Introduction to General Relativity – Preliminaries

PHYS 5770

paul.romatschke@colorado.edu

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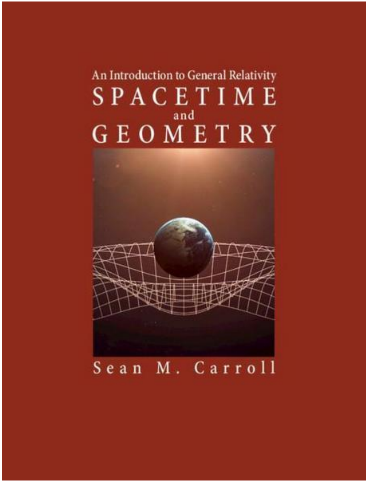
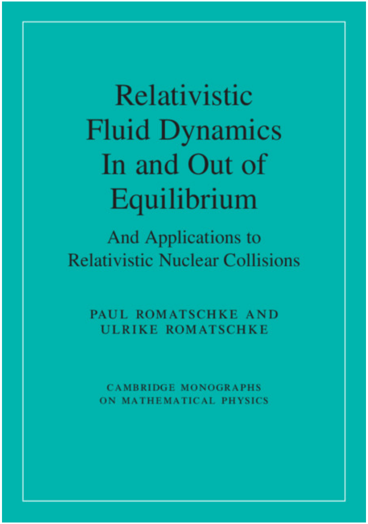
Textbook



- The course will loosely be based on Carroll's textbook
- I may deviate quite a bit

Optional Supplemental Textbook



- Relativistic Fluid Dynamics is an essential Ingredient to GR calculations
- I will use my fluid dynamics book to cover this part of the course
- This book is an optional supplement, not required for completing the course
- You can get full-text access to the book by clicking on this link

Units

- I will be using Natural Units
- This implies

$$c = \hbar = k_B = 1\,. \tag{1.1}$$

- Natural units imply that everything is measured in terms of energy (mass)
- The only unit appearing in GR will be Newton's constant G

Notation: Vectors

- Ordinary vectors are referred to as 3-vectors
- I may denote 3-vectors by a little arrow or bold-face font, e.g.

$$\vec{v}\,, \quad \mathbf{v} \tag{1.2}$$

- Frequently, I will also use index notation with a Latin index

$$v^i = \begin{pmatrix} v^1 \\ v^2 \\ v^3 \end{pmatrix} \tag{1.3}$$

- Products between vectors will be denoted alternatively as

$$\vec{v} \cdot \vec{u} = \mathbf{v} \cdot \mathbf{u} = v^i u^i\,, \tag{1.4}$$

where Einstein sum-convention is implied

Notation: 4-Vectors

- Spacetime vectors are referred to as 4-vectors
- 4-vectors will be denoted by a Greek index, e.g.

$$v^\mu = \begin{pmatrix} v^0 \\ v^1 \\ v^2 \\ v^3 \end{pmatrix} = \begin{pmatrix} v^0 \\ v^i \end{pmatrix} . \tag{1.5}$$

- Products between 4-vectors will be denoted using Einstein sum convention as

$$A_\mu A^\mu = A^\mu g_{\mu\nu} A^\nu = A_0 A^0 + A_1 A^1 + A_2 A^2 + A_3 A^3 \tag{1.6}$$

Notation: Derivatives

- Derivatives may be denoted as

$$\partial_x \equiv \frac{\partial}{\partial x}\,, \tag{1.7}$$

  and similar for other derivatives

- Gradients are similar to vectors that we can denote them as

$$\vec{\partial} = \partial_i = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix}\,. \tag{1.8}$$

- We will also use 4-gradients ∂_μ
- We can build things like the Laplacian from "contractions"

$$\nabla^2 = \partial_i \partial_i \tag{1.9}$$

Metric Tensor

- Metric Tensor denoted by $g_{\mu\nu}$
- Sign Convention: mostly positive

$$g_{\mu\nu} \equiv \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1.10}$$

- Note: occasionally I might change number of dimensions. I will clearly label these exceptions!!!

Integrals and Sums

- Integrals without any limits are taken to be improper integrals:

$$\int dx = \int_{-\infty}^{\infty} dx \tag{1.11}$$

- Sums are treated similarly:

$$\sum_n a_n = \sum_{n=-\infty}^{\infty} a_n \, . \tag{1.12}$$