Poisson-Lie T-duality, Courant algebroids, and their higher analogs

Pavol Ševera

Outline

1. What is Poisson-Lie T-duality?
2. Ricci flow and string effective action
3. Higher dualities

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What is Poisson-Lie T-duality?

[Klimčík, Š. 1995]

T-duality

Two different spacetimes $M_{1,2}$ can be equivalent from the string theory perspective

Requires an action of $U(1)$ (or of a torus) on M_1 by isometries

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- A non-Abelian generalization (symmetry is hidden, no Killing vector fields)
- M_1 and M_2 give isomorphic Hamiltonian systems (up to finitely many degrees of freedom)

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M_1 and M_2 (exact CAs) are shadows of the same "ideal world" (non-exact CA + a generalized metric)

Courant algebroids, or "generalized geometry"

[Liu, Weinstein, Xu 1997]

Courant algebroid: vector bundle $E \to M$, symmetric pairing $\langle\, , \rangle$ anchor map $\rho : E \to TM$, bracket $[\, ,] : \Gamma(E) \times \Gamma(E) \to \Gamma(E)$ such that $(\forall s, t, u \in \Gamma(E))$

$$[s, [t, u]] = [[s, t], u] + [t, [s, u]]$$
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Examples

- Lie algebras with invariant symmetric pairing ($M =$ point)
- **exact CAs** (classified by $H^3(M,\mathbb{R})$)

$$0 \to T^*M \xrightarrow{\rho^t} E \xrightarrow{\rho} TM \to 0$$

2d σ-models and generalized metrics

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2d σ-model

Ingredients: (M, g, H): g a Riemannian metric, $H \in \Omega^3(M)_{\text{closed}}$
Σ a surface with a Lorentzian metric

$$S(f) = \int_\Sigma g(\partial_+ f, \partial_- f) + \int_Y f^* H \qquad (f : \Sigma \to M,\ \partial Y = \Sigma)$$

CAs and Hamiltonian systems

- A CA $E \to M \rightsquigarrow$ an ∞-dim symplectic manifold $L_{CA}E$
- A generalized metric $V_+ \subset E \rightsquigarrow$ a function $\mathcal{H}_{V_+}$ on $L_{CA}E$

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A better explanation: a boundary field theory of an AKSZ model (see later)

Poisson-Lie T-duality

Backgrounds (M, g, H) of Poisson-Lie type

- a CA $\tilde{E} \to \tilde{M}$ (not exact), $\tilde{V}_+ \subset \tilde{E}$ a gen. metric
- a surjective submersion $f : M \to \tilde{M}$ and a compatible *exact* CA structure on $E := f^*\tilde{E} \to M$ **(not unique !)**
- pulled-back generalized metric: $V_+ := f^*\tilde{V}_+ \subset E$, gives rise to (g, H) on M

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PL T-duality

If (M_1, g_1, H_1) and (M_2, g_2, H_2) are obtained by pulling back the same gen. metric $\tilde{V}_+ \subset \tilde{E}$ then the corresponding 2-dim σ-models are (almost) isomorphic as Hamiltonian systems

…because they are (almost) isomorphic to $(L_{CA}\tilde{E}, \mathcal{H}_{\tilde{V}_+})$

How to construct CA pullbacks

No spectators (i.e. $\tilde{M} =$ point, $\tilde{E} = \mathfrak{d}$ a Lie algebra)

- $\mathfrak{g} \subset \mathfrak{d}$ a Lagrangian Lie subalgebra ($\mathfrak{g}^{\perp} = \mathfrak{g}$)
- $M = D/G$, $E = \mathfrak{d} \times M$, the anchor given by the action of $\mathfrak{d}$

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General $\tilde{M}$ (= spectators)

- A principal D-bundle $P \to \tilde{M}$
- Vanishing 1st Pontryagin class:
  $\langle F, F \rangle/2 = dC$ ($C \in \Omega^3(\tilde{M})$) gives a transitive CA $\tilde{E} \to \tilde{M}$
- $M = P/G$

A multiplicative gerbe over D trivial on G, acting on a gerbe on P

D a torus: the usual (Abelian) T-duality

Outline

"Quantum questions" - joint work with Fridrich Valach

[arXiv:1610.09004, arXiv:1810.07763]

σ-models:

is PL T-duality compatible with the renormalization group flow?

$$\frac{d}{dt} g = \mathrm{Ric}$$

– looking for suitable flow of generalized metrics in *general* CAs

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string theory:

other massless fields besides (g, H): dilaton, RR-fields, gauge fields. Do they make sense for arbitrary CAs? Is PL T-duality compatible with SUGRA equations?

Laplacian and the generalized string effective action

Laplacian

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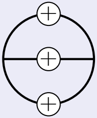

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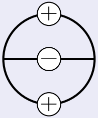

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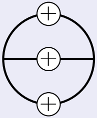
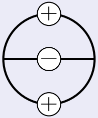
$$\Delta_{V_+} = 4\, L_{\rho(e_a)} L_{\rho(e_a)} + \frac{1}{6}$$  $$+ \frac{1}{2}$$ 

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exact CAs: $\Delta_{V_+} = 2\Delta_g - \frac{1}{2}R + \frac{1}{4}H^2$

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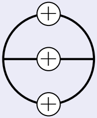
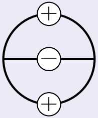
$$\Delta_{V_+} = 4\, L_{\rho(e_a)} L_{\rho(e_a)} + \frac{1}{6}$$  $$+ \frac{1}{2}$$ 

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Generalized string effective action $S(V_+, \sigma) = -\frac{1}{2}\int_M \sigma\, \Delta_{V_+}\, \sigma$

exact CAs: the (bosonic) string effective action ($\sigma = e^{-\phi}\mu_g^{1/2}$)

transitive CAs (with $\rho : V_+ \cong TM$): type I SUGRA action

Generalized Ricci flow

The **gradient flow** of $S(V_+, \sigma)$ in the space of generalized metrics in E (with a fixed σ):

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Generalized Ricci flow (of a generalized metric)

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$$\mathrm{GRic}_{V_+,\mathrm{div}}(u,v) := \mathrm{div}[v,u]_+ - v\cdot \mathrm{div}\, u - \mathrm{Tr}_{V_+}[[\,\cdot\,,v]_-,u]_+$$

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More generally $\mathrm{div} : \Gamma(E) \to C^\infty(M)$ such that $\mathrm{div}(fu) = f\,\mathrm{div}\, u + \rho(u)f$ [Alekseev,Xu 2001], [Garcia-Fernandes 2016].

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Other definitions of GRic: [Coimbra, Strickland-Constable, Waldram 2011], [Garcia-Fernandez 2014], [Jurčo, Vysoký 2016] (using auxiliary data)

PL T-duality is compatible with the renorm. group flow

- If E is exact, the GRicci flow is the renormalization group flow (Ricci flow) of (g, H)
- GRic is compatible with CA pullbacks (if we pull back div)
- Hence, Poisson-Lie T-duality is compatible with the renormalization group flow

PL T-duality and string background equations

without RR fields

Generalized string background equations

EOM of S: $\Delta_{V_+}\sigma = 0$, $\mathrm{GRic}_{V_+,\sigma} = 0$ (exact CAs: bosonic string background equations; some transitive CAs: type I/heterotic)

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Another approach: [Jurčo, Vysoký 2018]

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If no τ exists we get a solution of modified type II SUGRA of [Tseytlin, Wulff 2016], [Arutyunov, Frolov, Hoare, Roiban, Tseytlin 2016] (σ replaced by div)

Outline

Back to the worldsheet perspective and higher dualities

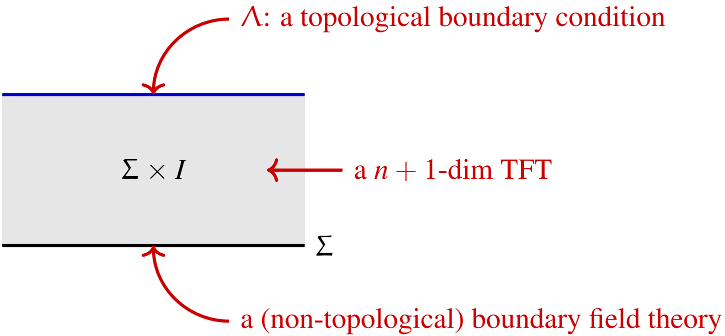
Joint work in progress with Ján Pulmann and Fridrich Valach

The problem

Abelian T-duality has an easy higher dimensional version: (higher) electric-magnetic duality. How to extend it to a non-abelian (Poisson-Lie) generalization?

Duality from boundary field theories

A "sandwich field theory" on a n-dim Σ



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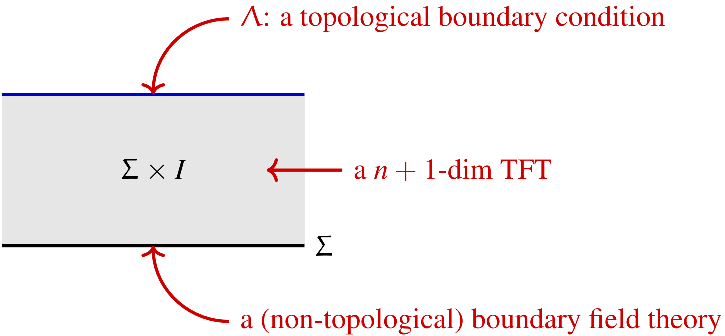

Duality of sandwiches

Different choices of Λ give "dual" field theories on Σ.
If Λ_1 and Λ_2 are classically different but quantum-mechanically equal, we get a true duality (equivalence of theories).

Abelian Chern-Simons $\rightsquigarrow$ (Abelian) T-duality

AKSZ sandwich (classical BV picture)

or ideal worlds and their shadows revisited

Ideal world (TFT + a boundary field theory)

- A dg symplectic manifold X, $\deg \omega_X = n$ (e.g. a CA ($n = 2$))
- An n-dimensional Σ
- A dg Lagrangian submanifold $\mathcal{L} \subset \mathrm{Maps}(T[1]\Sigma, X)$
  (e.g. a generalized metric)

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Different choices of Λ's $\rightsquigarrow$ mutually dual models

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A BV description of the σ-model with the target D/G given by $V_+ \subset \mathfrak{d}$

Σ a surface with a (pseudo)Riemannian metric

$X = \mathfrak{d}[1] \rightsquigarrow$ Chern-Simons $S(A) = \int_Y \left(\frac{1}{2}\langle A, dA\rangle + \frac{1}{6}\langle [A,A], A\rangle \right)$

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Example: PL T-duality without spectators

A BV description of the σ-model with the target D/G given by $V_+ \subset \mathfrak{d}$

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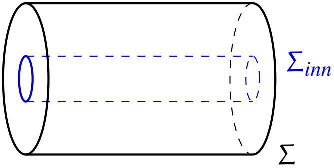

$A|_\Sigma \in \mathcal{L}$

$A|_{\Sigma_{inn}} \in \Omega(\Sigma_{inn}, \mathfrak{g})[1]$

$$\mathcal{L} = \left\{ A \in \Omega^1(\Sigma, \mathfrak{d}) \mid *A = RA \right\} \oplus \Omega^2(\Sigma, \mathfrak{d}) \subset \Omega(\Sigma, \mathfrak{d})$$

($R : \mathfrak{d} \to \mathfrak{d}$ the reflection wrt. V_+) $\quad \Lambda = \mathfrak{g}[1] \subset \mathfrak{d}[1] = X$

How to do calculations

- Resolve $\Lambda \hookrightarrow X$ to a (quasi-isomorphic) submersion $\Lambda' \to X$
- The sandwich is equivalent to the (much smaller) BV manifold $\mathrm{Maps}(T[1]\Sigma, \Lambda') \times_{\mathrm{Maps}(T[1]\Sigma, X)} \mathcal{L}$

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THANKS FOR YOUR ATTENTION!