Generalized geometry and topological string theory

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with A. Sinkovics and R. J. Szabo

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- The question what we study in this talk:

How the AKSZ formulations of topological strings fit into the framework of GG?

[Pestun,Zucchini,Stojevic,Ikeda,Tokunaga,. . .]

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Review

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- DFT approach to AKSZ models of topological strings
- Reduction to GG $\rightsquigarrow$ description w/ generalized complex structure
- Interesting feature: topological S-duality arises from the generalized complex structure

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   a) $Q_{\mathcal{W}}$ have degree 1

   b) $\mathcal{L}^2_{Q_{\mathcal{W}}} = 0$

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3. μ is invariant under $Q_{\mathcal{W}}$

   the choice: $\mu = \mathrm{d}^d \hat{z} = \mathrm{d}^d \sigma \mathrm{d}^d \theta$

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a coordinate $\phi \in \mathcal{M}$ corresponds to a field

$$\overset{|\phi|}{\boldsymbol{\phi}}(\sigma,\theta) = \overset{|\phi|}{\phi^{(0)}}(\sigma) + \overset{|\phi|-1}{\phi^{(1)}}{}_{\mu}(\sigma)\theta^{\mu} + \ldots + \frac{1}{d!}\overset{|\phi|-d}{\phi^{(d)}}{}_{\mu_1\ldots\mu_d}\theta^{\mu_1}\ldots\theta^{\mu_d}$$

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Target QP-manifold $\longleftrightarrow$ QP-manifold of superfields

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$\gamma = \gamma\left(q^a(\hat{z}), p_a(\hat{z})\right)$ Hamiltonian	$\longleftrightarrow$	$\boldsymbol{\mathcal{S}}_{\mathrm{int}} = \int_{T[1]\Sigma_d} \mathrm{d}^d\hat{z}\, \gamma\left(\boldsymbol{q}^a, \boldsymbol{p}_a\right)$ interaction term

full Hamiltonian: $\boldsymbol{\mathcal{S}} = \boldsymbol{\mathcal{S}}_{\mathrm{kin}} + \boldsymbol{\mathcal{S}}_{\mathrm{int}}$

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Hamiltonian		interaction term
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$Q_\gamma = \{\gamma,\,.\,\}$	$\longleftrightarrow$	$\boldsymbol{Q} = (\boldsymbol{\mathcal{S}},\,.\,)_{\mathrm{BV}}$ BV-BRST trans.

Gauge fixing of AKSZ models

- Theory is not specifeid yet: we need to choose **fields** ϕ^a and **antifields** ϕ_a^+

$$\omega = \int_{T[1]\Sigma_d} \mathrm{d}^d\hat{z}\, \overset{d-1-|a|}{\delta\phi_a^+(\hat{z})}\, \overset{|a|}{\delta\phi^a(\hat{z})}$$

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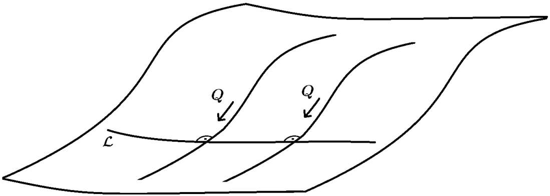
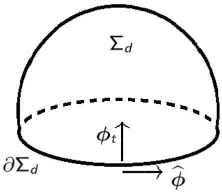
- In general: choice of a Lagrangian submanifold $\mathcal{L}$ with $\omega|_{\mathcal{L}} = 0$



Dimensional reduction to boundary

- Split the fields to **normal** ϕ_t and **transverse** $\widehat{\phi}$ modes wrt. the boundary $\partial\Sigma_d$ as

$$\phi(\sigma,\theta) = \widehat{\phi}(\sigma,\widehat{\theta}) + \phi_t(\sigma,\widehat{\theta})\,\theta^t$$



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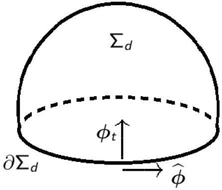

- Then ϕ_t or $\widehat{\phi}$ is gauge fixed

$\rightsquigarrow$ different choice gives different boundary theories

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$$\boldsymbol{\mathcal{S}}^{(2)}_\pi \;=\; \int_{T[1]\Sigma_2} \Big(\boldsymbol{\chi}_i\, \boldsymbol{D}\boldsymbol{X}^i \;+\; \frac{1}{2}\,\pi^{ij}(\boldsymbol{X})\,\boldsymbol{\chi}_i\,\boldsymbol{\chi}_j \Big)$$

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- How to define topological strings?

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$$g_{\mathrm{A}} \quad \overset{\mathrm{S}}{\longleftrightarrow} \quad \frac{1}{g_{\mathrm{B}}} \qquad \text{couplings}$$

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[Nekrasov, Ooguri, Vafa '04]

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[Ikeda,Tokunaga '07]

Doubled Poisson sigma-model for A/B-models

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- DFT-like description, but how to introduce GG and fluxes?

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$$\begin{aligned} \langle A+\alpha, B+\beta \rangle &= \iota_A\beta + \iota_B\alpha && \text{pairing} \\ \rho(A+\alpha) &= \iota_\alpha \Omega && \text{anchor} \\ [A+\alpha, B+\beta]_{\mathrm{D}} &= [\alpha,\beta]_\Omega + \mathcal{L}^\Omega_\alpha Y - \iota_\beta \mathrm{d}_\Omega X - \iota_\alpha \iota_\beta \mathcal{R} && \text{Dorfman br.} \end{aligned}$$

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- Higher formulation of a Poisson structure Ω with possible twists $\mathcal{R}$!

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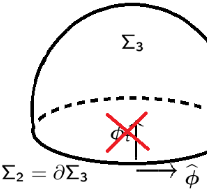

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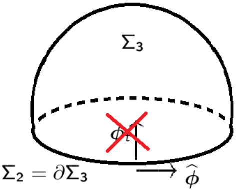

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Membrane level

- The action constructed by AKSZ

$$\mathcal{S}^{(3)}_{\Omega,\mathcal{R}} = \int_{T[1]\Sigma_3} \Big(\boldsymbol{F}_I \, \boldsymbol{D}\boldsymbol{X}^I - \chi_I \, \boldsymbol{D}\psi^I + \Omega^{IJ}(\boldsymbol{X}) \, \boldsymbol{F}_I \, \chi_J - \frac{1}{2} \partial_I \Omega^{JK}(\boldsymbol{X}) \, \psi^I \, \chi_J \, \chi_K + \frac{1}{3!} \mathcal{R}^{IJK}(\boldsymbol{X}) \, \chi_I \, \chi_J \, \chi_K \Big)$$

- How does it give the doubled Poisson sigma-model?

In the **partial gauge**

$$\boldsymbol{F}_I = \boldsymbol{D}\chi_I \quad \text{and} \quad \psi^I = -\boldsymbol{D}\boldsymbol{X}^I$$

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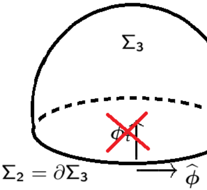

- It allows the definition of fluxes (DFT-like)

$$\mathcal{R}^{IJK} \longrightarrow H_{ijk}, \; F^i{}_{jk}, \; Q^{ij}{}_k, \; R^{ijk} \qquad [\Omega, \mathcal{R}]_{\rm S} = 0 \quad \rightsquigarrow \quad \text{Bianchi id.}$$

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- We impose the master eq. on Ω ($\rightsquigarrow$ SC?)

$$\Rightarrow \quad \mathbb{J}^I{}_J = \eta_{JK}\,\Omega^{IK} = \begin{pmatrix} J^i{}_j & \pi^{ij} \\ 0 & -J^j{}_i \end{pmatrix}$$ generalized complex structure

Generalized complex structure and A-/B-models

- The **integrability conditions** for $\mathbb{J}^I{}_J = \begin{pmatrix} J^i{}_j & \pi^{ij} \\ 0 & -J^j{}_i \end{pmatrix}$ are

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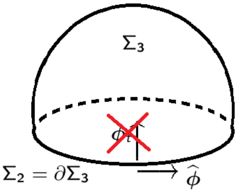
we need just **transverse** modes: $\widehat{X}^i, \widehat{\chi}_i$



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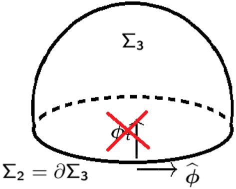

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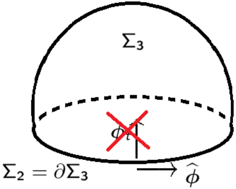
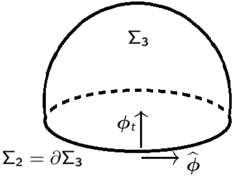


B-model ($\pi = 0$):

we need both **normal** and **transverse** modes: $\widehat{\boldsymbol{X}}^i, \widehat{\boldsymbol{q}}^i, (\boldsymbol{p}_t)_i, (\boldsymbol{F}_t)_i$



Topological S-duality from generalized complex structure

- The symplectic structure $\mathrm{d}q^i \wedge \mathrm{d}p_i$ has a diffeomorphism invariance (i.e. **canonical transformation**)

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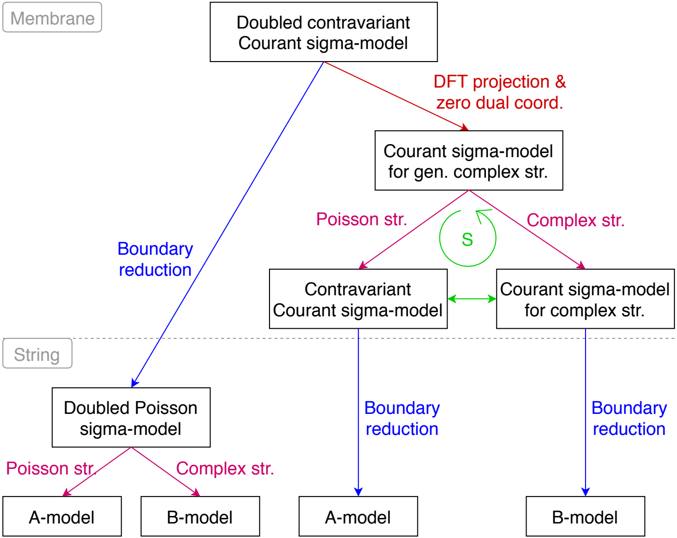
with $g_{\mathrm{A}} = \dfrac{1}{g_{\mathrm{B}}}$ $\rightsquigarrow$ Topological S-duality

Summary


Membrane
Doubled contravariant Courant sigma-model
DFT projection & zero dual coord.
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S
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Thank you for your attention!