Manifolds

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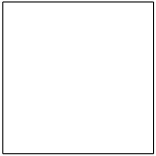
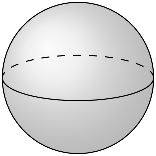
What are manifolds?

There probably is a formal definition, but I don't know it
So instead, I'll give you some examples of manifolds

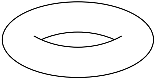
Example Euclidean space $\mathbb{R}^N$

- Consider Euclidean space in N-dimensions: $\mathbb{R}^N$
- Zero dimensions $\mathbb{R}^0$: $\cdot$
- One dimensions $\mathbb{R}^1$: ______________
- Two dimensions $\mathbb{R}^2$: 
- ...
- Coordinates on $\mathbb{R}^N$: $x_i = (x_1, x_2, \ldots, x_N)$
- Metric on $\mathbb{R}^N$: δ_{ij}

Example N-sphere S^N

- Next consider the sphere in N-dimensions S^N
- Zero dimensions S^0: $\cdot$ The same as $\mathbb{R}^0$!
- One dimension S^1: 
- Two dimensions S^2:  Austrian version: 
- S^2: coordinates $x_i = \begin{pmatrix} \phi \\ \theta \end{pmatrix}$; Metric $g_{ij} = \begin{pmatrix} R^2 & 0 \\ 0 & R^2 \sin\theta \end{pmatrix}$
- All S^N: genus 0

Example N-torus T^N

- Next consider the torus in N-dimensions T^N
- Two dimensions T^2: 

Example N-torus T^N

- Next consider the torus in N-dimensions T^N
- Two dimensions T^2 (American version):

- Torus is always genus 1
- Can view torus T^N as compactification of $\mathbb{R}^N$
- For instance T^2 as compactification of $\mathbb{R}^2$; can get toroidal coordinates from coordinate transformation of $\mathbb{R}^2$

Example: double-torus

- Next consider the double-torus
- It looks like this:

- Double torus is genus 2

Example: triple-torus

- Next consider the triple-torus
- German version:

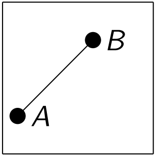
- Triple torus is genus 3

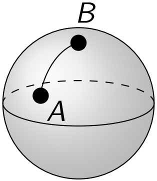
Example: Klein bottle

- More complicated 2-dimensional manifolds: Klein Bottle
- Looks like this:

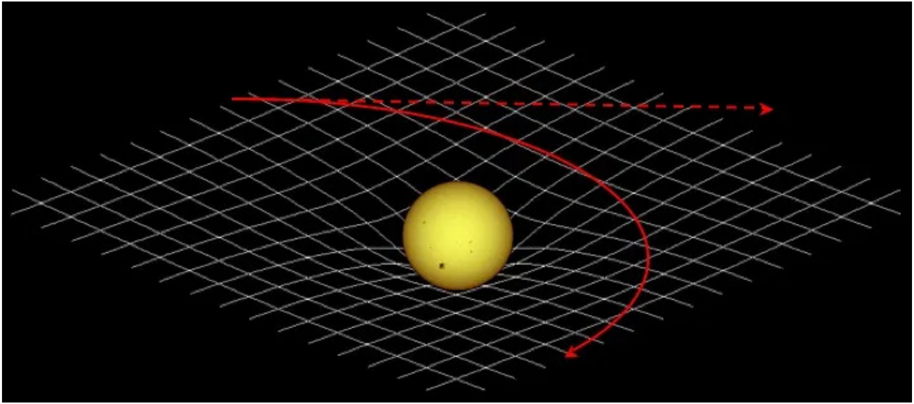
- Genus 2
- Non-orientable

Geodesics

- Geodesic is the path that takes minimal time from A to B on a manifold
- Example $\mathbb{R}^2$: 

- Example S^2: 

Our universe as a manifold



Manifolds

We've discussed several 2d manifolds, such as

- S^2:  T^2:  Triple torus: 
- Is there an edible version of the double-torus?