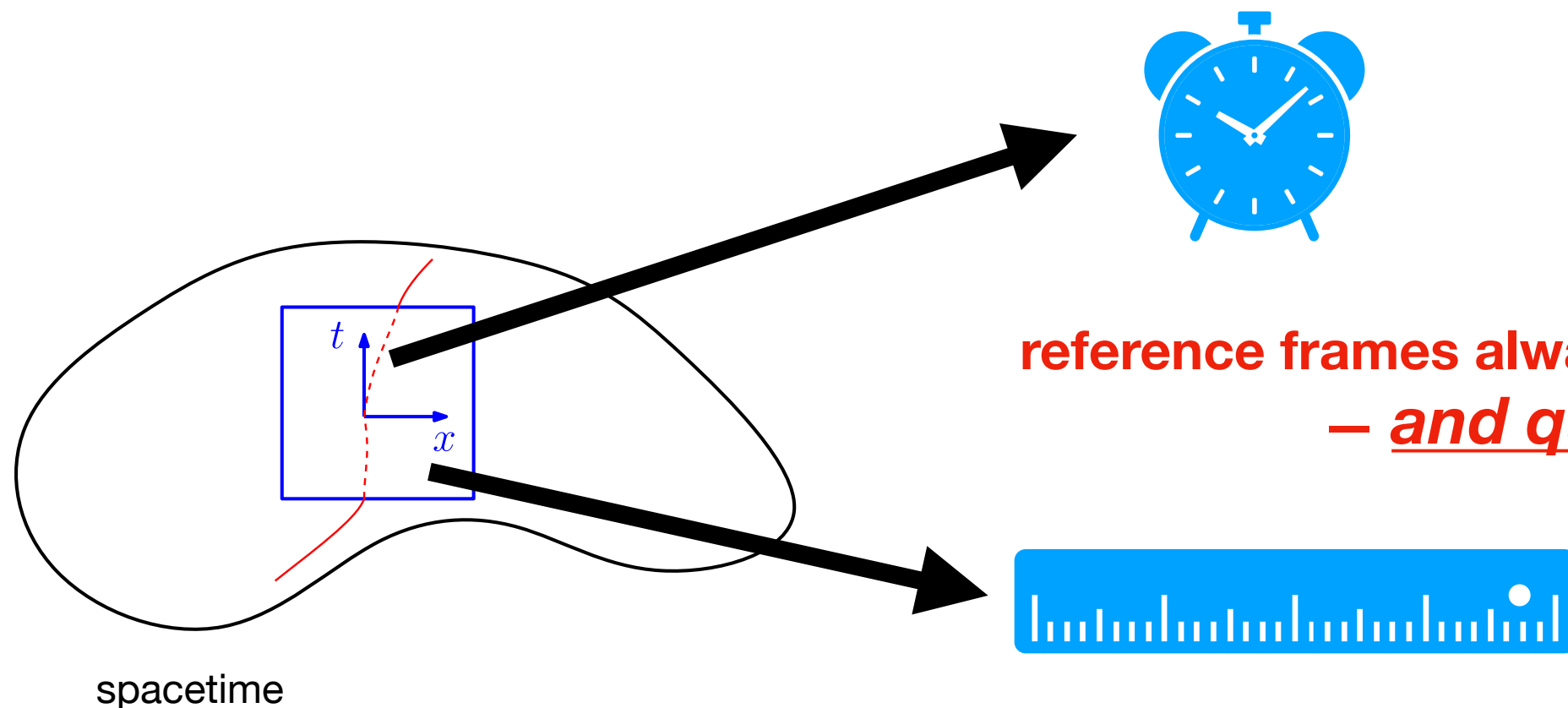


Towards quantum general covariance

Bojowald, PH, Tsobanjan, CQG 28, 035006 (2011)
Bojowald, PH, Tsobanjan, PRD 83, 125023 (2011)
PH, Kubalova, Tsobanjan, PRD 86, 065014 (2012)
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PH, arXiv:1811.00611

“All the laws of physics are the same in all reference frames.”



**reference frames always physical systems
– and quantum**

Towards quantum general covariance

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“All the laws of physics are the same in all reference frames.”

How to make sense of general covariance when frames are quantum?

AIM:
unifying framework for switches of
temporal and spatial
quantum reference systems

how?

symmetry principle



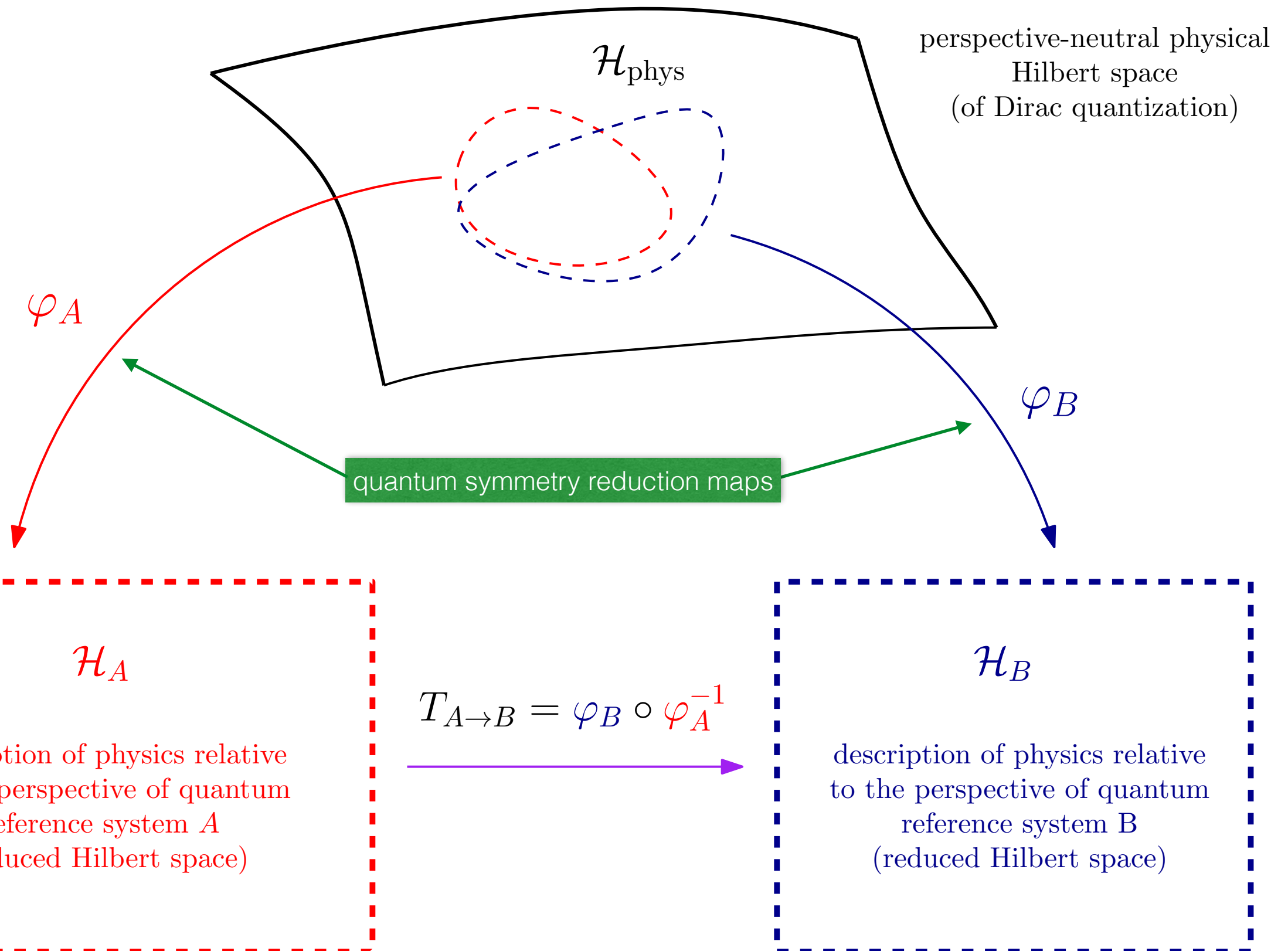
redundancy (constraints)



perspective-neutral framework

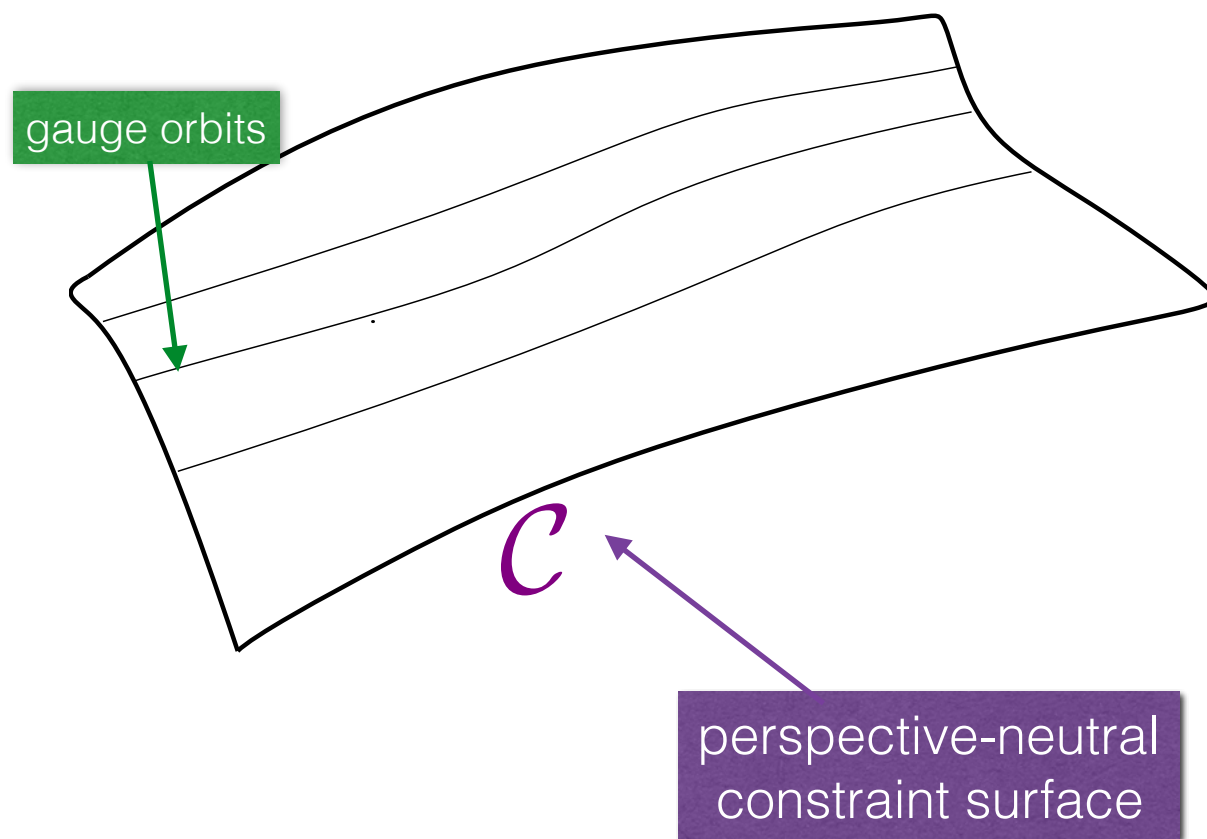
Vanrietvelde, PH, Giacomini, Castro Ruiz, arXiv:1809.00556
Vanrietvelde, PH, Giacomini, arXiv:1809.05093
PH, Vanrietvelde, arxiv:1810.04153
PH, arXiv:1811.00611

Result will be:



Symmetry & perspective-neutral structure

gauge symmetry $\xRightarrow{\text{canon. form.}}$ constraints



constraint surface encodes all frame choices at once

Toy model: relative motion in 1D

$$L = \frac{1}{2} (\dot{q}_A^2 + \dot{q}_E^2 + \dot{q}_F^2) - \frac{1}{6} \overbrace{(\dot{q}_A + \dot{q}_E + \dot{q}_F)^2}^{E_{\text{kin}}^{\text{cm}}} - V(\{q_a - q_b\})$$

L invar. under translations $q_a, \dot{q}_a \mapsto q_a + f(t), \dot{q}_a + f'(t)$

Legendre tr.

$$H = \frac{1}{2} (p_A^2 + p_E^2 + p_F^2) + V(\{q_a - q_b\}) + \lambda P$$

arbitrary

Localization in Newtonian space unphysical, only relational motion

Constraint $P = p_A + p_E + p_F \approx 0$ translation generator



q_A



q_E



q_F

q

Redundancy

gauge symmetry: $q_a(t)$ not physical

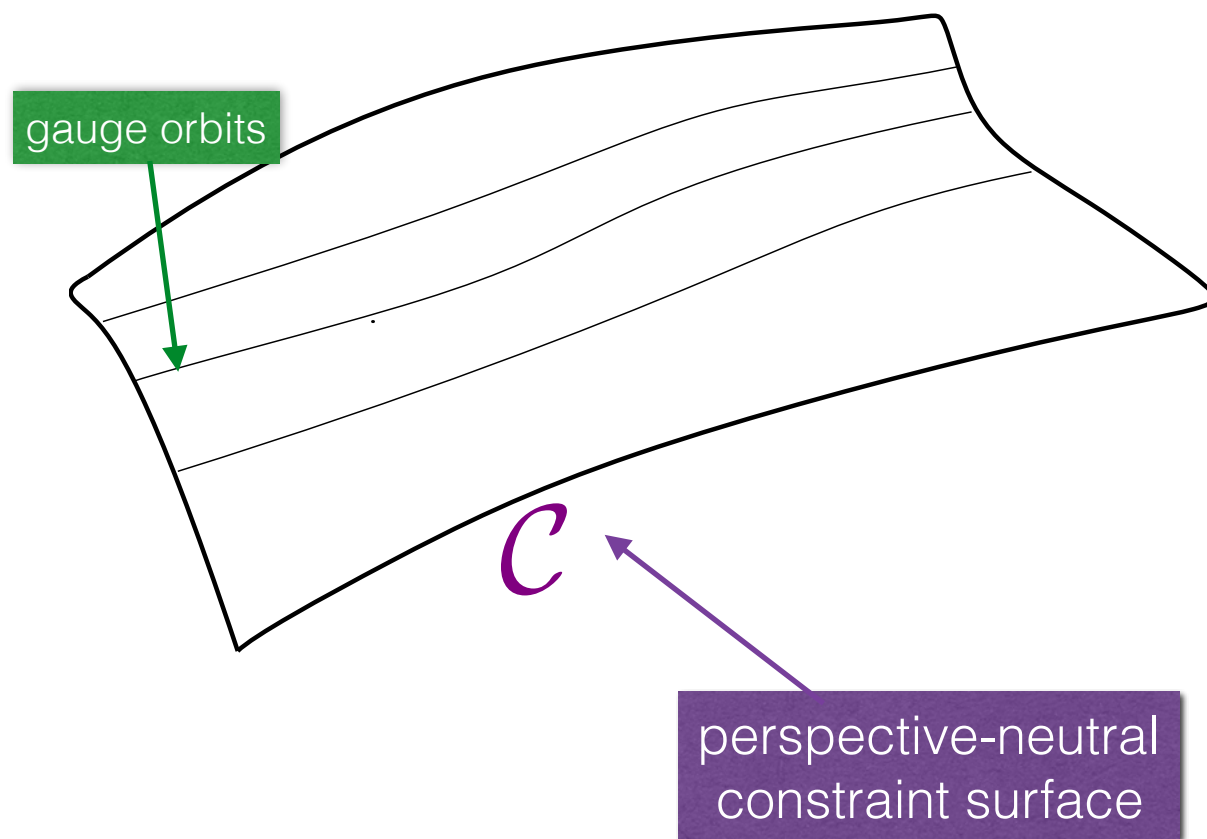
gauge inv. observables: $(q_E - q_A), (q_F - q_A), (q_E - q_F), p_A, p_E, p_F$

$\Rightarrow$ commute with $P = p_A + p_E + p_F$

perspective-neutral description

$\Rightarrow$ but redundant (only 4 indep. gauge inv. obs)

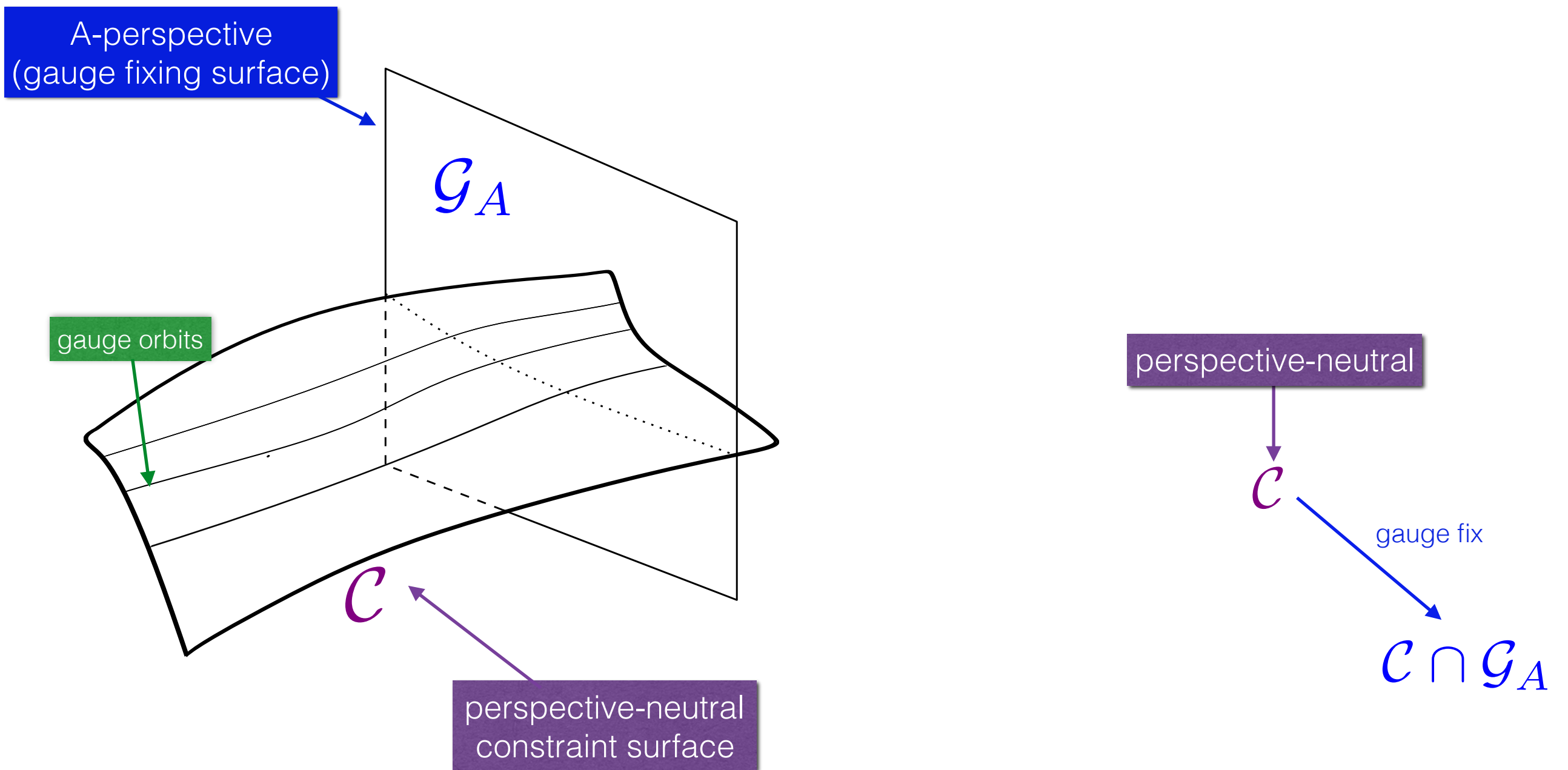
From perspective-neutral to perspectival



Classical reduction:

1. Choose reference system (e.g. A)

Frame choice as gauge



Classical reduction:

1. Choose reference system (e.g. A)
2. Gauge fix

Choosing perspective = choosing gauge

gauge symmetry: $q_a(t)$ not physical

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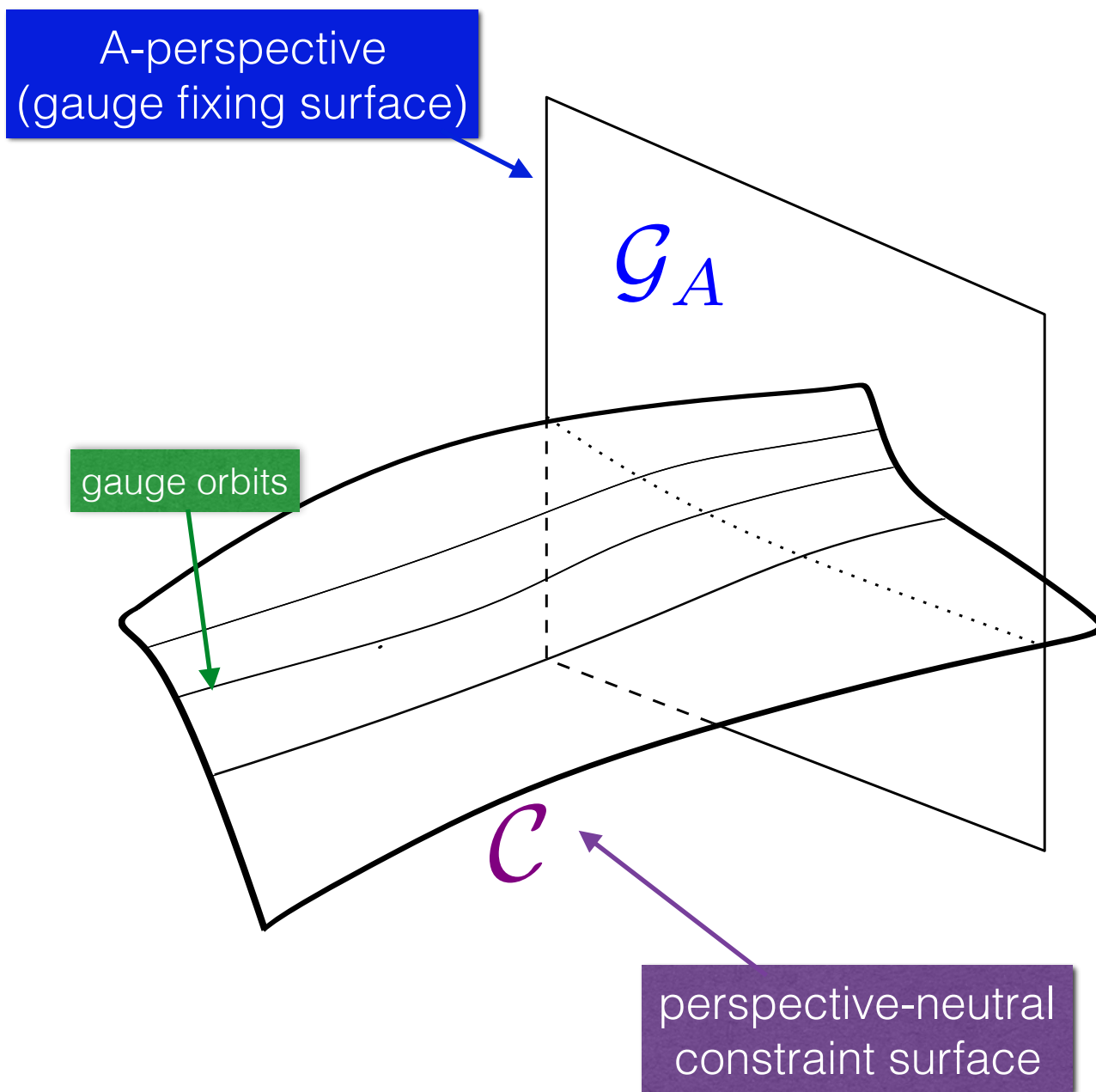
perspective-neutral description

$\Rightarrow$ but redundant (only 4 indep. gauge inv. obs)

- fix symmetry, **take A perspective** $q_A = 0$ (fixes $\lambda = -p_A$ so that $\dot{q}_A = 0$)

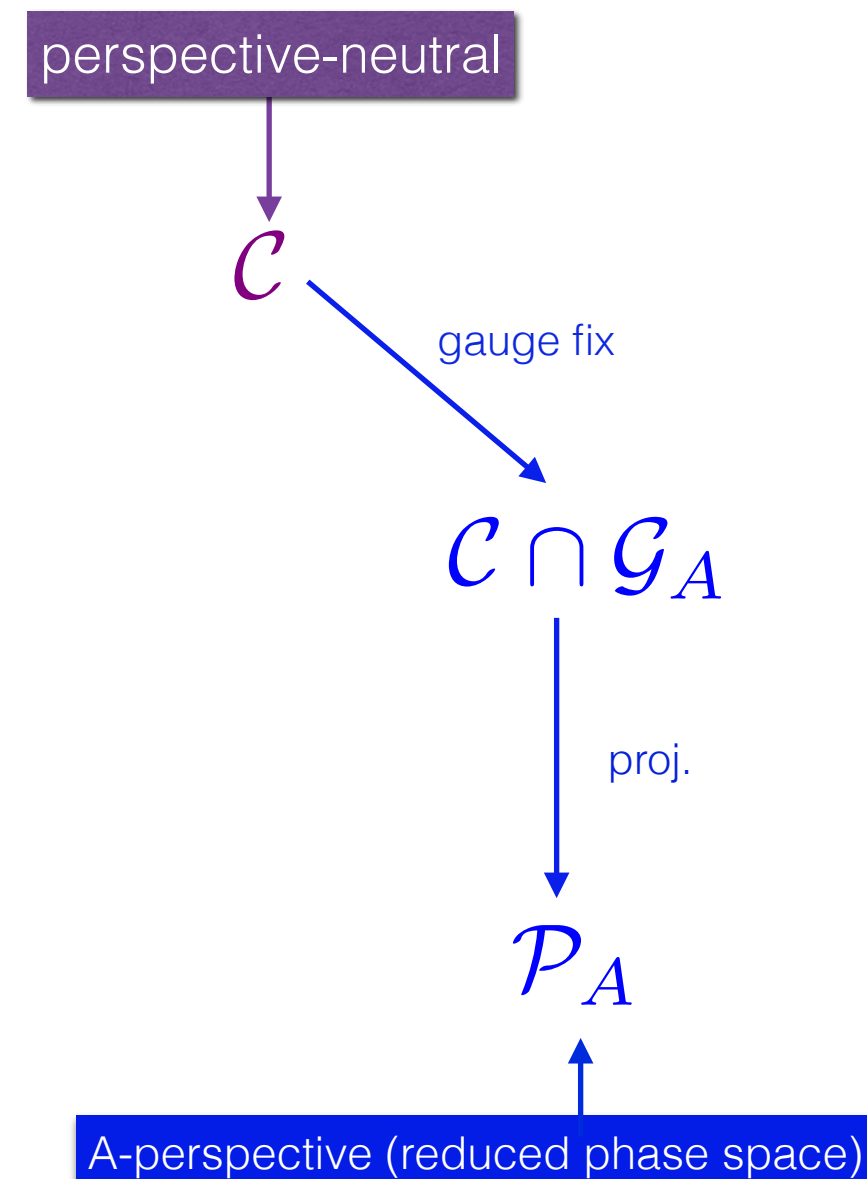
$(q_{E,F} - q_A) \mapsto q_{E,F}$ become rel. distance to A

Frame choice as gauge



Classical reduction:

1. Choose reference system (e.g. A)
2. Gauge fix
3. remove A's redundant DoFs



Choosing perspective = choosing gauge

gauge symmetry: $q_a(t)$ not physical

gauge inv. observables: $(q_E - q_A), (q_F - q_A), (q_E - q_F), p_A, p_E, p_F$

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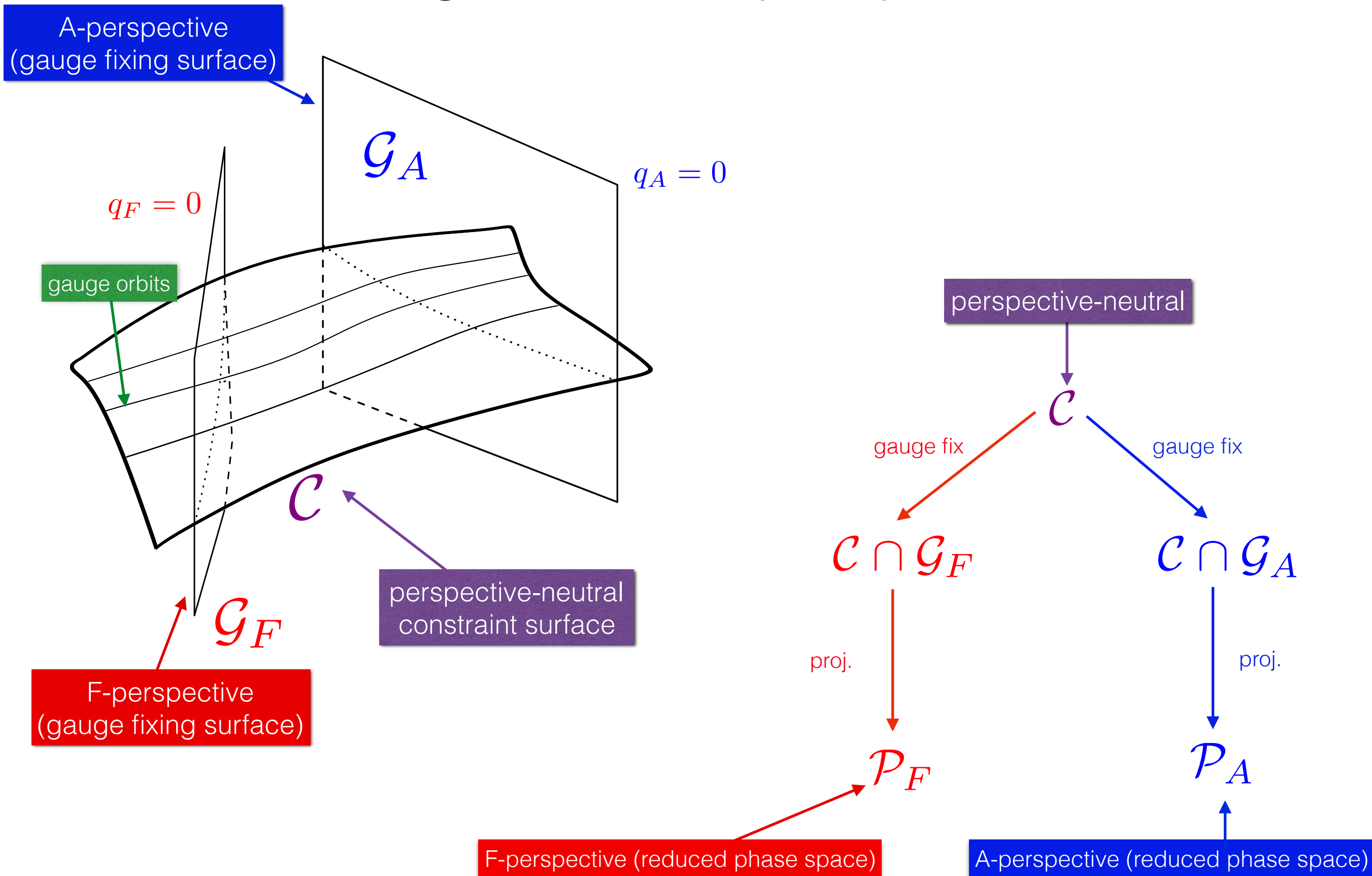
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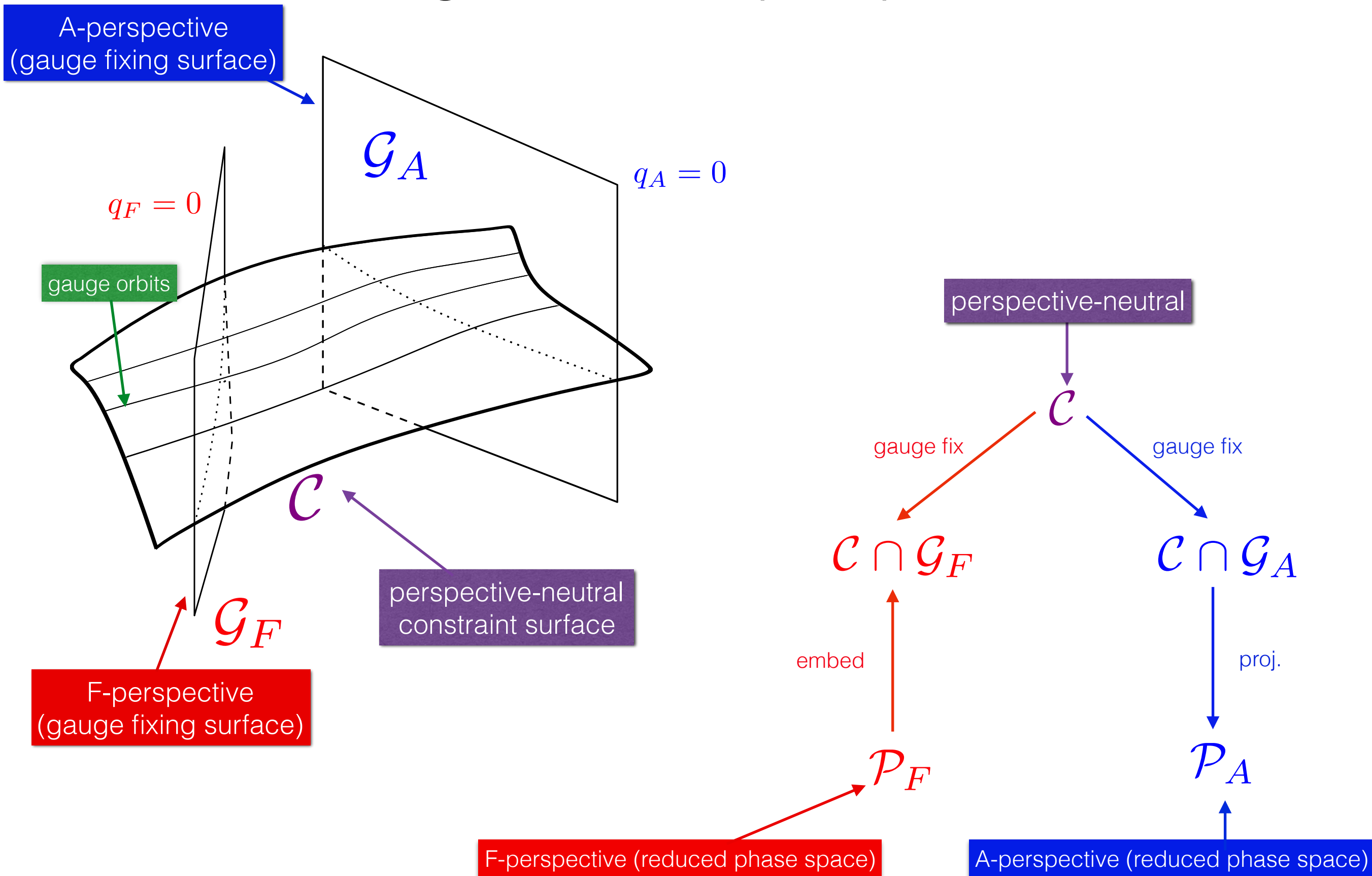
- Hamiltonian reduces to

$$H_{EF|A} = p_E^2 + p_F^2 + p_E p_F + V(q_E, q_F)$$

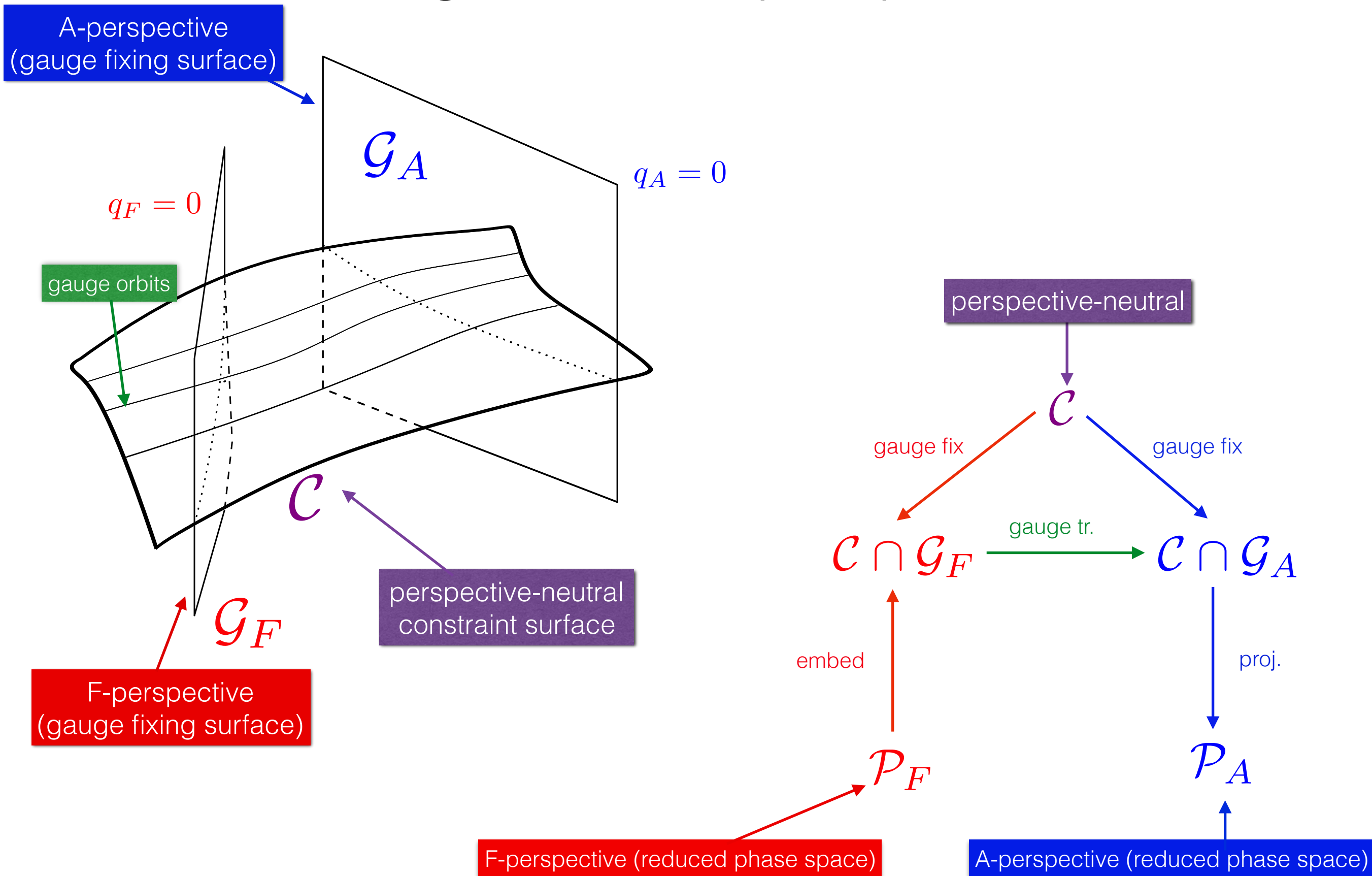
Change of frame perspective



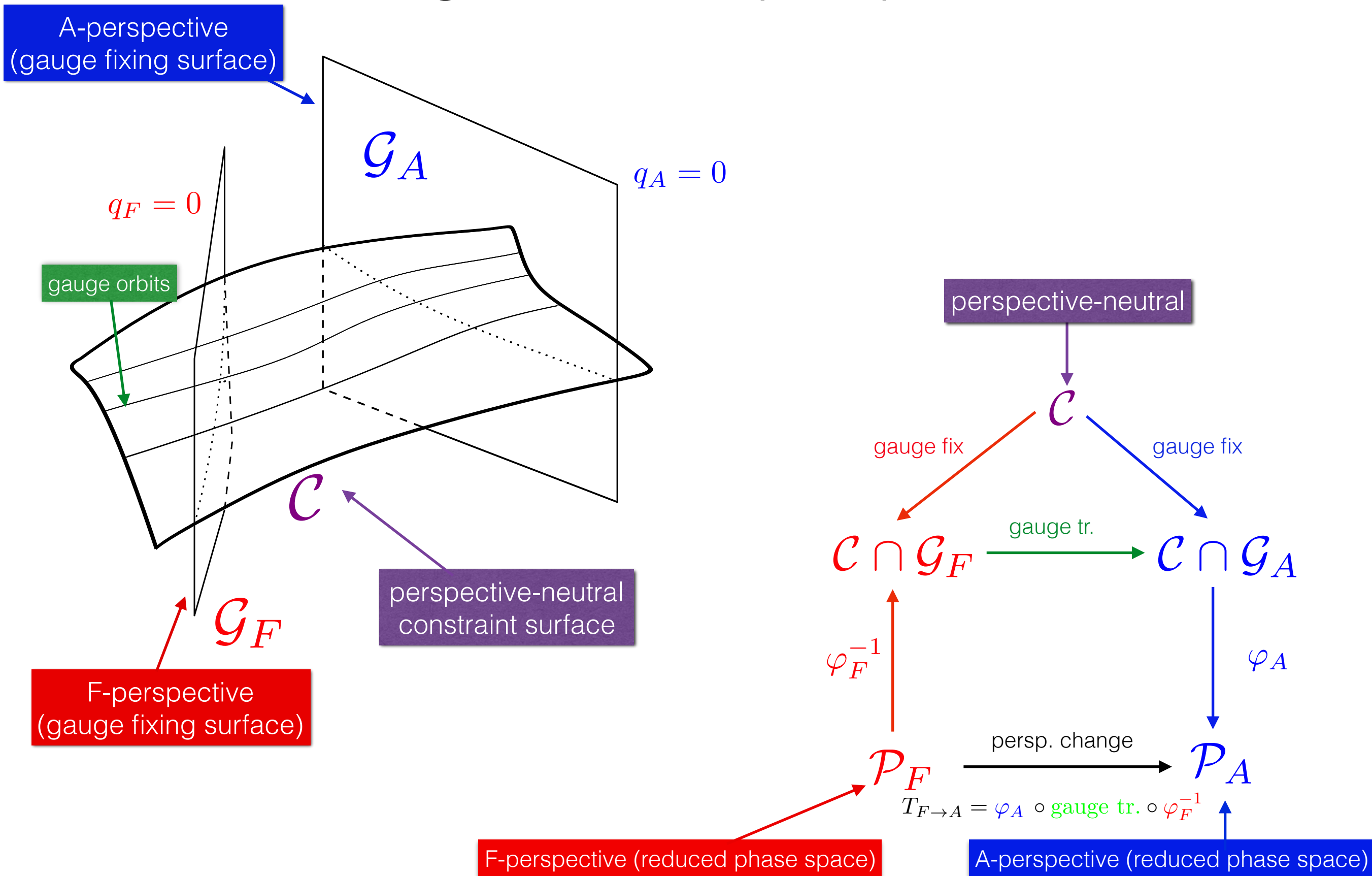
Change of frame perspective



Change of frame perspective



Change of frame perspective



Quantum reference frame switches

Reduced quantization: QT in a frame perspective

1. Solve constraints classically
2. Quantize

Reduced quantization: QT in A-perspective

- quantize reduced class. model in A-perspective

$$[\hat{q}_{E,F}, \hat{p}_{E,F}] = i \quad \text{on} \quad \mathcal{H}_A^{\text{red}} = L^2(\mathbb{R}^2)$$

$$|\psi\rangle_{EF|A} = \int dp_E dp_F \psi_{EF|A}(p_E, p_F) |p_E, p_F\rangle$$

with Hamiltonian

$$\hat{H}_{EF|A} = \hat{p}_E^2 + \hat{p}_F^2 + \hat{p}_E \hat{p}_F + V(\hat{q}_E, \hat{q}_F)$$

Dirac quantization - perspective-neutral

1. Quantize all DoFs
2. Solve Constraint in QT

Dirac quantization - perspective-neutral

quantize 1st on $\mathcal{H}_{\text{kin}} = L^2(\mathbb{R}^3)$ solve constraint in QT

$$\hat{P}|\psi\rangle_{\text{phys}} = (\hat{p}_A + \hat{p}_E + \hat{p}_F)|\psi\rangle_{\text{phys}} = 0$$

Dirac quantization - perspective-neutral

quantize 1st on $\mathcal{H}_{\text{kin}} = L^2(\mathbb{R}^3)$ solve constraint in QT

$$\hat{P}|\psi\rangle_{\text{phys}} = (\hat{p}_A + \hat{p}_E + \hat{p}_F)|\psi\rangle_{\text{phys}} = 0$$

Projection $\Pi : \mathcal{H}_{\text{kin}} \rightarrow \mathcal{H}_{\text{phys}}$

$$|\psi\rangle_{\text{kin}} = \int dp_A dp_E dp_F \psi_{\text{kin}}(p_A, p_E, p_F) |p_A, p_E, p_F\rangle$$

$$\begin{aligned} \mapsto |\psi\rangle_{\text{phys}} &= \int dp_E dp_F \psi_{\text{kin}}(-p_E - p_F, p_E, p_F) | -p_E - p_F, p_E, p_F\rangle \\ &= \int dp_A dp_F \psi_{\text{kin}}(p_A, -p_A - p_F, p_F) |p_A, -p_A - p_F, p_F\rangle \end{aligned}$$

Dirac quantization - perspective-neutral

quantize 1st on $\mathcal{H}_{\text{kin}} = L^2(\mathbb{R}^3)$ solve constraint in QT

$$\hat{P}|\psi\rangle_{\text{phys}} = (\hat{p}_A + \hat{p}_E + \hat{p}_F)|\psi\rangle_{\text{phys}} = 0$$

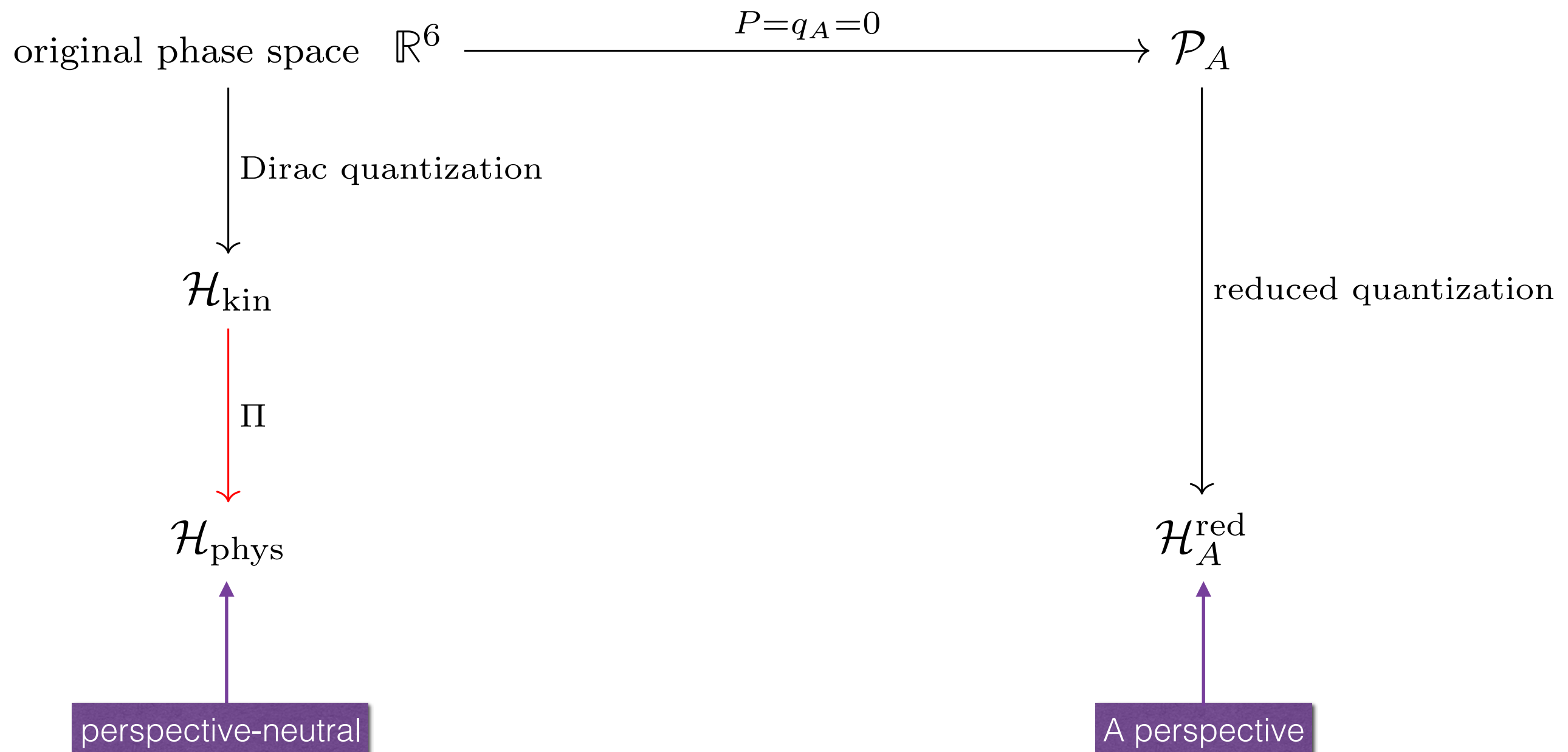
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- observables on $\mathcal{H}_{\text{phys}}$ $[\hat{O}, \hat{P}] = 0$ e.g. $\hat{q}_a - \hat{q}_b, \hat{p}_a$

Summary so far:



Quantum reference frame switches?

quantum reduction procedure?

- debate on relation reduced vs Dirac quantization
- no gauge fixing in QT

Perspective-neutral quantum structure



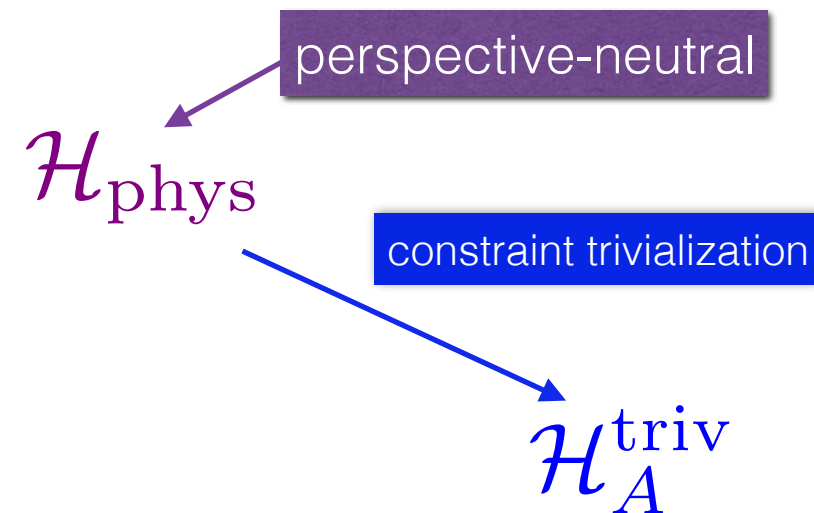
Dirac quantized theory

Perspective-neutral to perspectival in the QT



1. Choose reference system (e.g. A)

Quantum reduction



1. Choose reference system (e.g. A)
2. Trivialize constraints to reference system

Go to A-perspective in QT

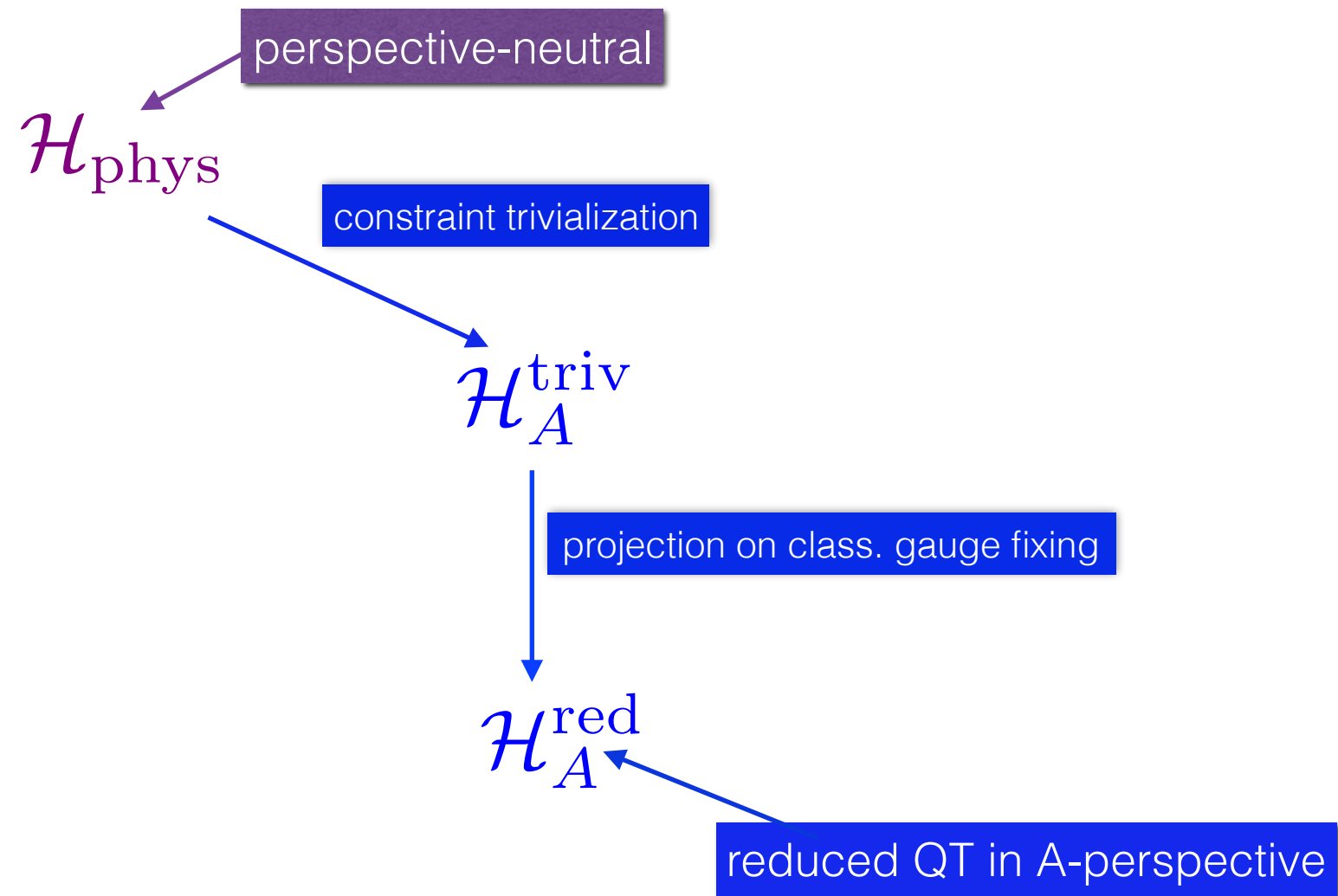
- remove redundant A-variables with isometry to red. QT

$$\mathcal{T}_A := e^{i\hat{q}_A(\hat{p}_E + \hat{p}_F)}$$

$$\Rightarrow \text{trivializes constraint to A} \quad \mathcal{T}_A \hat{P} \mathcal{T}_A^\dagger = \hat{p}_A$$

$$\begin{aligned} \Rightarrow |\psi\rangle_{A,EF} &:= \mathcal{T}_A |\psi\rangle_{\text{phys}} = |p=0\rangle_A \otimes \int \underbrace{dp_E dp_F \psi_{\text{kin}}(-p_E - p_F, p_E, p_F)}_{:= |\psi\rangle_{EF|A}} |p_E, p_F\rangle \\ &:= |\psi\rangle_{EF|A} \end{aligned}$$

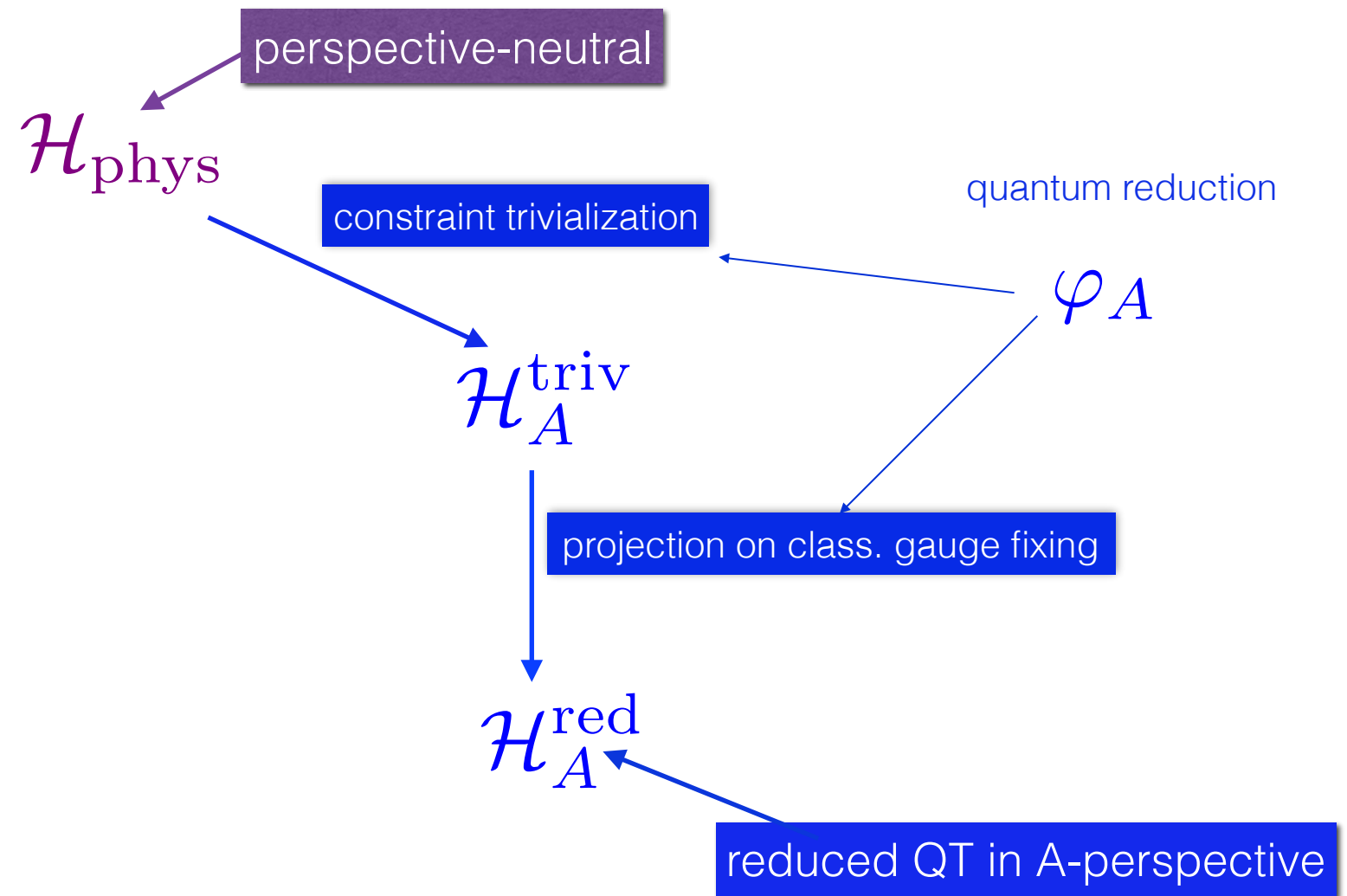
Quantum reduction



1. Choose reference system (e.g. A)
2. Trivialize constraints to reference system
3. Project onto classical gauge fixing condition

Quantum reduction

Vanrietvelde, PH, Giacomini, Castro Ruiz '18
Vanrietvelde, PH, Giacomini '18
PH, Vanrietvelde '18
PH '18



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Go to A-perspective in QT

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$$:= |\psi\rangle_{EF|A}$$

project out redundancy with $\sqrt{2\pi} \langle q_A = 0 |$

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- observables transform correctly

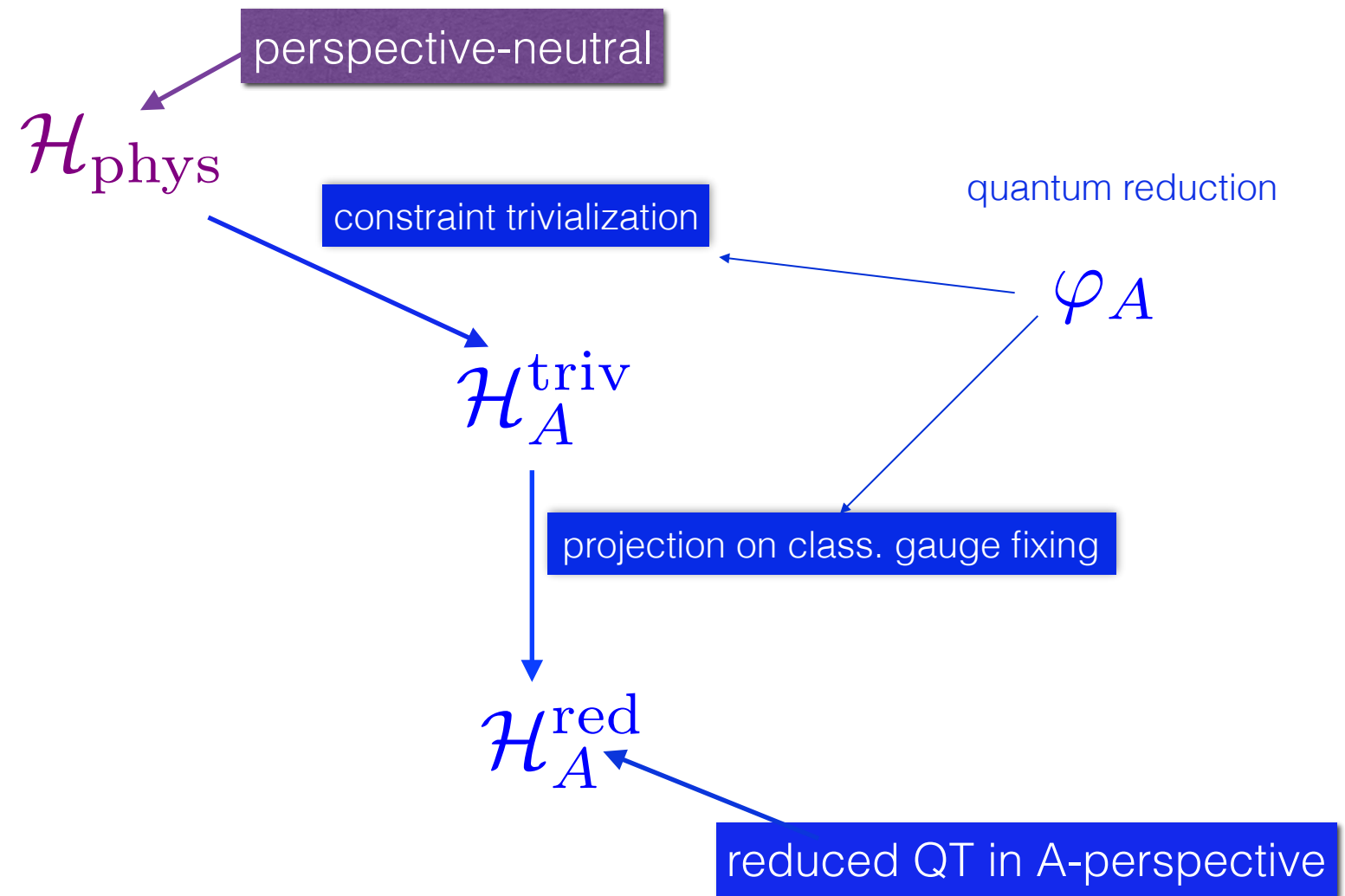
$$\mathcal{T}_A (\hat{q}_{E,F} - \hat{q}_A) \mathcal{T}_A^\dagger = \hat{q}_{E,F}$$

$$\mathcal{T}_A \hat{p}_{E,F} \mathcal{T}_A^\dagger = \hat{p}_{E,F}$$

$$\mathcal{T}_A \hat{H} \mathcal{T}_A^\dagger = \hat{H}_{EF|A} = \hat{p}_E^2 + \hat{p}_F^2 + \hat{p}_E \hat{p}_F + V(\hat{q}_E, \hat{q}_F)$$

$\Rightarrow$ defines isometry to A-perspective

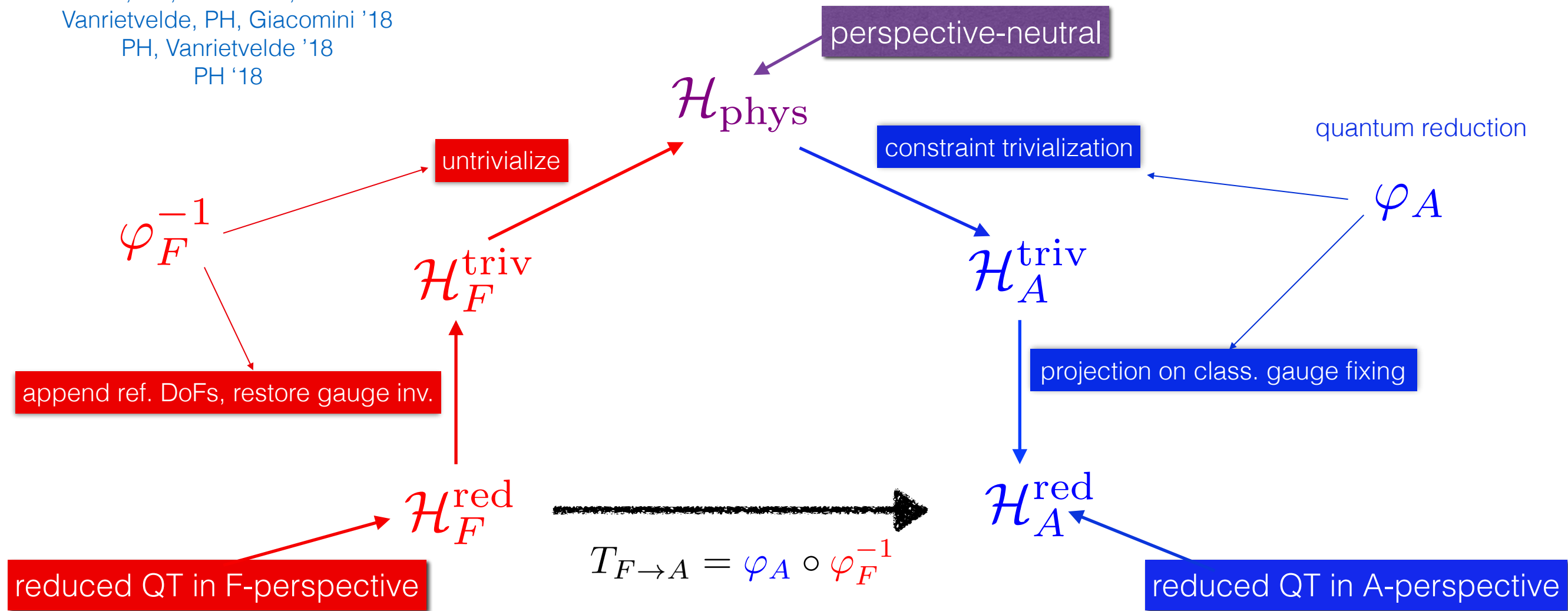
Switch from F- to A-perspective in QT



invert F-reduction φ_F^{-1} , concatenate
with A-reduction φ_A

Switch from F- to A-perspective in QT

Vanrietvelde, PH, Giacomini, Castro Ruiz '18
 Vanrietvelde, PH, Giacomini '18
 PH, Vanrietvelde '18
 PH '18



invert F-reduction φ_F^{-1} , concatenate
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Go to A-perspective in QT

- remove redundant A-variables with isometry to red. QT

$$\mathcal{T}_A := e^{i\hat{q}_A(\hat{p}_E + \hat{p}_F)}$$

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$$:= |\psi\rangle_{EF|A}$$

project out redundancy with $\sqrt{2\pi} \langle q_A = 0 |$

- observables transform correctly

$$\mathcal{T}_A (\hat{q}_{E,F} - \hat{q}_A) \mathcal{T}_A^\dagger = \hat{q}_{E,F}$$

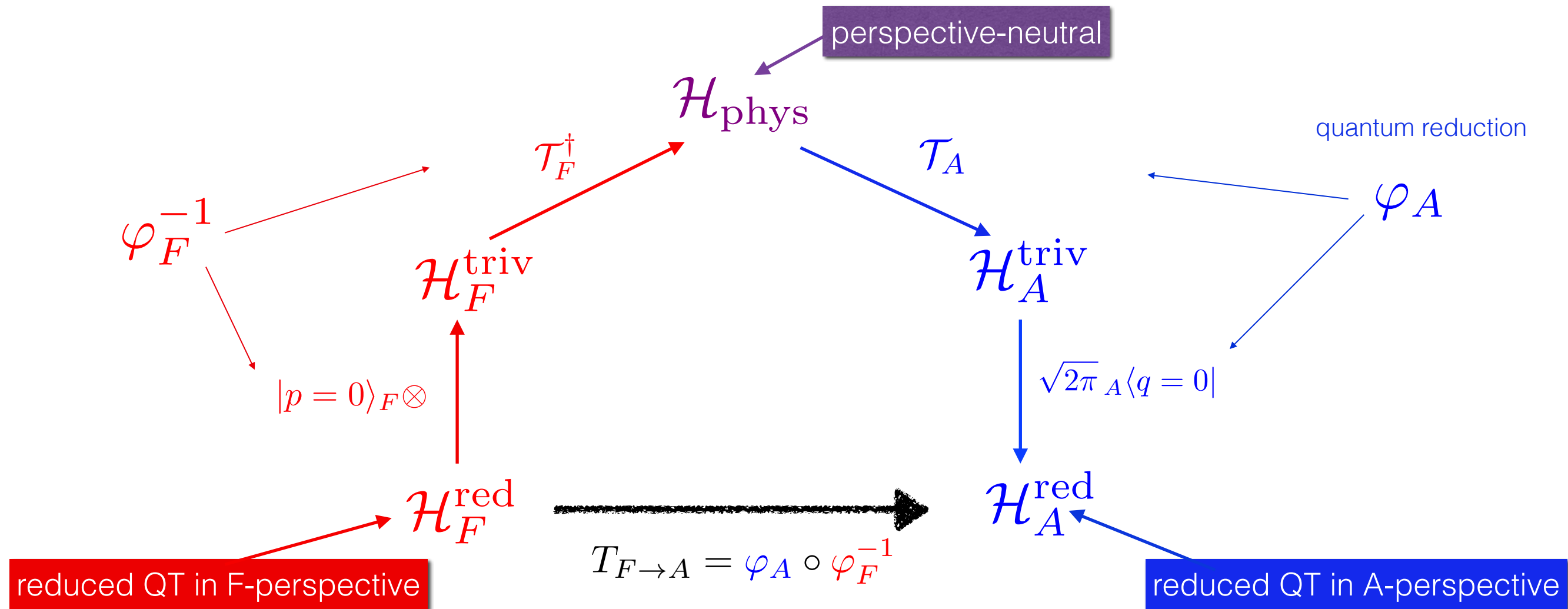
$$\mathcal{T}_A \hat{p}_{E,F} \mathcal{T}_A^\dagger = \hat{p}_{E,F}$$

$$\mathcal{T}_A \hat{H} \mathcal{T}_A^\dagger = \hat{H}_{EF|A} = \hat{p}_E^2 + \hat{p}_F^2 + \hat{p}_E \hat{p}_F + V(\hat{q}_E, \hat{q}_F)$$

$\Rightarrow$ defines isometry to A-perspective

Switch from F- to A-perspective in QT

Vanrietvelde, PH, Giacomini, Castro-Ruiz, arXiv:1809.00556



concretely

$$\begin{aligned}
 T_{F \rightarrow A} &= \sqrt{2\pi} {}_A\langle q=0| \mathcal{T}_A \mathcal{T}_F^\dagger |p=0\rangle_F \otimes \\
 &= \mathcal{P}_{AF} e^{i\hat{q}_A \hat{p}_E}
 \end{aligned}$$

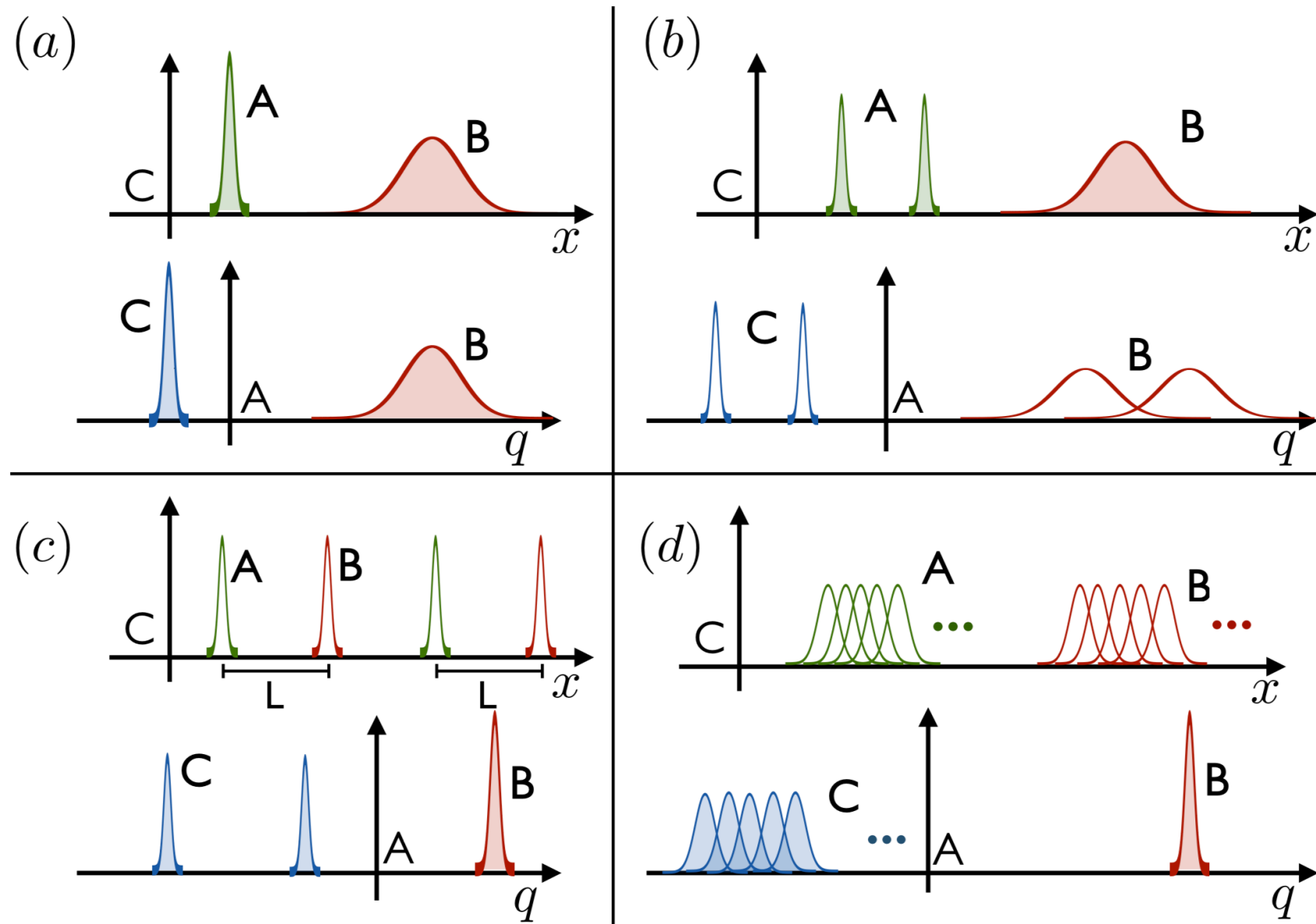
parity swap

$\Rightarrow$

recover QRF transf. proposed by Giacomini, Castro-Ruiz, Brukner (arXiv:1712.07207)

Operational consequences

(stolen from GC-RB paper)



see Giacomini, Castro-Ruiz, Brukner arXiv:1712.07207

Generalize to 3D with “Mach principle”

Vanrietvelde, PH, Giacomini, arXiv:1809.05093

- incl. also rotational invar.

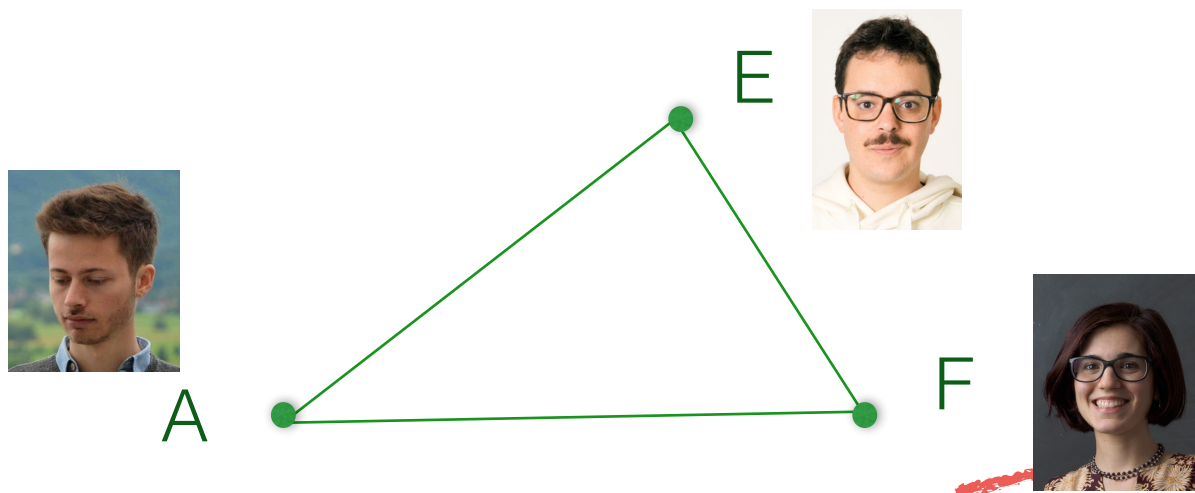
$$H = \sum_{a=A,E,F} \vec{p}_a^2 + V(\{|\vec{q}_a - \vec{q}_b|\}) + \vec{\lambda} \vec{P} + \vec{\mu} \vec{J}$$

where

$$J_i = \sum_a \epsilon_{ijk} q_a^j p_a^k$$

- gauge inv. motion and observables

$$|\vec{q}_a - \vec{q}_b|, \vec{p}_a^2$$



perspective-neutral

$\Rightarrow$ gauge fix to A-perspective

A: origin

E: in z-direction

F: in x-z-plane

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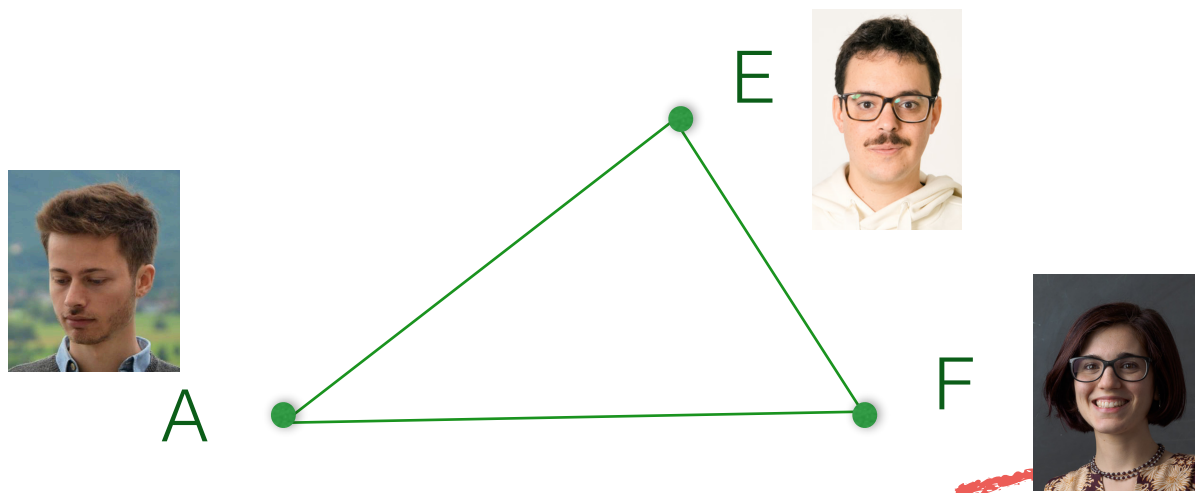
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$$|\vec{q}_a - \vec{q}_b|, \vec{p}_a^2$$



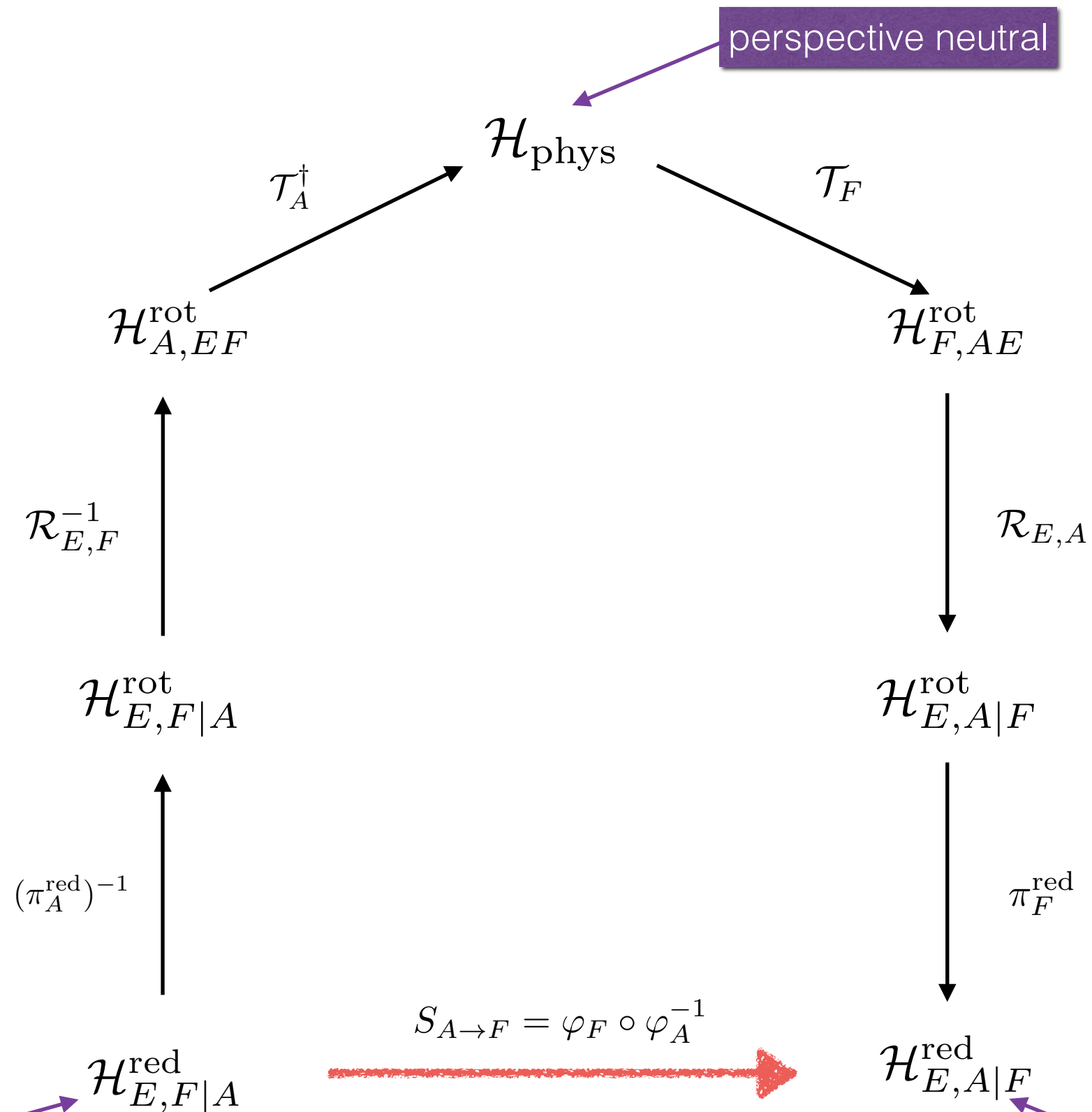
perspective-neutral

⇒ gauge fix to A-perspective

A: origin
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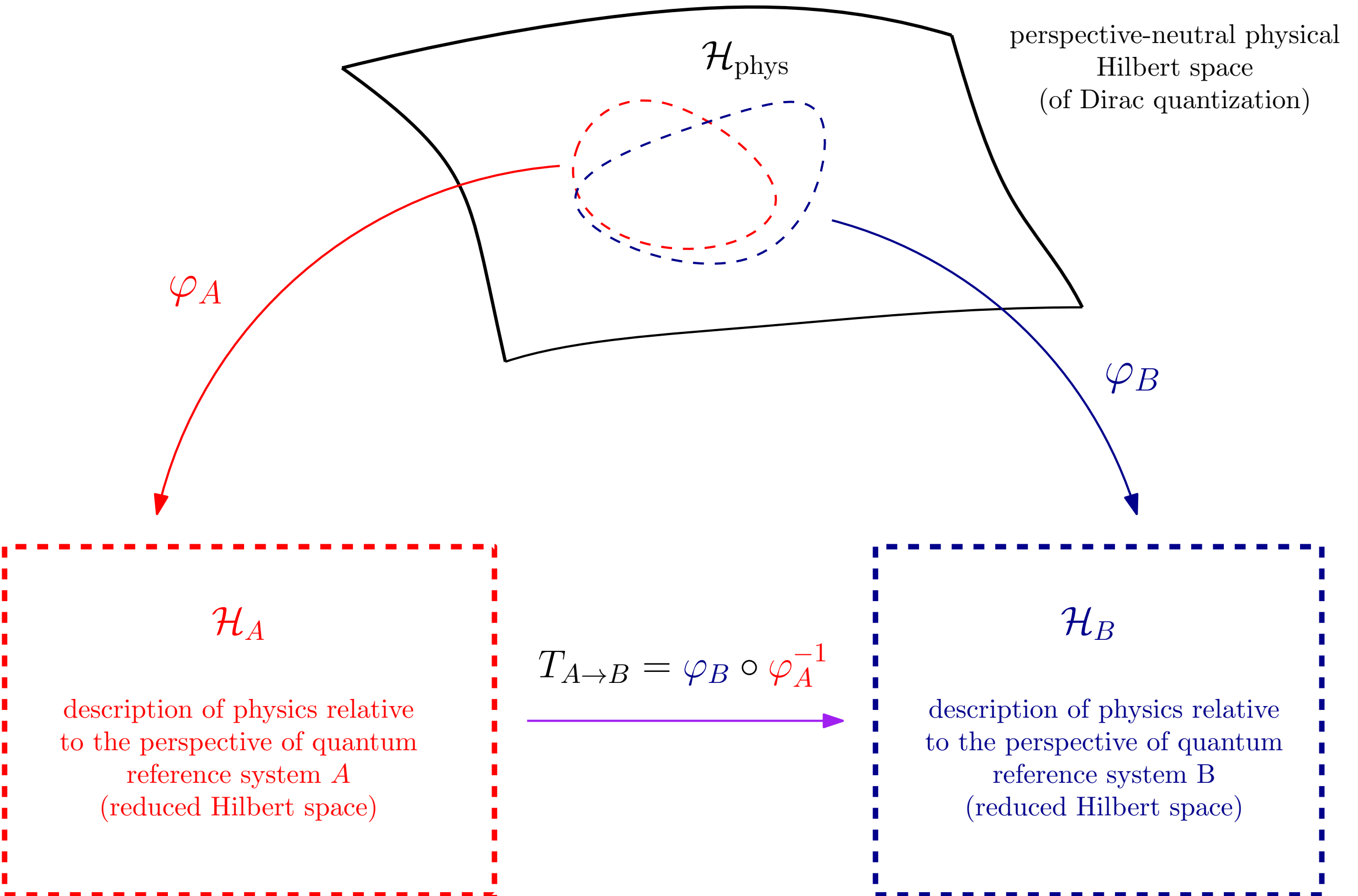
Gribov problem arises
⇒
no global relational perspectives

Perspective switch



reduced QT in A-perspective

reduced QT in F-perspective



Quantum clocks

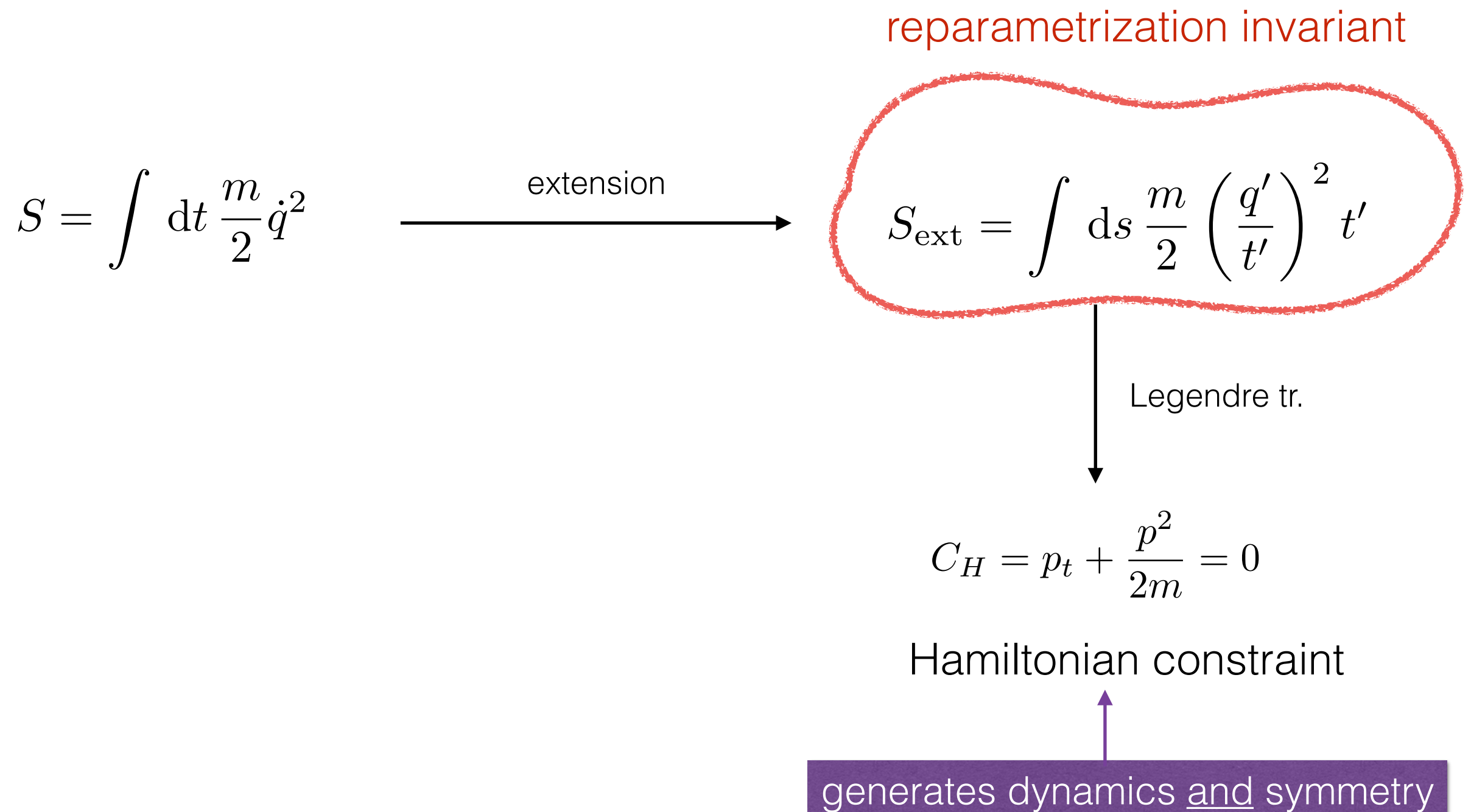
2 clocks variables t, q

$t(q)$ vs $q(t)$

What if t, q operators?

address “multiple choice problem” (Kuchar '91; Isham '92)

Example: parametrized particle



Example: parametrized particle

$$S = \int dt \frac{m}{2} \dot{q}^2$$

extension

reparametrization invariant

$$S_{\text{ext}} = \int ds \frac{m}{2} \left(\frac{q'}{t'} \right)^2 t'$$

Legendre tr.

$$C_H = p_t + \frac{p^2}{2m} = 0$$

Hamiltonian constraint

generates dynamics and symmetry

clock	relational Dirac observable
t	$Q(\tau) = q(s) \big _{t(s)=\tau} = 2p(\tau - t_0) + q_0$
q	$T(X) = t(s) \big _{q(s)=X} = t_0 + \frac{X - q_0}{2p}$

Example: parametrized particle

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Hamiltonian constraint

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clock	relational Dirac observable on constraint surface
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q	$T(X) = t(s) \big _{q(s)=X} = t_0 + \frac{X - q_0}{2p}$ time-of-arrival

Example: parametrized particle

Getting rid of redundancy through gauge-fixing

clock	relational Dirac observable on constraint surface
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Example: parametrized particle

Getting rid of redundancy through gauge-fixing

$$C_H = p_t + \frac{p^2}{2m} = \frac{1}{2m} (p + \sqrt{-p_t})(p - \sqrt{-p_t})$$

clock	relational Dirac observable on constraint surface	reduced observable	gauge fixing:
t	$Q(\tau) = q(s) \big _{t(s)=\tau} = 2p(\tau - t_0) + q_0$	$Q_{\text{red}}(\tau) = 2p\tau + q$	
q	$T(X) = t(s) \big _{q(s)=X} = t_0 + \frac{X - q_0}{2p}$	$T_{\pm}(X) = t \mp \frac{X}{2\sqrt{-p_t}}$	

$$(t_0 = 0)$$

$$(q_0 = 0)$$

Example: parametrized particle

Getting rid of redundancy through gauge-fixing

$$C_H = p_t + \frac{p^2}{2m} = \frac{1}{2m} (p + \sqrt{-p_t})(p - \sqrt{-p_t})$$

clock	relational Dirac observable on constraint surface	reduced observable on reduced phase space	gauge fixing:
t	$Q(\tau) = q(s) \big _{t(s)=\tau} = 2p(\tau - t_0) + q_0$	$Q_{\text{red}}(\tau) = 2p\tau + q$	
q	$T(X) = t(s) \big _{q(s)=X} = t_0 + \frac{X - q_0}{2p}$	$T_{\pm}(X) = t \mp \frac{X}{2\sqrt{-p_t}}$	

Example: parametrized particle

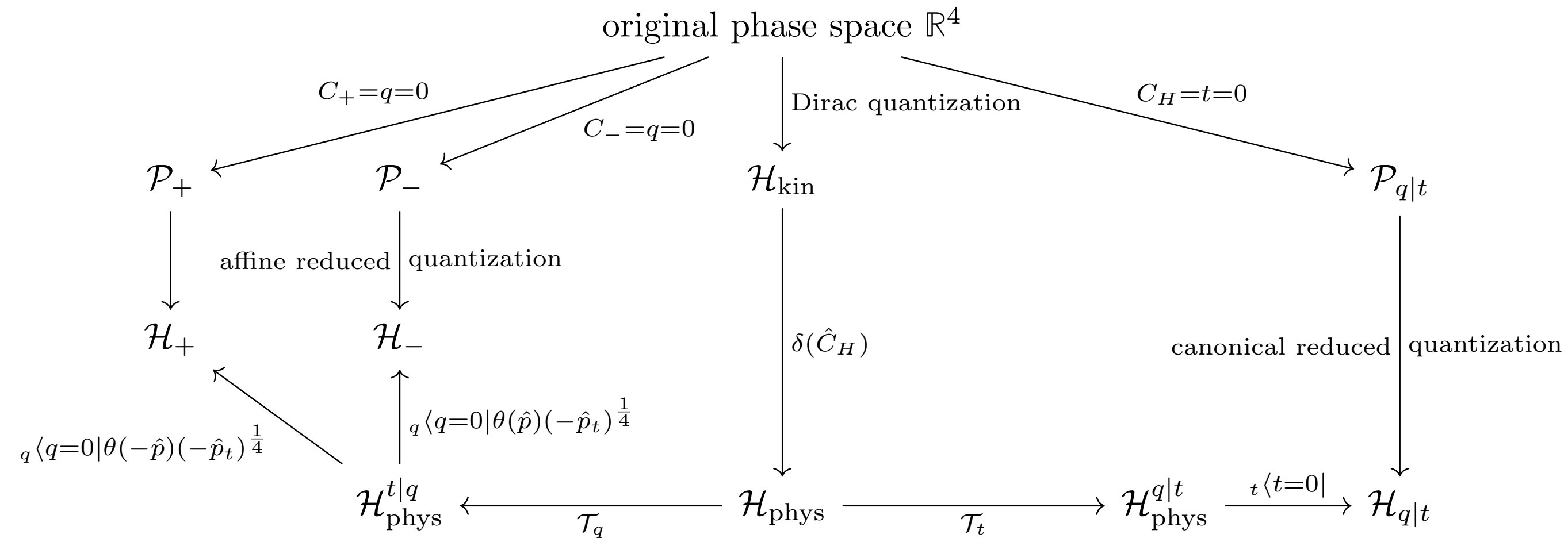
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q	$T(X) = t(s) \big _{q(s)=X} = t_0 + \frac{X - q_0}{2p}$	$T_{\pm}(X) = t \mp \frac{X}{2\sqrt{-p_t}}$ evolution w.r.t q

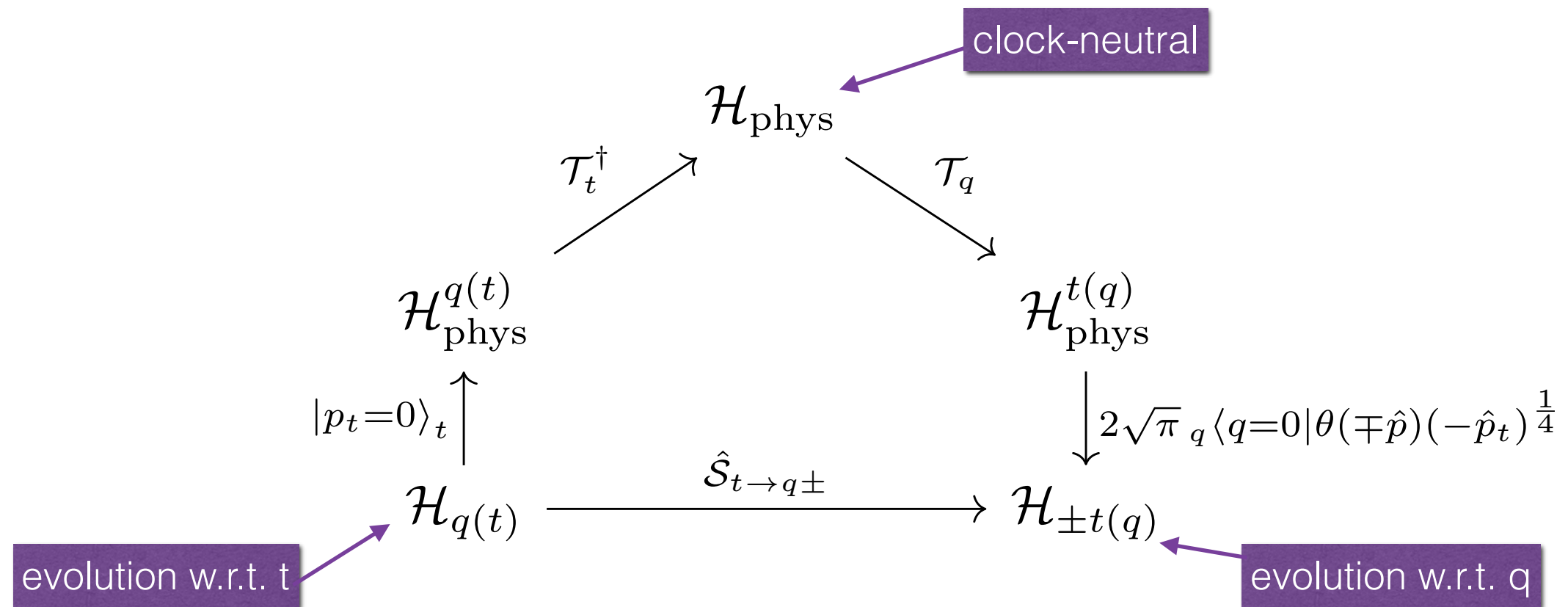
Summary of steps for parametrized particle

PH, Vanrietvelde, arxiv:1810.04153



Quantum clock switch

PH, Vanrietvelde, arxiv:1810.04153



where $\mathcal{T}_q := \mathcal{T}_{q+} + \mathcal{T}_{q-}$ $\mathcal{T}_{q\pm} := \exp \left(\pm i \hat{q} (\widehat{\sqrt{-p_t}} - \epsilon) \right) \theta(\mp \hat{p})$

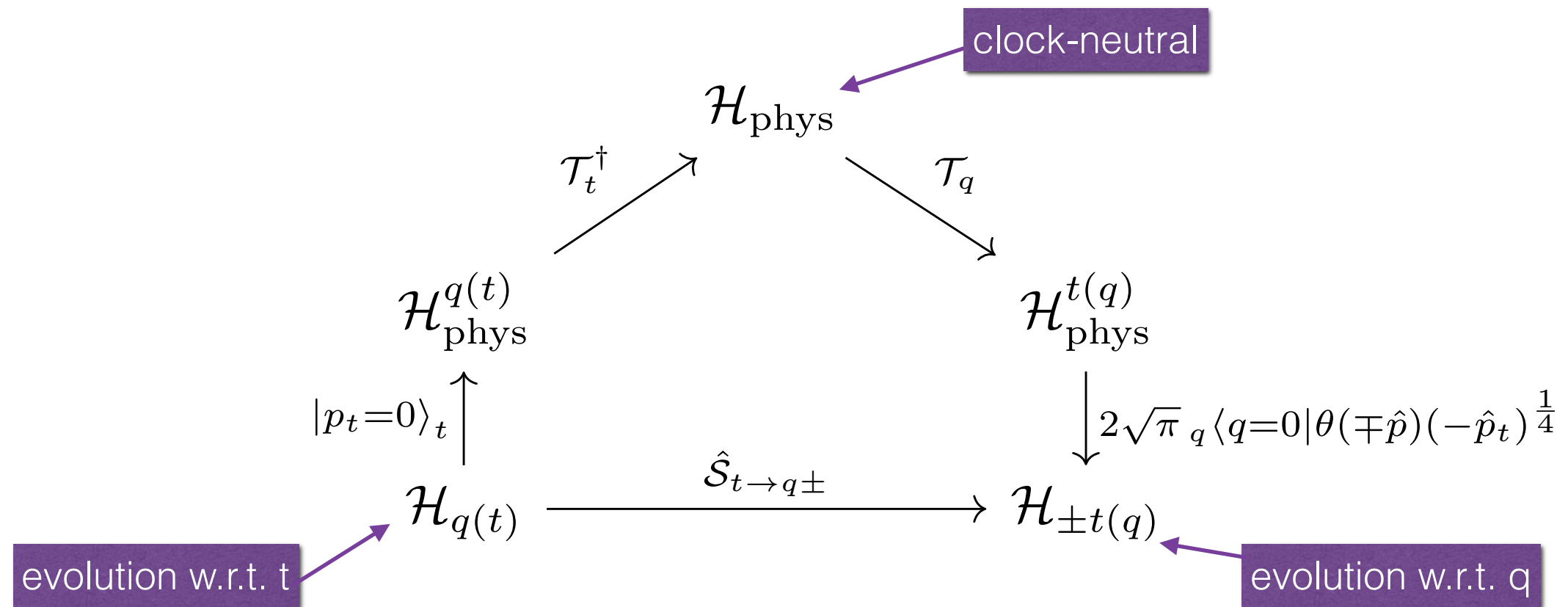
$$\mathcal{T}_t := \exp(i \hat{t} \hat{p}^2 / 2m) = \exp(i \hat{t} \hat{H})$$

$$\hat{\mathcal{S}}_{t \rightarrow q \pm} \equiv \sqrt{2} \hat{\mathcal{P}}_{q \rightarrow t} \theta(\mp \hat{p}) \widehat{\sqrt{|p|}}$$

clock swap

Quantum clock switch

PH, Vanrietvelde, arxiv:1810.04153



observables transform correctly

$$\mathcal{T}_t \hat{Q}(\tau) \mathcal{T}_t^\dagger = \hat{Q}_{\text{red}}(\tau)$$

$$\widehat{(-p_t)^{1/4}} \mathcal{T}_q \hat{T}_\delta(X) \mathcal{T}_q^{-1} \widehat{(-p_t)^{-1/4}} = \hat{T}_+^\delta(X) \theta(-\hat{p}) + \hat{T}_-^\delta(X) \theta(\hat{p})$$

Regularization details: time-of-arrival

Dirac:

affine reduced:

had: $\widehat{(-p_t)^{1/4}} \mathcal{T}_q \hat{T}_\delta(X) \mathcal{T}_q^{-1} \widehat{(-p_t)^{-1/4}} = \hat{T}_+^\delta(X) \theta(-\hat{p}) + \hat{T}_-^\delta(X) \theta(\hat{p})$

Dirac
quantization

$$\hat{T}_\delta(X) := \hat{t}_\delta + \frac{1}{4} \left(\widehat{(p)_\delta^{-1}} (X - \hat{q}) + (X - \hat{q}) \widehat{(p)_\delta^{-1}} \right)$$

$$\hat{t}_\delta := \frac{1}{2} \left(\hat{t} \hat{p}_t \widehat{(p_t)_\delta^{-1}} + \hat{p}_t \widehat{(p_t)_\delta^{-1}} \hat{t} \right)$$

$$\widehat{(p_t)_\delta^{-1}} |p_t\rangle := \begin{cases} \frac{1}{p_t} |p_t\rangle & p_t \leq -\delta^2, \\ -\frac{1}{\delta^2} |p_t\rangle & -\delta^2 < p_t \leq 0, \end{cases}$$

$$\hat{T}_\pm^\delta(X) := \hat{t}_{\delta\pm} \mp \frac{X}{2} \widehat{(\sqrt{-p_t})_\delta^{-1}}$$

Affine
Reduced
Quantization

$$\hat{t}_{\delta\pm} := \frac{1}{2} \left(\widehat{(p_t)_\delta^{-1}} \hat{\mathfrak{t}} + \hat{\mathfrak{t}} \widehat{(p_t)_\delta^{-1}} \right)$$

$$\widehat{(\sqrt{-p_t})_\delta^{-1}} |p_t\rangle_\pm := \begin{cases} \frac{1}{\sqrt{-p_t}} |p_t\rangle_\pm & p_t \leq -\delta^2, \\ \frac{\sqrt{-p_t}}{\delta^2} |p_t\rangle_\pm & -\delta^2 < p_t \leq 0. \end{cases}$$

Relational dynamics in FRW

- homogeneous & isotropic universe

$$ds^2 = -dt^2 + a(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

- Hamiltonian constraint incl. homog. field

$$C_H = p_\phi^2 - p_\alpha^2 - 4k e^{4\alpha} + 4m^2 \phi^2 e^{6\alpha} \approx 0$$

$$\alpha = \ln a$$

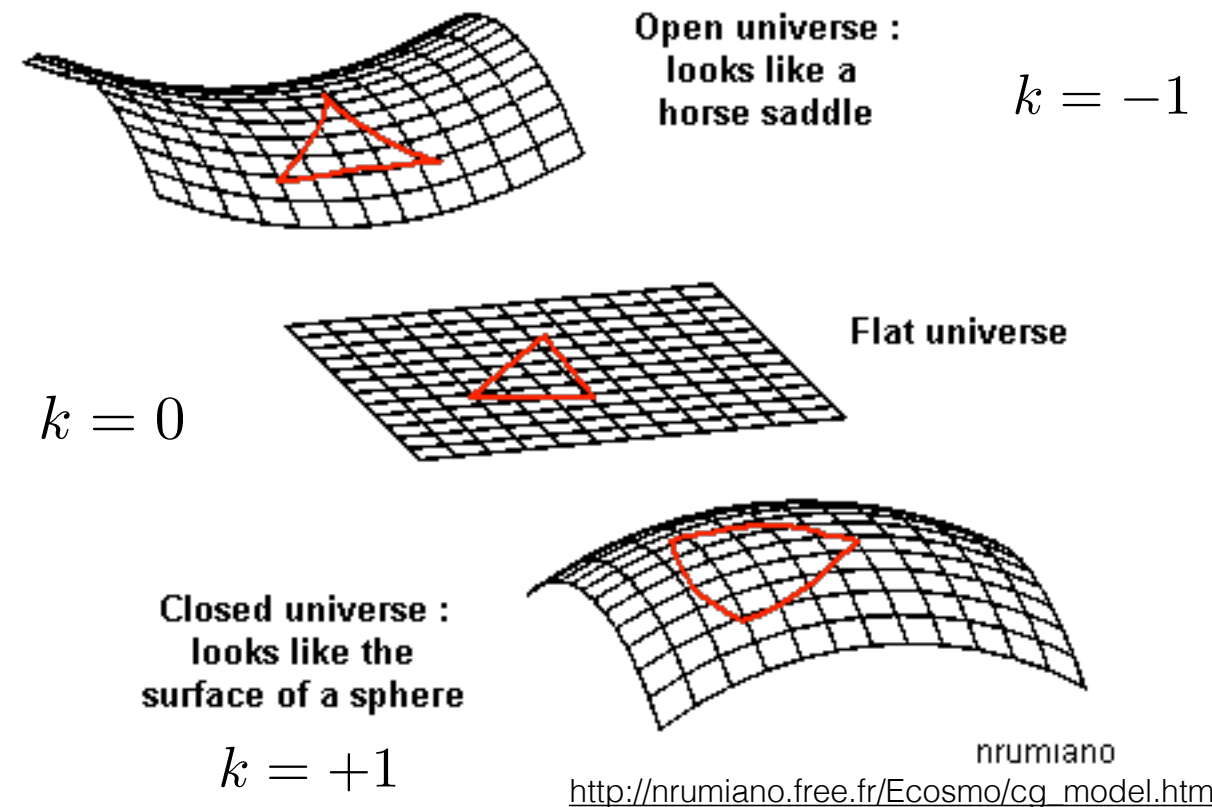
$\Rightarrow$ time evol. generated is gauge transf.

$$\dot{\phi} = \{\phi, C_H\} = 2p_\phi$$

$$\dot{\alpha} = \{\alpha, C_H\} = -2p_\alpha$$

can go through 0, thus
not necessarily monotonic

$\Rightarrow$ want to use α or ϕ as relational “clock”



Relational dynamics in FRW

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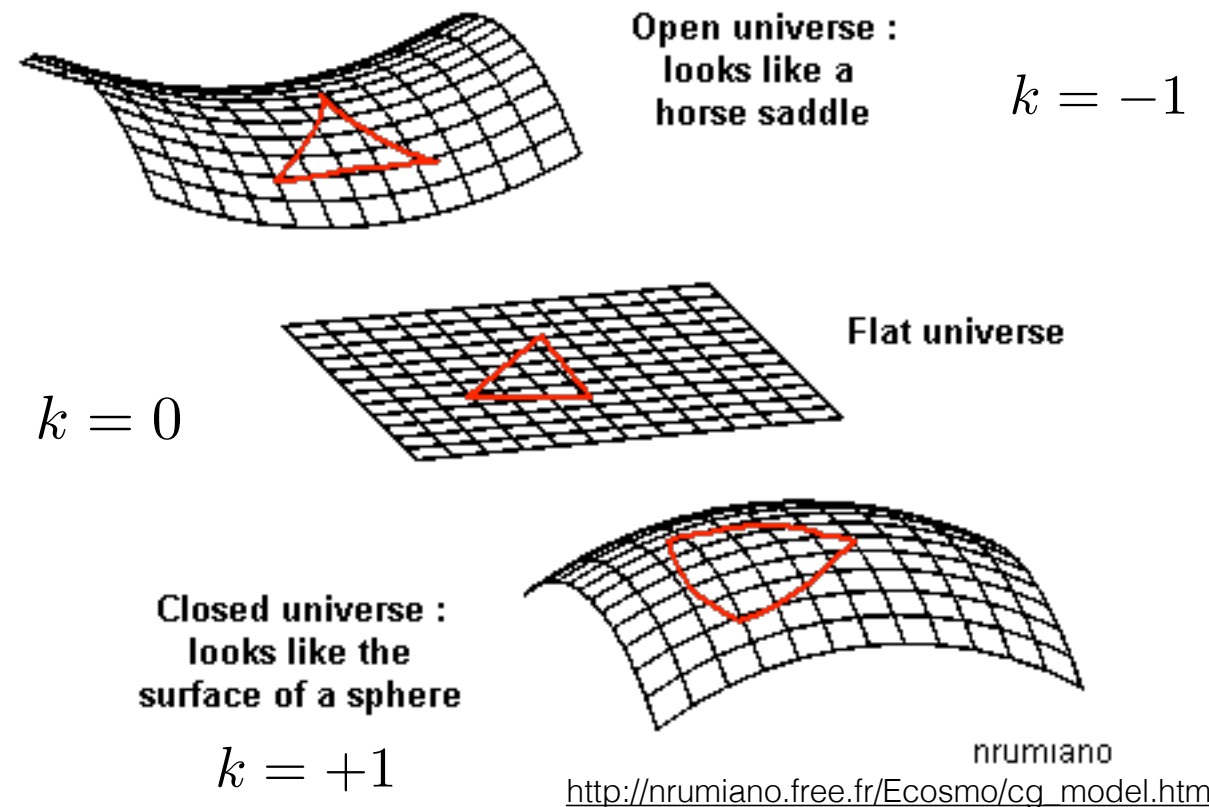
$$\alpha = \ln a$$

$\Rightarrow$ time evol. generated is gauge transf.

$$\dot{\phi} = \{\phi, C_H\} = 2p_\phi$$

$$\dot{\alpha} = \{\alpha, C_H\} = -2p_\alpha$$

$\Rightarrow$ want to use α or ϕ as relational “clock”



set

$$k = m = 0$$

$\Rightarrow$ want to use α or ϕ as relational “clock” $\leftarrow$ **both monotonic**

$k=0$ FRW with massless scalar

- Hamiltonian constraint of Klein-Gordon form

PH, arXiv:1811.00611

$$C_H = p_\phi^2 - p_\alpha^2 \approx 0$$

$$\Rightarrow \begin{aligned} \phi(t) &= 2p_\phi t + \phi_0 \\ \alpha(t) &= -2p_\alpha t + \alpha_0 \end{aligned}$$

- choose α as “clock” and affine evolving $\Phi = \phi p_\phi$

$$\Rightarrow \Phi(\tau) = -p_\alpha(\tau - \alpha) + \Phi \quad \text{rel. Dirac observable}$$

- get rid of redundant α, p_α

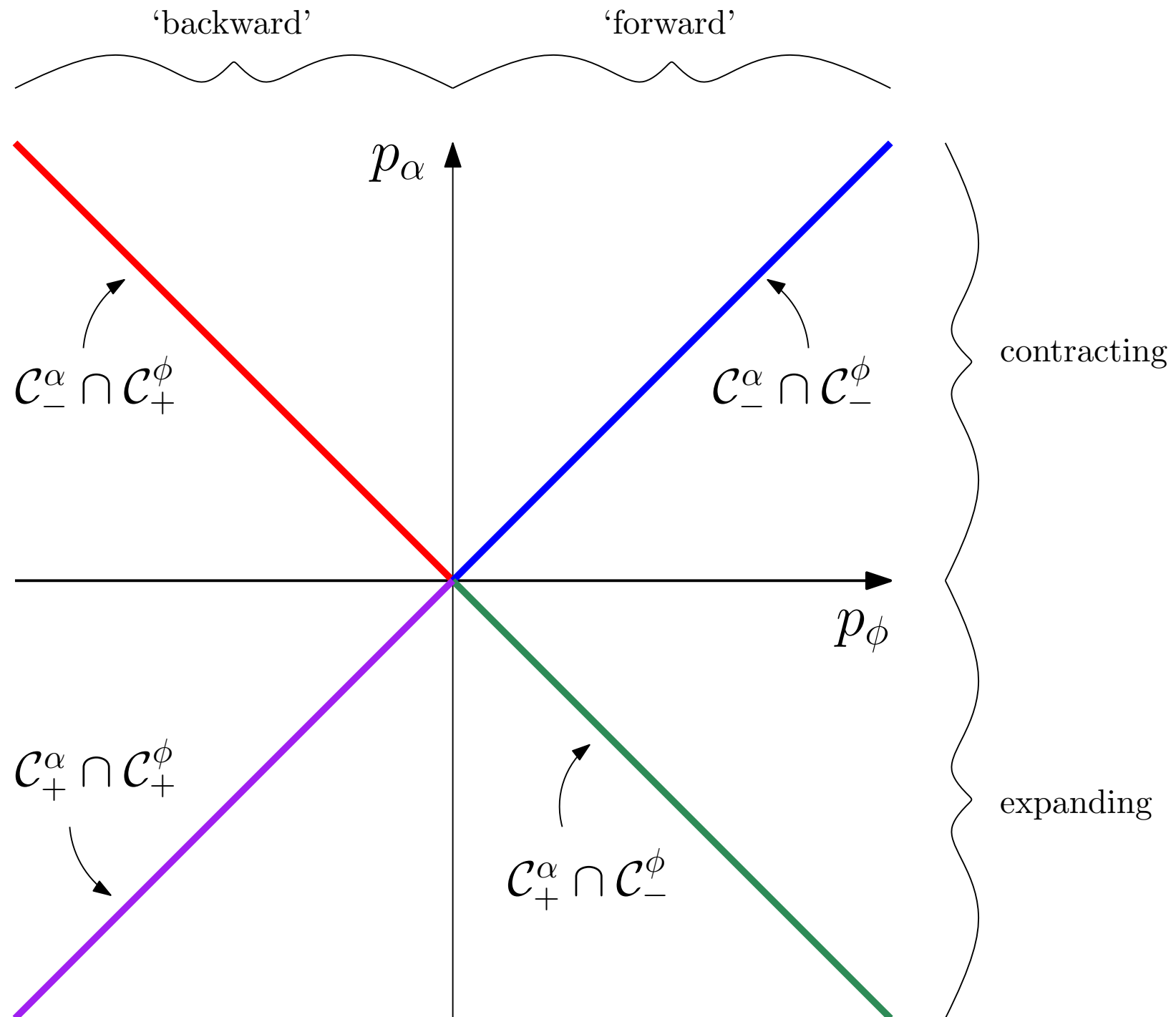
$$p_\alpha = \pm |p_\phi| = \pm H$$

generates fwd/bwd
evol.

$$\Phi_\pm(\tau) = \pm |p_\phi| \tau + \Phi$$

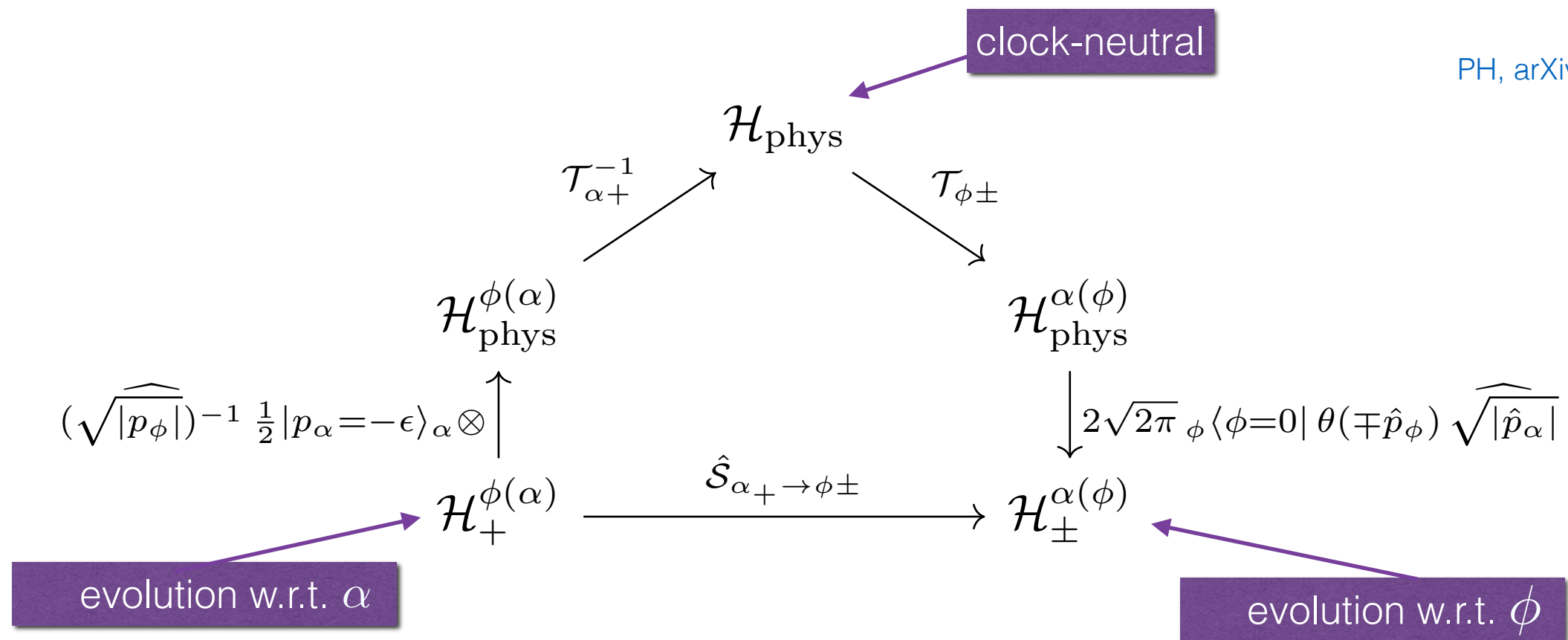
on reduced phase space

Positive and negative frequency sectors



Internal time switch in quantum cosmology

PH, arXiv:1811.00611



where

$$\mathcal{T}_\alpha = \mathcal{T}_{\alpha+} + \mathcal{T}_{\alpha-} \quad \mathcal{T}_{\alpha\pm} = e^{\pm i\hat{\alpha}(|\hat{p}_\phi| - \epsilon)} \theta(\mp \hat{p}_\alpha)$$

works the same way

Relational dynamics more generally

- can get arbitrarily complicated

$$C_H = p_\phi^2 - p_\alpha^2 - 4k e^{4\alpha} + 4m^2 \phi^2 e^{6\alpha} \approx 0$$

chaos for massive field

interacting clocks,
global problem of time,
non-unitarity, transient clocks

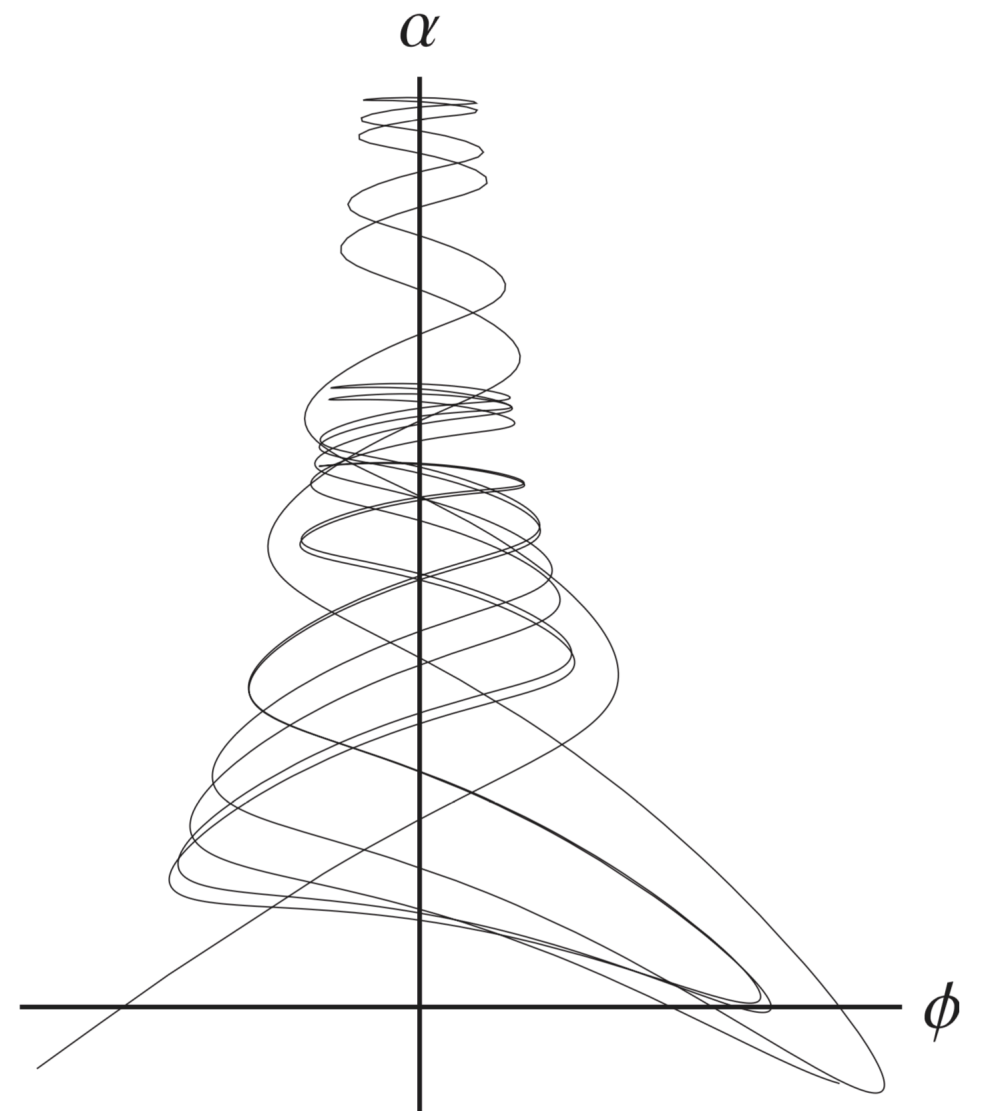
see

Bojowald, PH, Tsobanian, CQG 28,035006, (2011)

Bojowald, PH, Tsobanian, PRD 83,125023 (2011)

PH, Kubalova, Tsobanian, PRD 86, 065014 (2012)

Dittrich, PH, Koslowski, Nelson, PLB 769, 554 (2017)



New perspective on “wave function of the universe”

PH arXiv:1811.00611

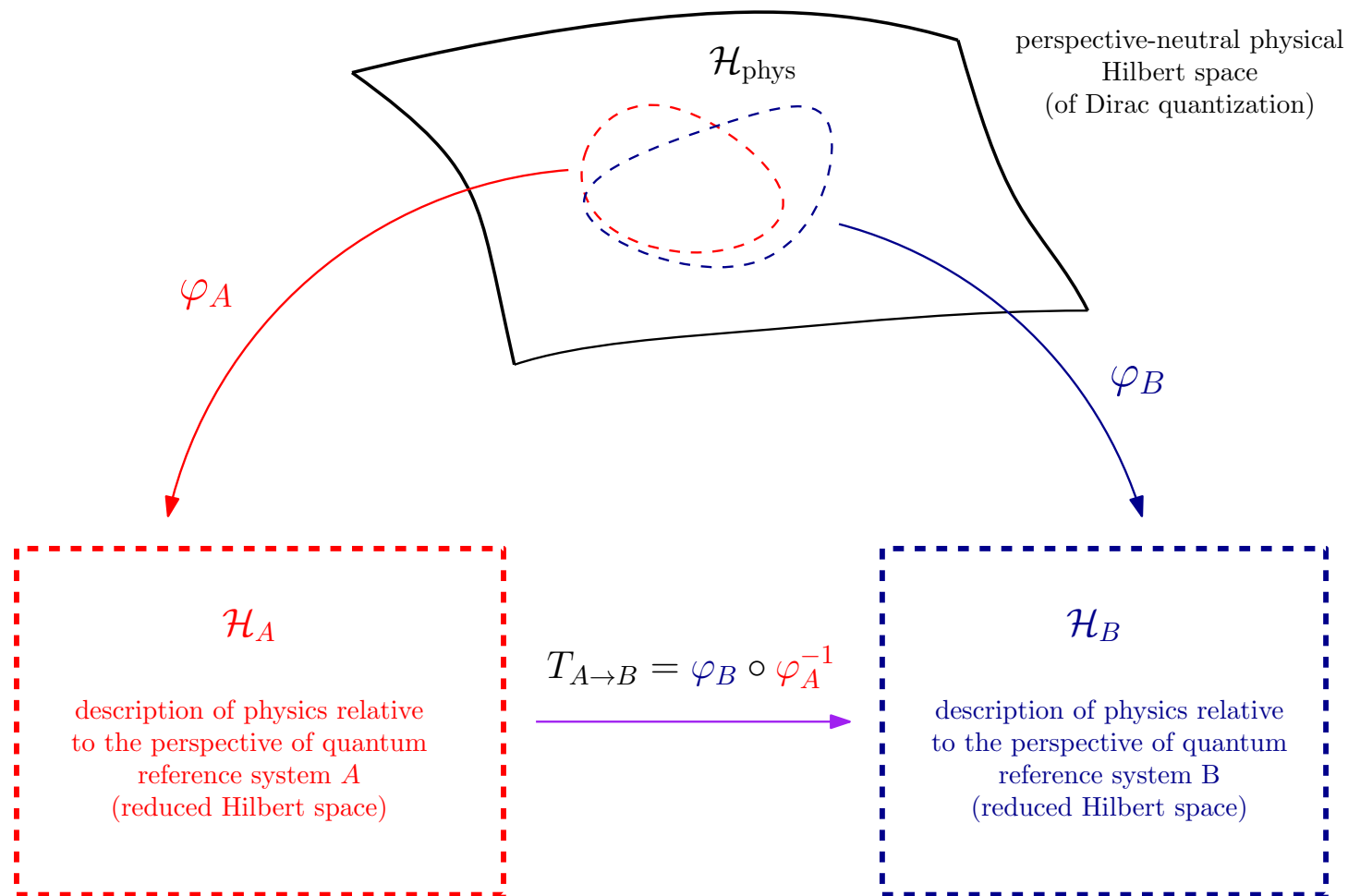
- no global operational state —> global state perspective neutral
- only relative operational states

“Wave function of the universe” as a perspective-neutral state

Operational interpretation from transformation to specific reduced theory

PH, Quantum 1, 38 (2017)
PH, JPCS 880, 012014 (2017)

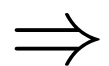
Conclusions



systematic switches of quantum
reference system perspectives

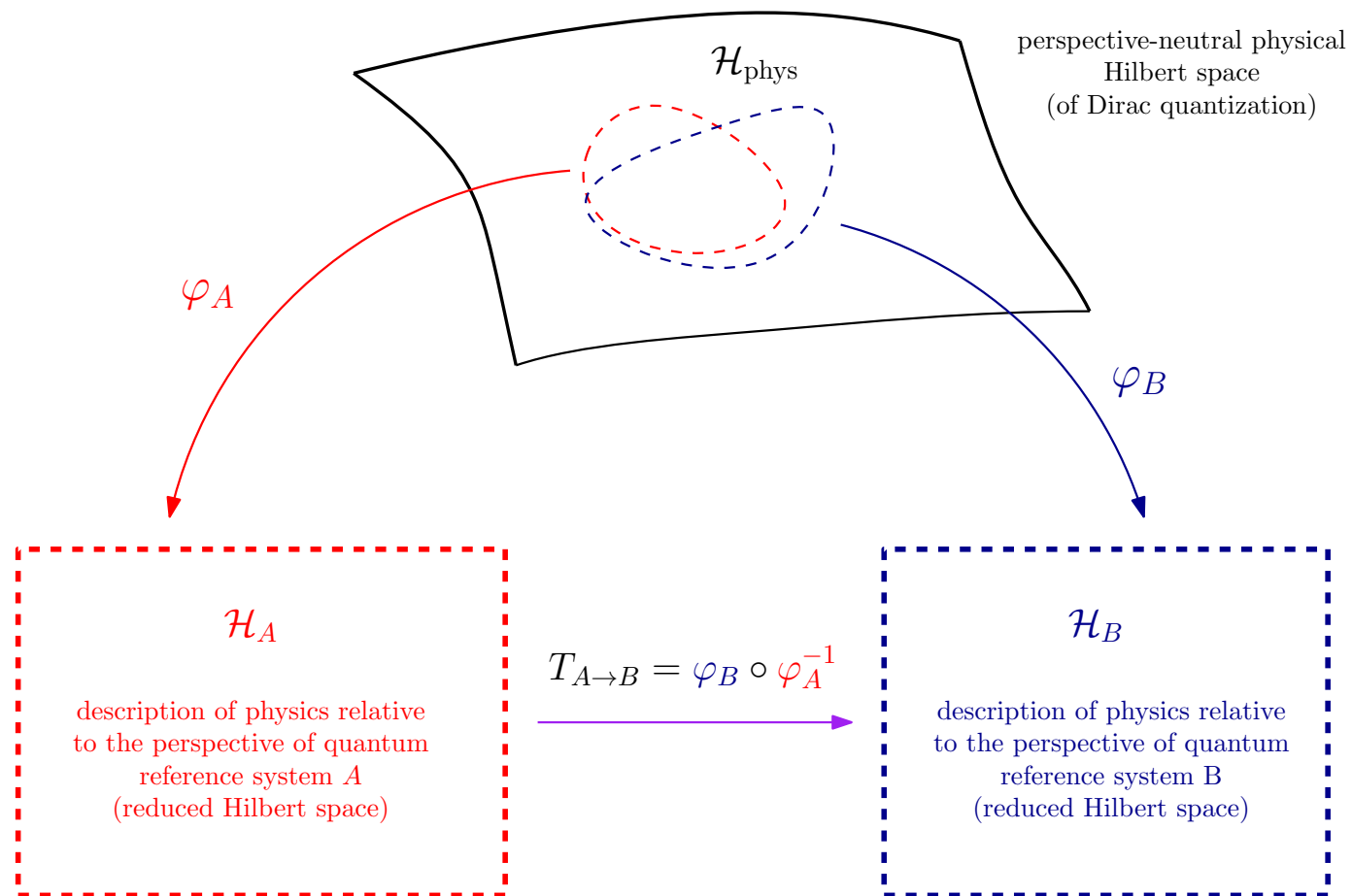
- new: quantum reduction method
- both spatial and temporal
reference systems

Not always global due to Gribov problem



complete relational QT, admitting quantum general covariance:
conjunction of Dirac and red. quantized theories

Outlook



Systematic method for switching
(spatial and temporal)
quantum reference system perspectives

applications:

- Field theory
- Quantum general covariance in QG (multiple choice problem of time,...)
- Measurement problem and Wigner's friend paradox
- Frame dependence of correlations in cosmology
- Import QRF machinery from QI into QG