Relativistic Time Delay

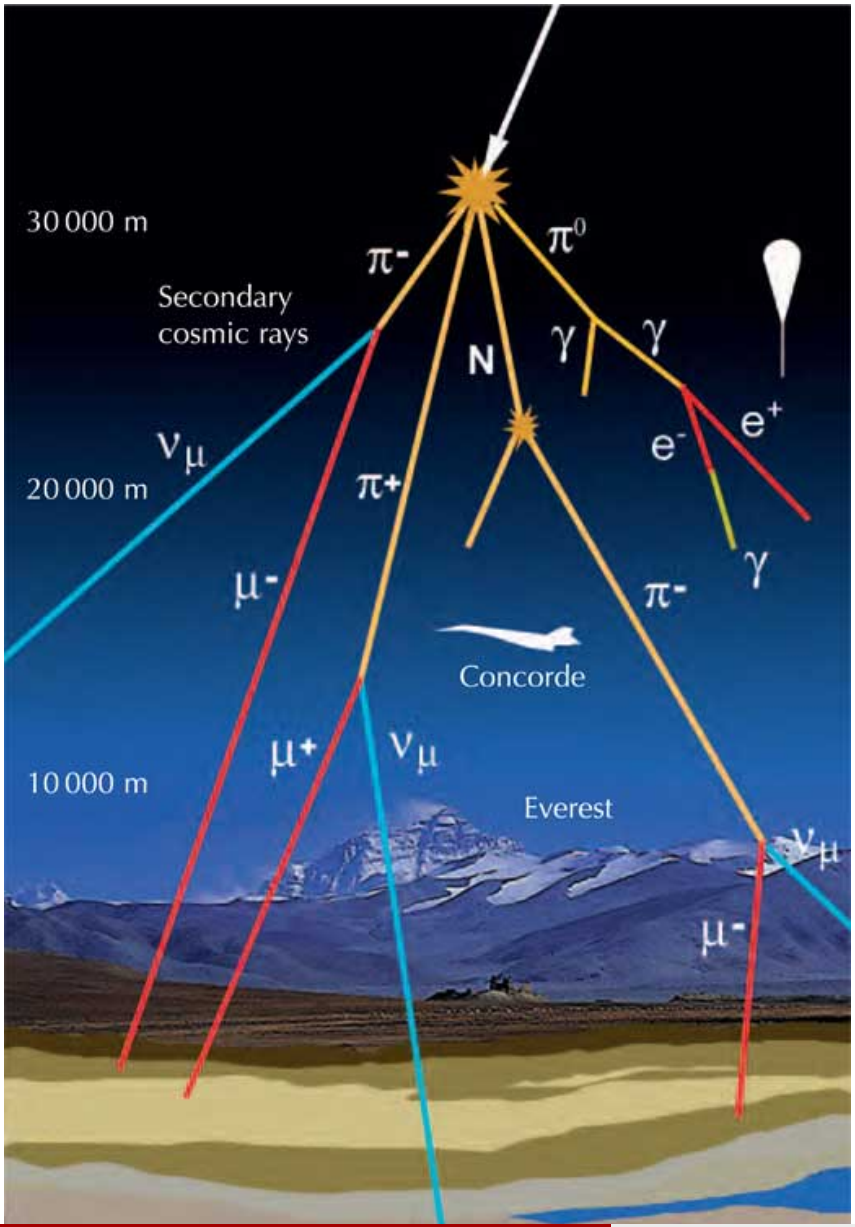
paul.romatschke@colorado.edu

Spring 2021

Relativistic Time Delay

- Last lectures we started a review of special relativity (SR)
- In this lecture, we consider a specific application of SR: relativistic time delay

Muon's in Earth's atmosphere



- Muons created at altitude $H \geq 10000\text{m}$
- Muon lifetime: $2.2 \times 10^{-6}\text{s}$
- Distance traveled at speed of light

$$L = 2.2 \times 10^{-6} \times c \simeq 660m$$

$$L \ll H$$

- How is it possible we can observe them at ground level???

SR to the rescue

Two coordinate systems:

- Observatory at rest wrt ground level: x^μ
- Muons x'^μ moving at velocity $v = \frac{dz}{dt}$ wrt ground level
- Relation between coordinate systems: $ds^2 = ds'^2$ or

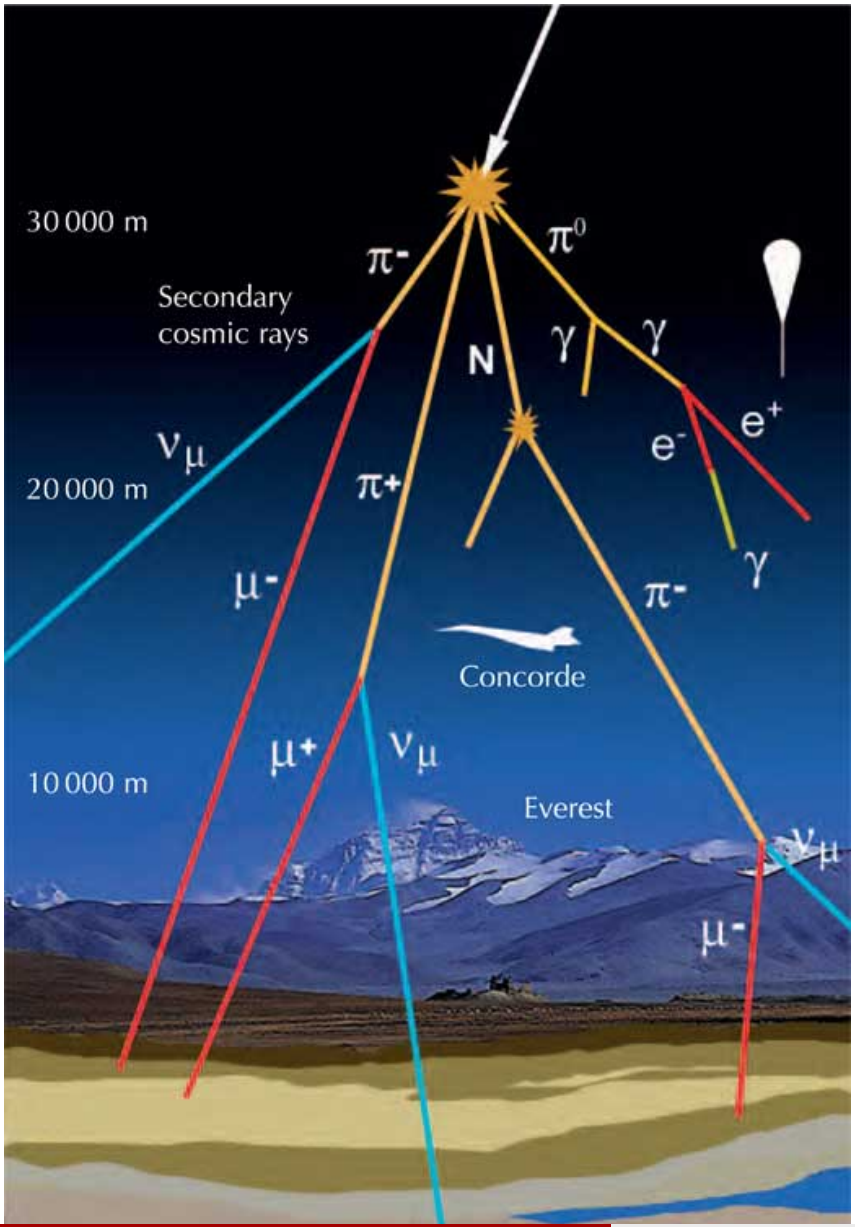
$$-dt^2 + dz^2 = -dt'^2 + dz'^2 \tag{5.1}$$

- Muons are at rest in their own coordinate system: $dz' = 0$

SR to the rescue

- We get the relation
$$-dt^2 + dz^2 = -dt'^2 \tag{5.2}$$
- Now "pull out" dt^2 to find
$$dt^2 \left(1 - \frac{dz^2}{dt^2}\right) = dt'^2 \tag{5.3}$$
- Recognize $\frac{dz}{dt} = v$ to find
$$dt = \gamma dt' \,, \quad \gamma = \frac{1}{\sqrt{1-v^2}} \,. \tag{5.4}$$
- Time interval on muon's clock dt' is observed as $\gamma dt'$ on observatory clock

SR to the rescue



- Time interval on muon's clock dt' is observed as $\gamma dt'$ on observatory clock
- For $v \to 1$, γ becomes very large
- How large γ do we need?

$$\gamma = \frac{dt}{dt'}\,, \quad dt' = 2.2 \times 10^{-6}$$

- $dt = 10000 m/c \simeq 3 \times 10^{-5} s$
- Therefore

$$\gamma \geq 15\,, \quad v \geq 0.9978c$$

Relativistic Time Delay

- We found that one time unit on the muon's clock is many more time units on the observatory clock
- The moving clock runs "slow"
- SR effect: time elapse slower for moving objects